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A

COMPLETE

SYSTEM

OF

PRACTICAL ARITHMETIC,

WITH

VARIOUS BRANCHES

IN THE

MATHEMATICS.

THE SECOND EDITION.

By WILLIAM TAYLOR,

TEACHER OF THE MATHEMATICS, and LAND SURVEYOR,

AUTHOR OF

A COLLECTION OF TABLES FOR THE USE OF HIS MAJESTY'S OFFICERS OF EXCISE; THE ARITHMETICIAN'S GUIDE; KEY TO THE ARITHMETICIAN'S GUIDE; THE MEASURER'S ASSISTANT; THE TRADESMAN'S CORRECT READY RECKONER; A TREATISE OF CHRONOLOGY; CORRECT TABLES OF DISCOUNT, &c. &c.

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PREFACE.

SCIENCE may be compared to a highly finished pile of building, all the parts of which being disposed in the most exact symmetry, they must affect our preception, and gratify our internal sensation with a more exquisite pleasure, than if viewed in a separate state: For, in such a state, to all but the learned, they would appear broken and unconnected materials of a mighty structure, which the mind, wanting power to conceive, could enjoy no satisfaction in the contemplation of such a train of imperfect and confused ideas. But, when thus exhibited in their true proportion, it will be easy, even for the youngest scholar, to gain a perfect notion of each; and, as he advances, a gradual comprehension of the beauty resulting from their connexion, and how they mutually assist and ornament each other.

When we consider the utility of *Arithmetic*, on which science almost all others do absolutely depend, we need not be surprised that so many efforts have been made to bring it to the utmost degree of perfection, since the real value of its use, certainly merits all the study and pains that can be bestowed upon it.

It must be owned, that the progress of mathematical sciences is but slow, owing to the difficulty of the several branches that come under consideration; but then, it is sure and certain: The acquision here gained is real knowledge. For this reason, it is the work of ages to bring even a single branch to perfection; therefore, it is no wonder if the ancients have, in many cases, made use of round-about methods to encompass their ends, and given us long and tedious demonstrations, laying down many propositions, either of no use, or too simple and trifling to be taken notice of; whence most of their

inventions may be demonſtrated ſhorter, propounded eaſier, diſpoſed in a better method, and taught in a more compendious way.

There are two things abſolutely neceſſary to make the acquiſition of any ſcience as eaſy as its nature will admit. Firſt, the diſpoſition of the work, ſo that the rules may be clear and diſtinct; ſecondly, the illuſtration of theſe rules, by a ſufficient number of proper and uſeful examples; and, as the great difficulty in this ſcience is acquiring the knowledge of ſtating and ſolving queſtions, I have given a great variety of theſe in all the different parts of this Treatiſe, in the moſt particular, diſtinct, and plain manner I poſſibly could, with their anſwers at full length, and explicit directions, where the leaſt difficulty ſeemed to occur.

The ſeveral rules follow in the ſame order, as ſpecified in the table of contents: thus, Part I. Book I. contains the four primary rules; i. e. Addition, Subtraction, Multiplication, and Diviſion, in integers, and Reduction aſcending and deſcending, with the tables of money, weights, meaſures, &c. which the learner ſhould be well acquainted with before he proceeds to the uſe of thoſe rules in compound numbers.

In Book II. the rules follow in the ſame order in which they are generally taught in ſchools; but they are all placed in ſuch a manner as to have little or no dependence on each other; therefore, they may be taught in what order every maſter chuſes.

In the ſecond and third parts, which treat of vulgar and decimal fractions, the rules and examples are laid down in ſo plain and intelligent a manner, as to be underſtood by the meaneſt capacities.—The fourth part treats of Geometry, Menſuration, Gauging, Land Surveying, and the Specific Gravity of Metals, &c. in which

which I have given every thing that is ufeful, taking all the care I poffibly could to make them plain and eafy to be underftood: and that the learner might not be at a lofs in the firft rudiments of Geometry, &c. I have given him the draught of every operation on a large Copper-plate, in order that he may the more eafily comprehend the Problems, having every where purpofely omitted the fpeculative part, or things that are ufelefs to beginners, and would prove ftumbling blocks, rather than any way to improve the mind.

As to thofe parts which treat of Chronology, Aftronomy, Geography, and Algebra, I have taken all the care poffible (within the compafs of fuch a limitation) to make them plain, and eafily underftood by young beginners.

And in order to make this Book as ufeful as poffible, I have added, firft, a courfe of Book-keeping, by fingle entry, with a difcription of the books, and directions for ufing them.--Secondly, Book-keeping, by double entry, according to the Italian method; with various Forms of Acquittances, Bills of Exchange, &c. &c.

Thefe are the fubjects of the enfuing work; which, if ferioufly purfued by a thoughtful mind, the reader may attain to a competent knowledge in thefe ufeful arts.

Perhaps it may be faid, there are books of this kind already, and therefore you are only doing the fame thing over again.—That there are books publifhed with the fame defign, is acknowledged, but that I have trod in the fame fteps with their authors, I muft beg leave to deny; for the chief reafon that induced me to write this Treatife was, becaufe very few had given the operations worked at full length: this was an article I have heard a great many complain of, even teachers themfelves.

As to the work itſelf, it is laid down upon the beſt foundation I could procure from the moſt celebrated authors; and the rules are built upon the beſt principles now taught and practiſed by the moſt eminent maſters of our private and public academies in this kingdom, every difficulty being explained in the moſt conciſe method, and the whole performance made perfectly eaſy to be underſtood; ſo that, by the help of this Treatiſe, any young man, of a tolerable capacity, may in a ſhort time make himſelf maſter of the moſt difficult parts here laid down.

The inſtruction of youth in ſchools and academies is certainly the moſt expeditious method of forming the minds of young perſons, and of bringing them acquainted with that kind of learning, which their intended ſtation and degree of life ſeems to require; thoſe, therefore, that are bleſt with affluent fortunes, and are under the care of prudent parents and guardians, will ſtand in no need of the aſſiſtance of this Treatiſe, unleſs it be to refreſh their memories with what they have formerly been taught, or to look into ſuch ſubjects as are quite foreign to the inſtitution of thoſe ſeminaries of learning; but there are a great many adult perſons, and grown up youth, who through the narrowneſs of their circumſtances, or the neglect of their friends, are forced to endeavour to improve their loſt time as well as they can.

To ſuch as theſe the following Treatiſe will be of great ſervice; for the variety of the ſubjects here treated of, muſt needs gain the attention of all who have the leaſt inclination to ſtudy arts and ſciences.

Perhaps ſome of our moſt eminent teachers may ſay, by inſerting the operations at length, I have encouraged dull and lazy boys, by this means, to copy out their anſwers, in order to deceive their teachers; but ſuch kind

kind of piracy may ſoon be detected, by varying the work of the queſtions according to the nature of the ſeveral rules.

But my ſole motive for undertaking this work, was purely for the inſtruction and benefit of the unlearned; ſo I hope every impartial reader will judge with candour of the merits of this performance; and if it meets with their approbation, I ſhall not think I have ſpent ſo much labour in vain, but rejoice at having done any thing for the ſervice and good of my country.

I hope my readers will excuſe all defects, and correct what errors they may occaſionally find herein; which will be eſteemed a particular favour:—

I have nothing further to add, but a return of my ſincere thanks to all thoſe Gentlemen, whoſe kind approbation and encouragement have now eſtabliſhed the uſe of this Book, and favoured me with their judicious remarks and ſignatures.

I am,

with the utmoſt eſteem,

Their's and the Public's

moſt obliged, obedient, humble Servant,

W. TAYLOR.

BIRMINGHAM,
Sept. 29, 1800.

No. 15, Spiceal-Street.

THE

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PART II.

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CHARACTERS USED IN THIS SYSTEM.

Mathematical, Algebraical, and Geometrical Signs.

∵ Ergo; therefore.

+ Plus, or more; the ſign of addition; as, $3+4$, is 3 added to 4.

— Minus, or leſs; the ſign of ſubtraction; as, $4-2$, is 2 from 4.

∽ The ſign expreſſing the difference between two quantities, when it is not known which is the greater of the two.

× The ſign of multiplication; as, 3×4, is 3 multiplied by 4.

. Is likewiſe a note of multiplication, and ſometimes uſed for ×.

÷ The ſign of diviſion; as, $9\div 3$, is 9 divided by 3. This is ſometimes written like a fraction; thus, $\frac{9}{3}$.

= The ſign of equality; as $a=8$, is a equal to 8.

: :: : The ſign of arithmetical proportion; as $3 : 6 :: 4 : 8$; is, 3 to 6 ſo is 4 to 8.

∺ The ſign of geometrical proportion.

⊏ or 7 The ſign of majority; as $a\sqsubset b$, ſignifies a is greater than b.

⊐ or ∠ The ſign of minority; as $a\sqsupset b$, ſignifies a is leſs than b.

√ The ſquare root; as, $\sqrt{16}$, is the ſquare root of 16.

$\sqrt[3]{\ }$ The cube root; as, $\sqrt[3]{8}$, is the cube root of 8. This character ſometimes affects ſeveral quantities, diſtinguiſhed by a line drawn over them; thus, $\sqrt{\overline{b+d}}$, denotes the ſquare root of the ſum of b and d.

$\overline{1+n}|^2$ Signifies that the number denoted by n is to be added to 1, and the ſum ſquared. $\overline{1+n}|^3$; that $1+n$ is to be raiſed to the third power, or cubed, &c.

∥ Parallel	⊥ Perpendicular	$\stackrel{v}{=}$ Equiangular, or Similar
△ Triangle	□ Square	$\stackrel{1}{=}$ Equilateral
∠ Angle	▭ Rectangle	
∟ Right Angle	○ Circle	

Zodiacal Signs.

♈ Aries	♋ Cancer	♎ Libra	♑ Capricornus
♉ Taurus	♌ Leo	♏ Scorpio	♒ Aquarius
♊ Gemini	♍ Virgo	♐ Sagitarius	♓ Piſces

Planetary Signs.

♄ Saturn	♂ Mars	☿ Mercury	☉ Sun
♃ Jupiter	♀ Venus	⊕ Earth	☽ Moon

Aspects.

🌑 New Moon	△ Trigonus	☍ Oppoſition
☽ Firſt Quarter	□ Quadril	☊ Dragon's Head
🌕 Full Moon	✶ Sextile	☋ Dragon's Tail
☾ Laſt Quarter	☌ Conjunction	

A SYSTEM OF Practical Arithmetic.

PART I.

BOOK I.

DEFINITIONS.

ARITHMETIC is the art of computing by numbers; it is called Vulgar or Common Arithmetic, when it treats of whole numbers.

Unit is any thing considered as one, or 1, and is the beginning of number.

Number is a multitude of units; by this every thing is reckoned.

A whole Number is a precise number, without any parts annexed.

A mixed Number is a whole number, with some part annexed.

A Fraction is a part or parts of a unit.

A proper Fraction is less than a unit.

An improper Fraction is greater than a unit.

An aliquot Part is that which is contained a precise number of times in another.

An aliquant Part is such as is contained in another, some number of times, with some part or parts over.

A prime Number is that which can only be measured by a unit.

Numbers are said to be prime to one another, when only a unit measures both. These are also called co-primes.

A perfect Number is that which is equal to the sum of all its aliquot parts.

Integers, or whole Numbers, are such as express a number or multitude of things, whereof each is considered as a unit. Thus 4 tons 6 yards, 20 miles, &c. each of which is called an integer, or whole number.

Compound Numbers are such as consist of different denominations, as tons, hundreds, quarters, pounds, ounces, &c. Thus 12 T. 3 C. 1 qr. 12lb. &c.

Arithmetic, with regard to art and science, consists both in theory and practice.

Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations.

The practice is that which shews the method of working by numbers, and consequently becomes the most useful and expeditious for business, and has five principal or fundamental rules for the operation, viz.

1. Numeration, or Notation; 2. Addition; 3. Subtraction; 4 Multiplication; and 5. Division.

S E C T. I.

N U M E R A T I O N.

TEACHETH to read or express the true value of any number when written down, and consequently, to write down any proposed number according to its true value; and this consisteth of two parts.

1. The true order of placing down figures.

2. The true valuing of each figure in its place; both of which are plainly exhibited in the following

T A B L E

Thousands of Millions.	Hundreds of millions.	Tens of millions.	Millions.	C. of thousands.	X. of thousands.	Thousands.	Hundreds.	Tens.	Units.	
									9	Nine.
								9	8	Eight.
							9	8	7	Seven.
						9	8	7	6	Six.
					9	8	7	6	5	Five.
				9	8	7	6	5	4	Four.
			9	8	7	6	5	4	3	Three.
		9	8	7	6	5	4	3	2	Two.
	9	8	7	6	5	4	3	2	1	One.
9	8	7	6	5	4	3	2	1	0	Cypher.

In this table each figure from the place of units, increaseth in a tenfold proportion; as 9 in the first place is nine; 9 in the second place is ninety; and so of the rest.

These ten places are as far as any common business will require; but when large numbers are expressed by figures, for the more easy reading of them, divide, or rather distinguish your number into periods, each period to contain six characters or figures, called grand periods; then the first period to the right hand will be units, the second millions, third billions, fourth trillions, fifth quadrillions, sixth quintillions, seventh sextillions, eighth septillions, ninth octillions; tenth nonillions, eleventh deci-millions, &c. As for instance, suppose I would number this train of figures, viz.

Septill.

Septill.	Sextill.	Quint.	Quadr.	Trill.	Billions.	Millions.	Units.
8	7	6	5	4	3	2	1
123456,	124121,	681213,	415682,	181643,	214168,	218164,	823812.

EXAMPLES.

Write down 34167 in words at length? Anſwer, Thirty-four thouſand, one hundred and fixty-ſeven.

Write down 790684218 in words at length? Anſwer, Seven hundred ninety millions, ſix hundred eighty-four thouſand, two hundred and eighteen.

Expreſs in figures, ſix thouſand and fifty-five? Anſwer 6055.

Expreſs in figures, nine hundred eighty-ſeven millions, ſix hundred and fifty thouſand? Anſwer 987650000.

Expreſs in figures, ſeventeen millions, ſeventeen thouſand, ſeventeen hundred and ſeventeen?

$$\begin{array}{r} 17000000 \\ 17000 \\ 1700 \\ 17 \\ \hline 17018717 \end{array}$$

17018717 Anſwer.

Expreſs in figures, forty-five billions, four hundred, forty-five thouſand and four millions, ſixty thouſand, ſix hundred and fifteen? Anſwer 45445004060615.

NOTATION, *by Roman Numerical Letters.*

One, five, ten, fifty, hundred, five hundred, thouſand.
I. V. X. L. C. D. M.

When a leſs numerical letter ſtands before a greater, it muſt be taken from it, as I before V or X, and X before L or C, &c. thus:

Four, nine, forty, ninety, &c.
IV. IX. XL. XC.

When a leſs numerical letter ſtands after a greater, it is to be added to it, thus:

Six, eleven, ſixty, one hundred and ten.
VI. XI. LX. CX.

A line drawn over any number leſs than a thouſand, ſignifies ſo many thouſands, as $\overline{\text{XL}}$. is forty thouſand; $\overline{\text{C}}$. is one hundred thouſand; $\overline{\text{M}}$. is one million, &c.

Write down in common figures the following numbers, expreſſed in numerical letters, viz.

XI. C. DC. DXL. MC. MDCCLXXXI. $\overline{\text{MC}}$.
Anſ. 11, 100, 600, 540, 1100 1781, 1100000.

Write down in numerical letters the following number, expreſſed in common figures, viz.

19, 50, 95, 101, 1000, 1500, 60000.
Anſwer, XIX. L. XCV. CI. M MD. $\overline{\text{VI}}$.

Write down in numerical letters the following numbers expreſſed in common figures, viz. 2060000. Anſw. $\overline{\text{MM}}$LX.

II. ADDITION.

TEACHETH to add sundry numbers together, into one sum, called the total.

RULE. 1. Place all the numbers of a like name so, that units may stand under units, tens under tens, hundreds under hundreds, &c.

2. Begin at the units place, and reckon up all the figures in that place from the bottom to the top, and what overplus there is above even tens, set down, carrying one for every ten to the next row, and so on, continuing to the last row, at which set down the total amount.

PROOF. Begin at the top of the sum, and reckon the figures downwards, in the same manner as you added them upwards; and if the sum comes the same as before, it is undoubtedly supposed to be right.

EXAMPLES.

Let these numbers be added together:

9482
590
307
85
10464

To add up this sum, begin at 5, say the sum of 5 and 7 is 12, and 2 is 14; set down 4 and carry 1. The sum of 1 and 8 is 9, and 9 is 18, and 8 is 26; set down 6 and carry 2. Then 2 and 3 is 5, and 5 is 10, and 4 is 14; set down 4 and carry 1. Lastly, 1 and 9 is 10, which being the last, set it down.

More examples for practice.

126893	12486	368121	21304
81621	2101	16012	2186
2118	652	5168	21
312	18	21	318
604	25	68102	21
24	3	245	5
211572	15285	457669	23855

234563	254312	689876	878761
41234	32168	2101	2180
312	1214	86	815
2	21	42	43
4286	24798	31234	124
302	314	1312	4210
56	45	421	432
280755	312872	725072	886565

A Gentleman

Questions to exercise Addition of Whole Numbers.

Quest. 1. A Gentleman had in his nursery one hundred million of oak, one hundred thousand ash, and one hundred fruit trees, and also sixty-nine elm trees; I demand how many trees were growing in the nusery?

Oak	100000000
Ash	100000
Fruit	100
Elm	69
Answer	100100169 Trees in all.

Quest. 2. A person was born in the year 1800, when will he (if he lives) be 65 years of age?

This is no more then to -	1800
Add - - - - - - -	65
And we have the year required	1865

Quest. 3.

Suppose a man was born in the year of our Lord 1800, in what year will he be 60 years of age?

To -	1800
Add -	60
Answer	1860

Quest. 4.

I was born in the year 1751, when shall I be 49 years of age?

To -	1751
Add -	49
Answer	1800 the year req.

Quest. 5. A draper has 7 pieces of cloth as under.

	Yards
No. 1 containing	87
2 ———	45
3 ———	14
4 ———	21
5 ———	36
6 ———	47
7 ———	20
Ans.	270 Yards.

How many yards are there in all?

Quest. 6. How is *eleven thousand, eleven hundred,* and *eleven* set down in 5 figures? 12111. Answer.

Proof.

11000
1100
11
12111

Quest. 7. In the second book of Kings Chap. 19th we read that an angel destroyed in the camp of the Assyrians, *an hundred, and fourscore, and five thousand:* How would you set this down? 185000, Answer.

PARADOXICAL QUESTIONS.

Queſt. 1. *If from ſix ye take nine and from nine ye take ten,*
(Ye wits, now the myſt'ry explain)
And if fifty from forty be taken; there then
Shall be juſt half-a-dozen remain.

SOLUTION.

If from SIX *you take nine* = IX, *there then remains* S
And if from IX *you take ten* = X, *there remains* I
And if from XL *you take fifty* = L, *there remains* X
= SIX.

Queſt. 2. *Four things I ſaw, but what they were,*
I beg, dear ladies, you'll declare;
And though they were but four exact,
Thirteen they were full as compact;
I cut off half, and then could find,
Exactly eight were left behind:
What ſeems more ſtrange, tho' very ſure,
Theſe eight remaining, were but four.

SOLUTION. *Divide thirteen by line that's ſtrait,*
The half is evidently eight.
~~XIII~~

Queſt. 3. *Three perſons at play, in a tavern were ſeated,*
Where none other play'd, nor any one betted;
Yet fortune prov'd kind, for each gain'd a guinea,
Who tells me this paradox, I hold him no ninney.

Anſwer Three Muſicians

Find how many Years it was from the Creation of Adam to the univerſal Deluge in the Days of Noah, called Noah's Flood; by the 5th Chapter, and 6th Verſe of the 7th Chapter of Geneſis.

When	Seth was born	Adam was	130	Years old.
	Enos	Seth	105	
	Cainan	Enos	90	
	Mahalaleel	Cainan	70	
	Jared	Mahalaleel	65	
	Enoch	Jared	162	
	Mathuſelah	Enoch	65	
	Lamech	Mathuſelah	187	
	Noah	Lamech	182	
And when the Flood happened Noah was			600	
		Anſwer	1656	Years.

III. SUBTRACTION.

TEACHETH to take a leſſer number from a greater, to find their difference or remainder.

RULE. 1. Place the greater number uppermoſt, and the other under it, ſo that units may be under units, tens under tens, &c. and draw a line under them.

2. Begin at the right hand or place of units, and ſubtract the lower figure from the upper, and ſet down the difference underneath them; do the ſame with the reſt of the figures.

3. When the under figure exceeds that which ſtands over it, you muſt borrow ten, and add it to the upper number, from which ſubtract the lower, and ſet down the remainder; carry one to be added to the next lower figure, and ſubtract the ſum from the upper, ſetting down the remainder; and ſo on from one row to another.

PROOF. The way to prove ſubtraction is no more than, to the leſſer number add the remainder; if the ſum be like the greater, the work is right.

EXAMPLE 1.		E. 2.	E. 3.	E. 4.		E. 5.	
From	94165	368419	86459	Bought	9876	From	869426017
Take	35641	126124	21821	Sold	1213	Take	214981764
Rem.	58524	242295	64638	unſold	8663	Rem.	654444253
Proof	94165	368419	86459	Proof	9876	Proof	869426017

To work example firſt, ſay, 1 from 5 and there remains 4, write down four in the place of units, and ſay 4 from 6 and there remains 2, which write down in the place of tens; then ſay 6 from 1 I cannot, but 10 that I borrow added to 1 is 11, 6 from 11 and there remains 5; then 1 that I borrowed and 5 is 6, 6 from 4 I cannot, but 6 from 14, and there remains 8; 1 that I borrowed and 3 is 4, 4 from 9 and there remains 5, which ſet down, and the work is done. But as theſe things are ſo eaſy, I think any farther explanation of the reſt would be looked upon as prolixity only.

Queſtions to exerciſe Subtraction of Whole Numbers.

Queſt. 1.

The age of a lady is fifty and three,
What year was ſhe born in, pray tell unto me?

From	1800
Take	53
Anſwer	1747

Queſt. 2.

Suppoſe a perſon was born in the year of our Lord ſeventeen hundred and forty-five, how old is he this preſent year, being 1800?

From	1800	
Take	1745	
Anſwer	55	Years.

Queſt. 3.

In the year of our Lord 508, Tarquin was baniſhed from Rome, for the raviſhing of Lucretia, how many years is it ſince to this preſent year, being 1800?

From	1800
Take	508
Anſwer	1292

Queſt. 4.

What is the difference betwixt twice twenty-five, and twice five and twenty?

This is no more then from twice $25=50$, ſubtract twice $5=10+20=30$.

Thus	25		5
	25		5
From	50		10
Take	30		20
	20	the difference.	30

Queſt. 5. The difference between two numbers is 36842, the greater is 864952, what is the leſſer?

	864952
	36842
Anſwer -	828110 the leſſer number.

Queſt. 6. Suppoſe this preſent year 1800, you were 14 years old, What year was you born in?

	1800
	14
Anſwer -	1786

Queſt. 7. There are two Numbers, the greater is 102, and the leſſer 72, what is their difference and ſum?

Firſt,	102	and	102
	72		72
Anſ.	30 diff.		174 ſum

Queſt. 8. When the air preſſes with its full weight, in very fair Weather, it may be demonſtrated, that there preſſes upon a human Body about 33905 Pounds of that fluid matter; and in foul Weather, when the Air is moſt light, but 30624 Pounds. What Difference of Weight lies on ſuch a Body, in the two greateſt Alterations of the Weather?

	lbs.
In fair Weather, - - - -	33905
In foul, - - - - -	30624
Anſwer	3281

IV. MULTIPLICATION.

TEACHETH how to increaſe any one number by another, ſo often as there are units in that number by which the one is increaſed; and ſerves inſtead of many additions.

There are three principal members belonging to this rule, viz.

1. The multiplicand, or number to be multiplied.

2. The multiplier, or number by which the multiplicand is multiplied.

3. The product, or number produced in multiplying.

Note. For the ready performance of this, and all the following rules, it is abſolutely neceſſary the following table ſhould be got by heart.

PROOF. The beſt way to prove multiplication is by diviſion, but as the learner is ſuppoſed not yet to know that rule, he cannot prove by it; he muſt therefore make the multiplicand the multiplier; then if the product is the ſame as the other, the work is right.

Note. There is a way of proving multiplication by caſting away the nines, which is moſtly taught in ſchools, and is very expeditious, but liable to error; but for the exerciſe and improvement of the learner, I ſhall ſhew him both ways in its proper place.

TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

CASE 1. To multiply by a ſingle figure.

RULE 1. Place the multiplier underneath the units place of the multiplicand.

2. Multiply from the right hand to the left, thus; begin with the units or loweſt figure of the multiplier, by which multiply the loweſt figure of the multiplicand, and ſet down the overplus, above the tens; and carry the tens: Then multiply the ſecond figure of the multiplicand by the ſame, adding ſo many units, as you had tens to carry; ſet down the overplus, and carry the tens as before. Do thus until you come to the laſt figure, whoſe product muſt be ſet down entire.

EXAMPLE 1.	
Multiplicand	6874
Multiplier	4
Product	27496

To work this example you muſt ſay, 4 times 4 is 16, ſet down 6 and carry one; then 4 times 7 is 28, and 1 that I carried is 29, ſet down 9 and carry 2; then ſay 4 times 8 is 32; and 2 that I carried is 34, ſet down 4 and carry 3; laſtly, 4 times 6 is 24, and 3 that I carried is 27, which ſet down, and the product will be 27496, as by the work.

To prove the foregoing work by caſting away the nines, make a croſs, and add all the figures of the multiplicand together, as units, thus $6+8+7+4=25$, throw away the nines, and ſet the remainder 7 on the ſide of the croſs; do the ſame with the multiplier 4, but as there are no nines to throw away, I ſet down 4 on the other ſide of the croſs. Do the like with the product, $2+7+4+9+6=28$, throw away the nines, and there remains 1 to be ſet at the top of the croſs. Laſtly, multiply the figures on the ſides, thus, 4 times 7 is 28, throw away the nines and ſet the remainder 1 at the bottom of the croſs, which is the ſame as the top, and proves the work to be right.

$$\begin{matrix} & 1 & \\ 7 & \times & 4 \\ & 1 & \end{matrix}$$

The other way to prove multiplication is, by making the multiplicand the multiplier.

E. 2.	E. 3.	E. 4.	E. 5.
32142352	53124564	21684218	16841509
2	3	4	5
64284704	159373692	86736872	84207545

E. 6.	E. 7.	E. 8.	E. 9.
6802124	3214568	3456102	1406178
6	7	8	9
40812744	22501976	27648816	12655602

CASE 2. When the multiplier conſiſts of ſeveral figures;

RULE. Multiply each figure through the line, and write down the products according to their places, that is each reſpective product underneath that figure of the multiplier, by which you multiply, then add them up, and you will have the true product in one line.

E. 10.	E. 11.	E. 12.	E. 13.
31246812	34216812	3841265	21806847
16	23	63	84
187480872	102650436	11523795	87227388
31246812	68433624	23047590	174454776
499948992	786986676	241999695	1831775148

E. 14.

E. 14.	E. 15.	E. 16.	E. 17.
281642	864927	302614	621452
458	653	362	984
2253136	2594781	605228	2485808
1408210	4324635	1815684	4971616
1126568	5189562	907842	5593068
128992036	564797331	109546268	611508768

E. 18.	E. 19.	E. 20.	E. 21.
368121456	460136527	746542	253648
2345	3615	253648	746542
1840607280	2300682635	5972336	507296
1472485824	460136527	2986168	1014592
1104364368	2760819162	4479252	1268240
736242912	1380409581	2239626	1521888
		3732710	1014592
863244814320	1663393545105	1493084	1775536
		189358885216	189358885216

CASE 3. When cyphers are intermixed with the figures in the multiplier.

RULE. Omit the cyphers, and place the first figures of each particular product under its respective multiplier; the following examples, wrought at full length, will sufficiently explain this rule.

E. 22.	E. 23.
804700625	314020065
207008009	200405006
7242305625	1884120390
6437605000	1570100325
5632904375	1256080260
1609401250	628040130
166579474222305625	62931193010445390

CASE. 4. When there are cyphers at the right hand of either, or both the multiplier and multiplicand.

RULE. Multiply as before, neglecting the cyphers until all the particular products are added together, and to that sum place the number of cyphers that are on the right hand of both factors.

 E. 24.

E. 24.	E. 25.	E. 26.
234000	36840	3684000
2600	230	306000
1404	11052	22104
468	7368	11052
608400000	8473200	1127304000000

If you have any number to multiply by 10, 100, 1000, &c. annex as many cyphers thereto as there are in the multiplier, and the work is done.

Multiply	1781	1781
By - -	10	100
Product -	17810	178100

CASE 5. When the multiplier is ſuch a number as any two figures (in the multiplication table) being multiplied together will produce it.

RULE. Multiply the given number by one of thoſe figures, and that product by the other, and you will have the true product.

E. 27.

Multiply 36421 by 16?
$4 \times 4 = 16$

145684
4

Product 582736

E. 28.

Multiply 48612 by 36?
$6 \times 6 = 36$

291672
6

1750032

E. 29.

Multiply 4364213 by 72?
$8 \times 9 = 72$

34913704
9

Product 314223336

E. 30.

Multiply 32410642 by 144?
$12 \times 12 = 144$

388927704
12

Product 4667132448

CASE 6. When the multiplier is any number between 10 and 20.

RULE. Multiply each figure in the multiplicand, by the figure in the unit's place, adding to each its back figure, which ſtands next on the right hand of that you multiplied, only mind, that the firſt figure you begin with, is the thing itſelf, there being none to be added to it.

E. 31.

E. 31. Let it be required to multiply 365 by 12 in one line?

```
 365
  12
----
4380
----
```

To work this example, according to the above rule, ſay, 2 times 5 is 10; ſet down 0 and carry 1; then 2 times 6 is 12, and 1 I carried is 13, and the 5 in the units place of the multiplicand makes 18; ſet down 8 and carry 1, then 2 times 3 is 6, and 1 I carried is 7, and 6 in the multiplicand makes 13; ſet down 3 and carry 1, which is added to 3 in the multiplicand, it makes 4 to be ſet down: So the product of 365 × 12 =4380, as appeareth by the work.

```
E. 32.    E. 33.    E. 34.    E. 35.
 4263     36124     48965     32145
   16        18        17        19
-----    ------    ------    ------
68208    650232    832405    610755
```

NOTES *upon the* NINE DIGITS.

1. The digit One, hath a property which no other number hath, for it neither multiplieth nor divideth, but leaveth the number to be multiplied or divided the ſame.

2. Multiplying any number by Two, is the ſame with doubling of it; and whether you add two to itſelf, or multiply it in itſelf, the ſum and product are equal. And likewiſe obſerve, that no ſquare number can ever terminate, or end with the digit two.

3. If you would multiply any given number by Three, to that number add the double thereof, and the ſum will be equal to the product.

4. If you would multiply any number by Four, you muſt double the doublication, and the ſum is the product.

5. If you would multiply any number by Five, add a cypher to the given number, and take half that ſum for the product.

6. If you would multiply any number by Six, add a cypher to the given number, and take half thereof, to which add the given number, and the ſum will be equal to the product.

Note. Between theſe two laſt-mentioned digits Five and Six, there is a ſecret property; for if you multiply either of them into themſelves, the number produced by ſuch multiplications will terminate in themſelves.

The number Six hath another eminent property, for all its aliquot parts are equal to itſelf, as its half, its third, and its ſixth, being all added together, make ſix. And of numbers that have this property, there are but ten to be found, between one, and one million of millions, which are thoſe exhibited in this table;

Numbers

Numbers of aliquot parts $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$	6
	28
	486
	8128
	120816
	2096128
	33550336
	536854528
	8586869056
	137438691328

Note, If the number 28, 8128, and ſeveral of the other numbers, be divided by 2, 3, and 6, ſeverally, there will remain $\frac{1}{3}$ and $\frac{2}{3}$, which fractions are equal to unity which one being added to the three ſeveral quotients, will make up the given number.

As, ſuppoſe the laſt number in the table;

2)137438691328(68719345664
Remains 0
3)137438691328(45812897109
Remains $\frac{1}{3}$
6)137438691328(22906448554
Remains $\frac{4}{6}$, or $\frac{2}{3}$ ———————
Sum of the quotients 137438691328 = to the dividend.

7. If you would multiply any number by Seven, add a cypher to the given number, and take half thereof, to which half add the double of the number given; the ſum of them will be the product of the given number, multiplied by ſeven.

8. To multiply any number by Eight, to the given number add a cypher, and from thence ſubtract the double of the number given, the remainder is the product.

9. To multiply by Nine, add a cypher to the given number, and from that number ſubtract the given number, the remainder is the product.

This digit Nine hath a privilege above all other digits; for if you take, any number, the nines taken out of the groſs ſum of that number, or of all the parts thereof ſeverally, the remaining digit will be ſtill the ſame.

EXAMPLE. The number 45 hath five nines contained therein; ſo if you multiply 9 by 5, the product is 45. In like manner, if you take the nines out of this number 843, it is the ſame as if you took the nines out of the ſingle figures 8, 4, 3, which make 15, from which 9 being taken, there will remain the digit 6; and alſo if you divide 843 by 9, the quotient will be 93, and 6 remaining.

From hence proceeds the way of proving multiplication, by caſting away the nines out of the factors and product, as taught in ſect. 4, page 10th.

I might introduce various other methods of contracting and working multiplication by ſhort, though tedious and inſignificant rules; but theſe

I ſhall

I shall omit for a future work (as they are more curious than useful) and give the learner a few questions for practice and improvement in this rule.

QUESTION 1. If 1 hogshead of tobacco cost 16 pounds, what will 18 cost?

£.
16
18

Answer 288 pounds.

Q. 2. Suppose 200 men take a prize, and each man's share amounts to 160 pounds; what is the value of the prize?

160
200

Answer 32000 pounds.

Q. 3. In Egypt, there was an ancient city called Babylon, which stood upon a square of 15 miles each way; how much ground did the whole city stand on?

Multiply 15 } The length of
By - 15 } a side.

75
15

Answer 225 miles.

Q. 4. A certain country village, it is said, hath 500 houses in it; now, allowing 5 persons to each house, what number of people are there in all?

500
5

Answer 2500

Q. 5. What is the difference, and what the sum, of six dozen dozen, and half a dozen dozen?

12 a dozen
12

144 a dozen dozen
6

From - 864 six dozen dozen
Take - 72 half a dozen dozen

Difference 792

To 864 six dozen dozen.
Add 72 half a dozen dozen.

Sum 936

V. DIVISION.

TEACHETH us to find how often one number, called the divisor, is contained in another called the dividend, and serves instead of many subtractions. In this rule there are three real numbers, and a fourth accidental, viz.

1. The divided, or number to be divided.
2. The divisor, or number by which you divide.
3. The quotient, or number that shews how often the divisor is contained in the dividend.
4. The remainder, which is always less than your divisor.

CASE 1. When the divisor is not greater than 12.

RULE.

RULE. Firſt, ſeek how often the diviſor is contained in the firſt figure of the dividend; or if the firſt figure of the dividend be leſs than the diviſor, then in the two firſt figures of the dividend, and ſet the quotient figure down, and if any thing remain, carry it to the next figure in the dividend, where it muſt be reckoned as ſo many tens, that is, if 1 remain you muſt call it ten; if 2, 20; if 3, 30, and ſo on, bearing in mind the remainder of each figure, and adding it to the next, until you have made uſe of all the figures in the dividend.

PROOF. Multiply the quotient by the diviſor, and as you multiply, add in the remainder (if any) or add the whole remainder to the product at laſt, and if it comes the ſame as the dividend, the work is right.

EXAMPLE 1.

```
          8)456789
           ------
Quotient    57098 — 5
                8
           ------
Proof  -   456789
           ------
```

To work this example, ſeek how often 8 is in 45, which is 5 times; ſet down 5 under the dividend, and ſay 5 times 8 is 40, then 40, from 45, and there remains 5, which makes the following 56; then ſay, how often 8 in 56, 7 times 8 is 56, from 56, and there remains 0; then ſay, how often 8 in 7, (no time) ſet down 0, and there remains 7, which makes the next figure 78; then ſay, how often 8 in 78, 9 times 8 is 72, from 78, there remains 6, which makes the next figure 69; then, how often 8 in 69; 8 times 8 is 64, from 69, there remains 5, which ſet down at the end of the quotient, as a remainder, and the work is compleated.

E. 2.	E. 3.	E. 4.
2)368421	3)423968	4)986428
184210 — 1	141322 — 2	246607

E. 5.	E. 6.	E. 7.
5)867501	6)612135	9)819684
173500 — 1	102022 — 3	91076

CASE 2. When the diviſor conſiſts of many places of figures;

RULE. 1. Set down the dividend, and the diviſor on the left hand of it.

2. Enquire how often the firſt figure of the divſor is contained in the firſt figure of the dividend; or in the two firſt figures when that of the diviſor is greater, and place the anſwer in the quotient, by which multiply the diviſor, and place the product under the ſaid figure of the dividend, drawing a line underneath it; ſubtract it therefrom, and to the remainder annex the following figure of the dividend, proceeding as before; but if this product be greater than that part of the dividend, a leſs figure muſt be placed in the quotient.

3. If

3. If the remainder ſhould be ſo ſmall, that when the figure of the dividend joined with it make a ſum leſs than the diviſor, then a cypher is to be placed in the quotient, and another figure brought down; for every figure brought down, a cypher or figure muſt be placed in the quotient. This is called

LONG DIVISION.

EXAMPLE 1. What is the quotient of 14122 divided by 46?

Diviſor. Dividend. Quotient.

```
46) 14122 (307
    138
    ----
    ..322
      322
      ---
      ...
```

To work this example, ſay how often can I have 4 in one (no times) then, how often 4 in 14, which is 3 times, then place 3 in the quotient, and multiply 46 by it, ſetting the product 138 under 141, and ſubtract it therefrom, and there remains 3. Then bring down the next figure 2 from the dividend, and annex it to 3, which makes 32; then enquire how often 4 in 3, the anſwer is 0, which I place in the quotient, and bring down the next figure 2, the dividual is then 322; then ſeek how often 4 in 32, the anſwer would be 8; but 46 multiplied by 8 would exceed 322; therefore I place 7 in the quotient, by which I multiply 46, and the product is 322; that, ſubtracted from 322, leaves nothing, therefore 307 is the quotient.

SCHOLIUM. There are various ways of proving diviſion, and for the exerciſe of the learner I ſhall prove it by three different ways; firſt, by multiplying the quotient by the diviſor; ſecondly, by caſting away the nines; and laſtly, by addition, thus:

Take the quotient of the laſt example, and multiply it by the diviſor.

```
  307
   46
 ----
 1842
1228
-----
14122 the ſame as the dividend.
```

E. 2.

```
47) 8460 (180
    47
    ---
    376
    376
    ---
    ..0
```

Take this example, and caſt away the nines in the diviſor and quotient, which put on each ſide of the croſs, and caſt away the nines out of the dividend; put the remainder at the top of the croſs; then multiply the ſide figures thereof into each other, and caſt the nines out of the product, and if the work be right, the remainder to be written at the bottom of the croſs will be the ſame as the top, as appears by this example.

```
 0
2×0
 0
```

E. 3.

345)746789(2164
*690

567
*345

2228
*2070

1589
*1380

* 209 Remainder.

Proof 746789

To prove this example, add up all the lines marked thus* ; and as there is nothing but a cypher to be added to 9 in the remainder, put down 9, and for the same reason put down 8 : then say, 2 and 3 is 5, and 7 is 12, and 5 is 17 ; set down 7 and carry 1. Then 1 and 1 is 2, and 4 is 6 ; set down 6, and say 2 and 3 is 5, and 9 is 14 ; set down 4 and carry 1 ; 1 and 6 is 7, which set down, and the sum is the same as the dividend, which proves the work to be right.

Note. If there be a remainder when you prove by the cross, it must be added to the product on the sides of the cross, and the nines thrown out as before.

E. 4.

6023)1897258(315
18069

9035
6023

30128
30115

13

E. 5. 61745)392628787(6358
370470

221587
185235

363528
308725

548037
493960

54077

E. 6.

684573)323323869(472
2738292

4949466
4792011

1574559
1369146

205413

E. 7.

476085)988390547(2076
952170

3622054
3332595

2894597
2856510

38087

CASE 3. When the divisor has cyphers on the right hand;

RULE. Strike them off, and likewise strike off as many places of the dividend on the right hand; and perform the division by the remaining

maining figures. And when the division is finished, annex the figures cut off to the remainder.

E. 1.

```
304|00)7456|78(24
       608
       ----
       1376
       1216
       ----
       16078 Remainder.
```

E. 2.

```
41|00)864987|32(21097
      82
      --
       44
       41
       --
        398
        369
        ---
         297
         287
         ---
         1032 Remainder.
```

When the dividend has the same number of cyphers on the right hand, as the divisor, strike them off from each, and the remainder will be so many of what you divide by, without annexing the cyphers that were cut off.

E. 3.

```
123|00)6397|00(52
       615
       ---
        247
        246
        ---
          1
```

E. 4.

```
312|000)9864|000(31
        936
        ---
         504
         312
         ---
         192
```

CASE 4. When the divisor is such a number, that any two figures (in the multiplication table) multiplied together, will produce it.

RULE. Divide the given number by those numbers or component parts, which is much easier than dividing by all the divisor at once; see the following examples worked at full length.

NOTE. If there be a remainder in the last division, it will be so many times the first divisor, which added to the first remainder (if any) will give the true remainder sought.

E. 1.

```
8|45876912306 ÷ by 32.
4| 5734614038—2 }
 | 1433653509—2 } 18 Rem.
```

E. 2.

```
7|17862493508 ÷ by 42.
6| 2551784786—6 }
 |  425297464—2 } 20 Rem.
```

To prove by multiplication all examples of this kind, you must add, or take in seperately the two remainders, when you multiply by their respective divisors that produced them.

E. 3.

8 | 789065432187 ÷ by 72.

9 | 98633179023—3 }
  | 10959242113—6 } 51

E. 4.

9 | 819186100212 ÷ by 81,

9 | 91020677801—3 }
  | 10113408644—5 } 48

E. 5.

12 | 244801864013 ÷ by 144.

12 | 20400155334—5 }
   | 1700012944—6 } 77 Remainder

Thoſe who are well acquainted with the nature of diviſion, may ſubtract each figure of the product as it is produced, and only write down the remainders; this will ſhorten the work, and is commonly called Italian diviſion; to perform which the following examples will ſufficiently explain:

E. 1.

34)368492(10838
284
129
272
...

E. 2.

324)6842189(2111
362
381
578
254 Remainder

E. 3.

6125)8649753(1412
25247
7475
13503
1253 Remainder

E. 4.

406502)4169031497(10255
1040114
2271109
2385997
353487 Remainder

VI. REDUCTION.

TEACHETH to reduce all great names into ſmall, by multiplying continually the given number with ſo many of the next lower name, as makes one of the higher, keeping them equivalent in value; this is called Reduction Deſcending. On the contrary, where the quantity is to be reduced to a higher denomination, divide continually the given number by ſo many of the leſſer name as makes one of the greater; this is termed Reduction Aſcending.

TABLES

TABLES of ENGLISH COINS.

Marked.							*q.*
q.	4 Farthings	make one	Penny	$\frac{1}{4}$	is wrote for		1
d.	12 Pence	make one	Shilling	$\frac{1}{2}$	is wrote for		2
s.	20 Shillings	make one	Pound £.	$\frac{3}{4}$	is wrote for		3

Note. The reaſon for placing £. *s. d. q.* over every denomination, ſignifies, Libra, Solidi, Denarii, Quadrantes; that is, Pounds, Shillings, Pence, Farthings.

PENCE TABLE.

s.		*d.*			*d.*		*s.*	*d.*	
1	are	12			20	are	1	8	
2	are	24			30	are	2	6	Half a Crown
3	are	36			40	are	3	4	
4	are	48			50	are	4	2	
5	are	60			60	are	5	0	A Crown
6	are	72			70	are	5	10	
7	are	84			80	are	6	8	Noble
8	are	96			90	are	7	6	
9	are	108			100	are	8	4	
10	are	120			110	are	9	2	
11	are	132			120	are	10	0	Angle
12	are	144			130	are	10	10	
13	are	156			140	are	11	8	
14	are	168			150	are	12	6	
15	are	180			160	are	13	4	Mark
16	are	192			170	are	14	2	
17	are	204	Piſtole		180	are	15	0	
18	are	216			190	are	15	10	
19	are	228			200	are	16	8	
20	are	240	Pound		210	are	17	6	
21	are	252	Guinea		220	are	18	4	
27	are	324	Moidore		230	are	19	2	
					240	are	20	0	Pound

The WEIGHTS and VALUE of ſuch GOLD and SILVER COINS, as are moſt commonly uſed in England.

	WEIGHT.			VALUE.		
	Dwts.	*gr.*	*mites.*	£.	*s.*	*d.*
A Guinea - -	5	9	9	1	1	0
Half ditto - -	2	16	14	0	10	6
A Quarter ditto	1	8	7	0	5	3
SILVER.						
A Crown - -	19	8	$10\frac{3}{4}$	0	5	0
Half ditto -	9	16	$5\frac{1}{6}$	0	2	6
A Shilling -	3	20	18	0	1	0
A Sixpence -	1	22	9	0	0	6

Note. 20 mites make one grain.

A pound weight avoirdupoiſe of copper, is coined into twenty-three pence; conſequently a halfpenny is nearly $\frac{1}{3}$ of an ounce, and a farthing $\frac{1}{6}$.

EXAMPLE 1.

EXAMPLE 1.

In 24 pounds, how many ſhillings and pence?

£.
24
20

480 Shillings
12

5760 Pence

E. 2. How many ſhillings and pounds are there in 5760 pence?

d.
12)5760

2|0) 48|0 Shillings

24 Pounds

E. 3. In 36*l.* 10*s.* how many ſhillings, pence, and farthings?

£. s.
36 10
20

730 Shillings
12

8760 Pence
4

35040 Farthings

In this example multiply as before, but obſerve to take in the 10*s.* in their proper place, that is, when you multiply by 20, the ſhillings in a pound.

E. 4. In 35040 farthings, how many pounds?

qrs.
4)35040

12)8760

2|0)73|0

Anſwer £. 36 10*s.*

E. 5. Reduce 302*l.* 16*s.* 4¾*d.* to farthings?

£. s. d.
302 16 4¾
20

6056
12

72676
4

Anſwer 290707 Farthings

E. 6. In 290707 farthings, how many pounds?

4)290707

12)72676 ¾ *qrs.*

2|0)605|6 4*d.*

Anſwer £. 302 16 4¾

WEIGHTS and MEASURES.
TROY WEIGHT.

Marked.			
gr.	24 Grains	make one	Penny weight
dwts.	20 Penny weights	make one	Ounce
oz.	12 Ounces	make one	Pound

By Troy weight is weighed gold, ſilver, jewels, corn, bread and all liquors; from this weight all meaſures for wet and dry commodities are taken.

N. B.

N. B. 14 *oz.* 11 *dwts.* $15\frac{1}{2}$ *grs.* Troy, is equal to one pound Avoirdupoise.

E. 1. In 36 *lb.* of silver, how many ounces, penny weights, and grains?

```
lb.
 36
 12
-----
432 Ounces
 20
-----
8640 Dwts.
  24
-----
 34560
17280
------
207360 Grains
------
```

E. 2. Reduce 207360 grains to penny weights, ounces, and pounds?

```
     grs.
  4)207360
  --------
  6) 51840
  --------
2|0) 864|0
 12) 432
     ---
Answer - 36 Pounds
```

E. 3. In an ingot of silver, weighing 14 *lb.* 10 *oz.* 16 *grs.* how many grains?

```
lb. oz. dwts. grs.
 14  10   0    16
 12
----
 178
  20
----
3560
  24
-----
14246
7121
-----
Ans. 85456 Grains
```

E. 4. Let it be required to reduce 85456 grains to pounds?

```
              grs.
4×6=24 { 4)85456
         -------
         6)21364      } =16
       -------
    2|0) 356|0 —4
       -------
     12) 178
    -------- oz. dwts. grs.
Answer lb. 14 10  0   16
```

APOTHECARIES WEIGHT.

Marked.			
grs.	20 Grains	make one	Scruple
℈	3 Scruples	make one	Dram
ʒ	8 Drams	make one	Ounce
℥	12 Ounces	make one	Pound, *lb.*

Apothecaries, in making up their medicines, use this weight; but they buy and sell their drugs by avoirdupoise weight.

E. 1.

E. 1. In 18 pounds, how many ounces, drams, ſcruples and grains ?

```
     18 Pounds
     12
    ----
    216 Ounces
      8
   -----
   1728 Drams
      3
   -----
   5184 Scruples
     20
 -------
 103680 Grains
 -------
```

E. 2. Reduce 103680 grains to ſcruples, drams, ounces, and pounds ?

```
2|0)10368|0
   --------
    3)5184
     -----
    8)1728
     -----
    12)216
      ----
        18 Pounds
        --
```

E. 3. In 2 *lb.* 4 ℥. 3 ʒ 2 ℈. 12 *grs.* how many grains ?

```
lb. ℥. ʒ. ℈. grs.
 2  4  3  2  12
12
--
28
 8
---
227
  3
---
683
 20
-----
```

Anſw. 13672 Grains

E. 4. In 13672 grains, how many pounds ?

```
2|0)1367|2
   -------
  3)683 — 12
   -----
  8)227 — 2
   -----
  12)28 — 3
   -----
Anſwer lb. 2 4 3 2 12
```

AVOIRDUPOISE WEIGHT.

Marked.			
dr.	16 Drams	make one	Ounce
oz.	16 Ounces	make one	Pound
lb.	28 Pounds	make one	Quarter of Cwt.
qr.	4 Quarters, or 112 lb.	make one	Hundred
Cwt.	20 Hundred	make one	Ton

By avoirdupoiſe weight is weighed all manner of grocery, and chandler's wares, and all metals, except ſilver and gold; alſo bread, butter, cheeſe, butcher's meat, &c.

The denominations in ſome of which are as follows, viz

WOOL WEIGHT.

7 Pounds	make one	Clove	6 ½ Todds	make one	Wey
2 Cloves	make one	Stone	2 Weys	make one	Sack
2 Stones	make one	Todd	12 Sacks	make one	Laſt.

HAY.

HAY.

56 Pounds of old hay } are one truss
60 Pounds of new dit. }
36 Trusses - - are one last

BREAD WEIGHT.

	lb.	oz.	dr.
Peck loaf - -	16	6	1
Half ditto - -	8	11	$\frac{1}{2}$
Quarter ditto - -	4	5	8

8 Pounds	make one	Stone of butcher's meat
14 Pounds	make one	Stone of horseman's weight
19 $\frac{1}{2}$ Hundreds	make one	Fodder of lead

Note. There are some sorts of silk, which are weighed by a great pound of 24 ounces.

EXAMPLE 1. In 20 *Cwt.* how many quarters, pounds, ounces, and drams?

```
 Cwt.
  20
   4
  80 Quarters
  28
 640
160
2240 Pounds
  16
35840 Ounces
   16
573440 Drams
```

E. 2. Let it be required to reduce 573440 drams to hundreds?

```
        { 4)573440 Drams
4×4=16 {
        { 4)143360
           4)35840
            4)8960
            4)2240
             7)560
              4)80
Answer         20 Cwt.
```

E. 3. In 10 *T.* 10 *Cwt.* 14 *lb.* 11 *oz.* 5 *drs.* how many drams?

```
 T.  C. qrs. lb. oz. drs.
 10  10   0   14  11   5
 20
210 Hundreds
  4
840 Quarters
 28
6724
1681
23534 Pounds
   16
141205
23535
376555 Ounces
    16
6024885 Drams
```

E. 4. In 6024885 drams, how how many ounces, pounds, quarters, hundreds, and tons?

```
4)6024885 Drams
4)1506221 — 1 }
              } = 5
4)376555 — 1  }
4)94138 — 3   }
              } = 11
4) 23534 — 2  }
7) 5883 — 2   }
              } = 14
4) 840 — 3    }
2|0) 21|0
             Cwt. qr. lb. oz. drs.
Answer T. 10  10   0   14  11   5
```

E. CLOTH

CLOTH MEASURE.

			Marked.
4 Nails	make one	Quarter of a yard	§ *na. qrs.*
3 Quarters		Ell Flemiſh	§ *Ell Fl.*
4 Quarters		Yard	§ *yd.*
5 Quarters		Ell Engliſh	§ *Ell. Eng.*
6 Quarters		Ell French	§ *Ell. Fr.*

Scotch and Iriſh linens are bought and ſold by the yard; but the Dutch linens are bought by the ell Flemiſh, and ſold by the ell Engliſh.

EXAMPLE 1. In a piece of cloth, containing 36 yards, how many quarters and nails?

36 Yards
4
———
144 Quarters
4
———
576 Nails

E. 2. In 576 nails, how many yards?

4)576
———
4)144
———
Anſwer 36 Yards

E. 3. How many nails are there in 84 ells Engliſh 4 qrs. 2 nails?

Ell.	*qrs.*	*na.*
84	4	2

5
———
424
4
———
Anſwer 1698 Nails

E. 4. In 1698 nails, how many ells Engliſh?

4)1698
———
5)424 - 2
———
Anſwer 84 4 2

E. 5. In 201 ells Flemiſh, how many nails?

201
3
———
603
4
———
Anſwer 2412 Nails

E. 6. Reduce 2412 nails to ells Flemiſh.

4)2412
———
3)603
———
Anſwer 201 Ells Flemiſh

E. 7. How many nails in 64 ells French?

64
6
———
384
4
———
Anſwer 1536 Nails

E. 8. In 1536 nails, how many ells French?

4)1536
———
6)384
———
Anſwer 64

LONG

LONG MEASURE.

Marked			
b. c.	3 Barley corns	make one	Inch
in.	12 Inches - - - -	make one	Foot
f.	3 Feet, or 36 inches	make one	Yard
yd.	2 Yards, or 6 feet - -	make one	Fathom
	$5\frac{1}{2}$ Yards, or 11 half yards	make one	Pole, rod, or perch
p.	40 Poles, or 220 yards	make one	Furlong
fur.	8 Furlongs, or 1760 yards	make one	Mile
m.	3 Miles - - - -	make one	League
lea.	$23\frac{1}{6}$ Leagues, or $69\frac{1}{2}$ Miles	make one	Degree. *Deg.*

360 Degrees are the circumference of the globe.
5 Feet is a geometrical pace.
$16\frac{1}{2}$ Feet is a pole.

ALSO,

4 Inches	make one	Hand, or hand's breadth
3 Hand's breadth	make one	Foot
$1\frac{1}{2}$ Foot	make one	Cubit
2 Cubits	make one	Yard

By this meaſure diſtances of places, or any thing elſe, that has length only, are meaſured.

EXAMPLE 1. How many yards, feet and inches, are there in 300 miles?

300 Miles
1760 The yards in a mile
528000 Yards
3
1584000 Feet
12
19008000 Inches

E. 2. In 19008000 inches, how many miles?

12)19008000
3) 1584000
176|0)52800|0)300 Miles
528
000

E. 3. Let it be required to reduce 12 leagues, 1 mile, 6 furlongs, 28 poles, and 4 yards, to barley-corns?

Lea.	*m.*	*fur.*	*p.*	*yds.*
12	1	6	28	4

3
37
8
302
40
12108
$5\frac{1}{2}$
60544
6054
66598
36 Inches in a yard
399588
199794
2397528
3
Anſ. 7192584 Barley corns

LAND MEASURE.

Marked			
	$5\frac{1}{2}$ Yards	make one	Perch, rod, or pole
p.	40 Poles	make one	Rood
r.	4 Roods	make one	Acre
a.	30 Acres	make one	Yard of land
	100 Acres	make one	Hide of land

Land is commonly meaſured by a chain called Gunter's, whereof 10 in length and 1 in breadth, are an acre of land = 4840 yards.

7 Inches 92 parts - - - -	make one	Link
25 Links - - - - - - -	make one	Pole
4 Poles, or 100 links, or 22 yards	make one	Chain
10 Chains - - - - - - -	make one	Furlong

EXAMPLE 1. In 84 acres, how many rods and poles?

```
    84 Acres
     4
   ---
   336 Roods
    40
  ----
 13440 Poles
```

E. 2. Let it be required to reduce 13440 poles to acres?

```
        4|0)1344|0
          4)336
Anſwer - -   84 Acres
```

E. 3.

To meaſure a neighbouring plain,
I took up my croſs ſtaff and chain;
Having found th' content of the whole,
Eighty acres, two roods, and a pole,
What roods and perches were there
Be pleaſed to make to appear?

```
  A.  r.  p.
  80  2   1
   4
 ---
 322 Roods
  40
 -----
12881 Perches
```

E. 4. In 12881 perches, how many acres?

```
          4|0)1288|1
            4) 322
Anſwer - -     80 Acres 2 roods 1 perch.
```

WINE MEASURE.

Marked			
pts.	2 Pints	make one	Quart
qts.	4 Quarts	make one	Gallon
	10 Gallons	make one	Anchor of br. or rum
	18 Gallons	make one	Runlet
	31½ Gallons	make one	Barrel

Marked			
	42 Gallons	make one	Tierce
tier.	2 Tierce, or 84 gallons	make one	Puncheon, *punch.*
	63 Gallons	make one	Hogſhead
h.	2 H. or 126 ga.	make one	Pipe or butt
p.	2 P. or 252 ga.	make one	Tun

Note. A tun of wine is 18 hundred weight avoirdupoiſe.

A gallon is 231 ſolid inches.

By wine meaſure all ſpirits, mead, perry, vinegar, oil, cyder, and honey, &c. are meaſured.

E. 1,

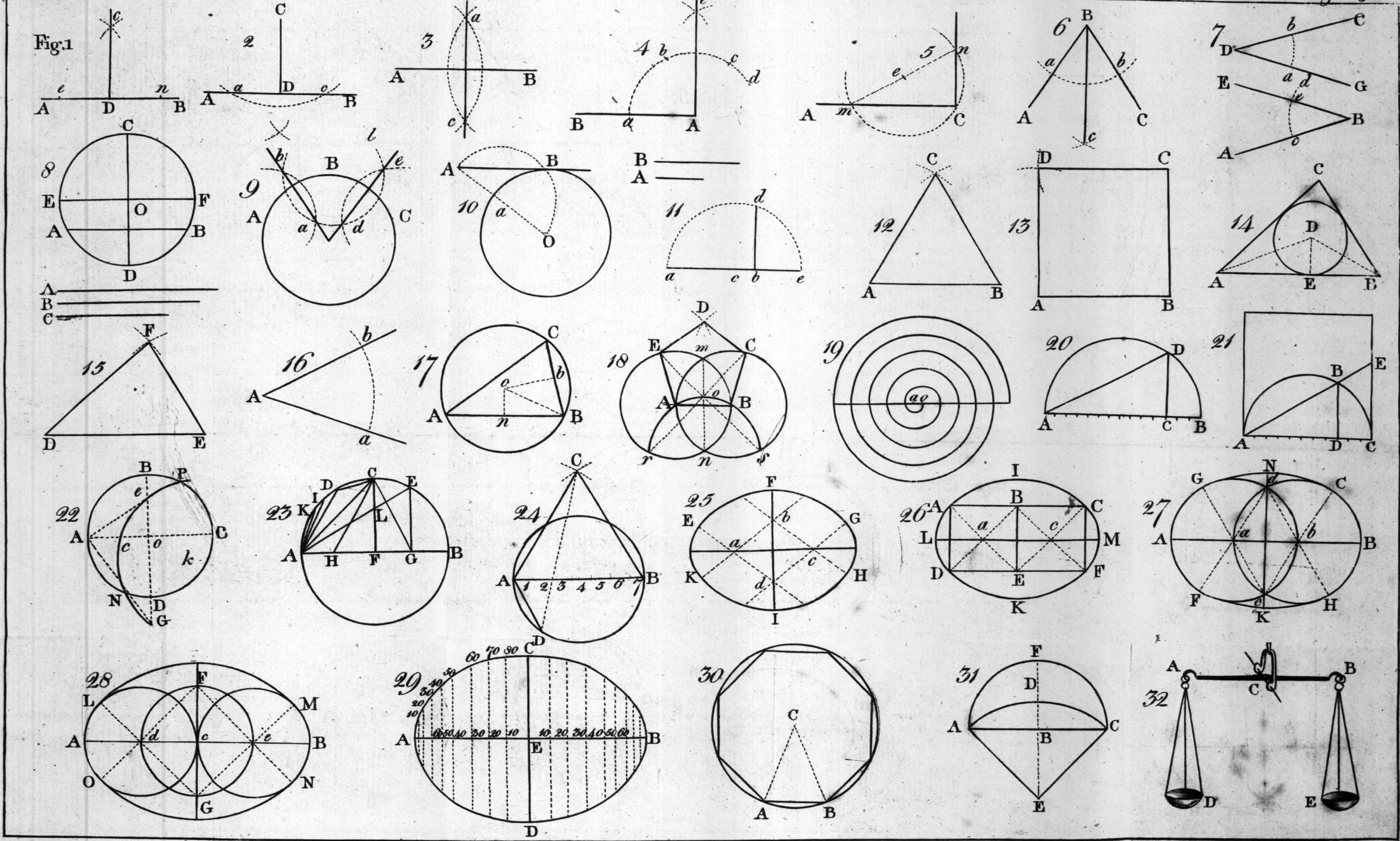


Fig. 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32

E. 1. In 10 anchors of brandy, how many gallons and quarts?

$$\begin{array}{r} 10 \\ 10 \\ \hline 100 \text{ Gallons} \\ 4 \\ \hline 400 \text{ Quarts} \end{array}$$

E. 2. In 400 quarts, how many anchors?

$$\begin{array}{r} 4)400 \\ \hline 10)100 \text{ Gallons} \\ \hline \text{Anſwer} \quad 10 \text{ Anchors} \end{array}$$

E. 3. In 8 hogſheads of wine, how many gallons and pints?

$$\begin{array}{r} 8 \\ 63 \\ \hline 24 \\ 48 \\ \hline 504 \text{ Gallons} \\ 8 \\ \hline 4032 \text{ Pints} \end{array}$$

E. 4. In 4032 pints of wine, how many hogſheads?

$$\begin{array}{r} 8)4032 \\ \hline 9 \times 7 = 63 \begin{cases} 9)504 \\ 7)\ 56 \end{cases} \\ \hline \text{Anſwer} \quad 8 \text{ Hogſheads} \end{array}$$

WINCHESTER MEASURE.

Called ALE and BEER MEASURE.

Marked			
pts.	2 Pints - - - -	make one	Quart
qts.	4 Quarts, or 8 pints - -	make one	Gallon
gal.	8 Gallons ale, or 9 gallons beer	make one	Firkin
fir.	2 Firkins - - -	make one	Kilderkin
kil.	2 Kilderkins, or 4 Firkins, or 32 gallons ale, 36 gallons beer	make 1	barrel
bar.	1½ Barrel, or 3 Kilderkins, or 48 gallons ale, 54 gallons beer	make one	hogſhead
hhds.	2 Hogſheads, or 3 barrels, or 108 gallons	make one	Butt
	2 Butts, or 236 gallons - - -	make one	Tun

Note. 8½ Gallons is a firkin in all parts of England except London, where the ale firkin contains 8 gallons, and the beer firkin 9.

A gallon of ale or beer is 282 ſolid inches.

E. 1. In 14 barrels of ale, how many gallons and quarts?

$$\begin{array}{r} 14 \\ 32 \\ \hline 28 \\ 42 \\ \hline 448 \text{ Gallons} \\ 4 \\ \hline 1792 \text{ Quarts} \end{array}$$

E. 2. In 1792 quarts of ale, how many barrels?

$$\begin{array}{r} 4)1792 \\ \hline 8 \times 4 = 32 \begin{cases} 8)448 \\ 4)\ 56 \end{cases} \\ \hline \text{Anſwer} \quad 14 \text{ Barrels} \end{array}$$

E. 3.

E. 3. In 34 barrels of beer, how many pints?

```
  34
  36
 ----
 204
102
-----
1224
   8
-----
```

Anſwer 9792 Pints

E. 4. In 9792 pints of beer, how many barrels?

```
             8)9792
             ------
           { 6)1224
6 × 6 = 36 {  -----
           { 6) 204
                ---
Anſwer           34 Barrels
                ---
```

DRY MEASURE.

Marked.

pts.	2. Pints	make one	Quart §
qts.	4 Quarts, or 8 pints	make one	Gallon §
gal.	2 Gallons	make one	Peck §
pks.	4 Pecks, or 8 gallons	make one	Buſhel §
bu.	4 Buſhels	make one	Comb §
c.	2 Combs, or 8 buſhels	make one	Quarter §
qrs.	5 Quarters	make one	Wey §
	2 Weys, or 10 quarters	make one	Laſt §

ALSO,

4 Quarters, or 32 buſhels, make 1 chaldron of corn, and 2 buſhels make 1 ſtrike.

A cart load of corn is 40 buſhels.

2 Quarts are one pottle, both in liquid and dry meaſure.

A gallon contains $268\frac{4}{5}$ ſolid inches.

In meaſuring ſea coal, 5 pecks are one buſhel, water meaſure.

3 Buſhels	make one	Sack
9 Buſhels	make one	Vatt
36 Buſhels, or 12 ſacks	make one	Chaldron
21 Chaldrons	make one	Score

By dry meaſure, corn, ſalt, and all other dry goods are meaſured.

The ſtandard buſhel is $18\frac{1}{2}$ inches wide, and 8 inches deep.

EXAMPLE 1. In 36 quarters of corn, how many buſhels, pecks, gallons, and quarts?

```
  36 Quarters
   8
 ----
 288 Buſhels
   4
 ----
1152 Pecks
   2
 ----
2304 Gallons
   4
 ----
9216 Quarts
```

E. 2. In 9216 quarts, how many quarters?

```
       4)9216
         ----
       2)2304
         ----
       4)1152
         ----
       8) 288
          ---
Anſwer     36 Quarters
          ---
```

E. 3.

E. 3. Reduce 36 chaldron, 26 buſhels of coals, to pecks?

cha.	bu.
36	26
36	
222	
110	
1322	
5	
6610 Pecks	

E. 4 In 6610 pecks of coals how many chaldrons?

5)6610
6)1322
6)220 - 2
36 - 4 } =26

Anſwer 36 chald. 26 buſh.

TIME

Marked.				
'''	60 Thirds	make one	Second	
ſec.	60 Seconds		Minute	*Thirty days hath September,*
m.	60 Minutes		Hour	*April, June, and November;*
h.	24 Hours		Day	*February hath twenty-eight alone,*
d.	7 Days		Week	*And all the reſt have thirty-one;*
w.	4 W. or 28*d.*		Month	*Except leap-year, and then's the time,*
				February's days are twenty-nine.

52 Weeks, 1 day, 6 hours; or 13 months, 1 day, 6 hours; or 365 days, 6 hours, make one Julian year.

According to the beſt computation a ſolar year contains 365 days, 5 hours, 48 minutes, 57 ſeconds, 39 thirds.

The year is alſo divided into 12 unequal calendar months, according to the above verſe.

EXAMPLE 1 How many days, hours, minutes and ſeconds, are there in 4 weeks?

4
7
28 Days
24
112
56
672 Hours
60
40320 Minutes
60
2419200 Seconds

E. 2. In 2419200 ſeconds, how many weeks?

6|0)241920|0
6|0)4032|0
$4\times6=24$ { 4)672
6)168
7)28
Anſwer 4 Weeks

E. 3.

E. 3. How many ſeconds are there in a Julian year, or 365 days 6 hours?

```
  365  6
   24
 ----
 1466
 730
 ----
 8766 Hours
   60
 ------
 525960
     60
 --------
Anſ. 31557600 Seconds
```

E. 4. In 31557600 ſeconds, how many years?

```
6|0)3155760|0
  6|0)525960
   ------
   4)8766
   ------
   6)2191 - 2 }
   ------      } = 6 h.
  d. 365 - 1  }
```

Anſwer 365 *days* 6 *hours*.

SQUARE MEASURE.

144	Square inches	make one	Square foot
9	—— feet	make one	—— yard
$30\frac{1}{4}$	—— yards	make one	—— pole
40	—— poles	make one	—— rood
4	—— roods	make one	—— acre
640	—— acres	make one	—— mile

$172\frac{1}{4}$ Feet, is 1 rod of brick work. 100 Sq. feet is 1 ſquare of flooring.

EXAMPLE 1 In 32 ſquare yards, how many ſquare inches?

```
    32
     9
   288
   144
  -----
  1152
 1152
 288
 ------
Anſwer 41472 Inches
```

E. 2. How many ſquare yards are there in 41472 ſquare inches?

```
           { 12)41472
12×12=144 {    -----
           { 12) 3456
                -----
                9)288
                -----
Anſwer          32 yds.
```

E. 3. Reduce 4 ſquares, 31 feet, 38 inches of flooring to inches.

```
Sq.  f.  in.
 4   31  38
100
---
431
144
-----
1732
1727
431
------
Anſ. 62102 Inches
```

E. 4. How many ſquares are there in 62102 ſquare inches?

```
1× 12  { 12)62102
=144  {    -----
       { 12)5175 - 2 }
            -----      } =38i.
  1|00)4|31 - 3      }
```

Anſwer *Sq.* 4 31*f.* 38 *Inches*

SOLID MEASURE

1728 Solid inches - - -	make one	Solid foot
27 Feet - - - - - -		—— yard
40 Feet of round timber, or 50 of hewn dit.		Ton or load

A solid yard of earth is called a load.

A statute cord of wood, is a pile 8 feet long, 4 feet broad, and 4 feet high; consequently its content is 128 feet; for $8 \times 4 \times 4 = 128$. This sort of cord is used in most of the northern counties of England; but in Sussex, and most of the southern counties, a pile of wood 3 feet high, 3 feet wide, and 14 feet long, is called a cord. The content of this is two feet less than the other; for $14 \times 3 \times 3 = 126$.

By this measure are measured all things, in which are considered length, breadth, and depth or thickness.

EXAMPLE 1. In 28 solid yards, how many solid inches?

```
     28
     27
    ---
    196
    56
    ---
    756
   1728
   ----
   6048
  1512
 5292
 756
-------
1306368 Inches
```

E. 2. In 1306368 solid inches, how many solid yards?

```
                    12)1306368
                    ----------
12×12×12=1728       12)108864
                    ----------
                     12)9072
                    ----------
3×9=27               3)756
                    ----------
                     9)252
                    ----------
Answer Yards            28
```

Practical Arithmetic.

PART I.

BOOK II.

VII. COMPOUND ADDITION.

TEACHETH to add sundry sums or numbers together, having divers denominations, as in money, weights, measures, &c.

RULE. 1. Place the numbers so, that those of the same denomination may stand directly under each other, viz. pounds under

pounds,

pounds, ſhillings under ſhillings, pence under pence, farthings under farthings, &c.

2. Begin to add at the loweſt denomination firſt, as in integers; then divide that ſum by as many of the ſame denomination, as makes one of the next greater. ſetting down the remainder under the row added, and carry the quotient to the next greater denomination, whoſe ſum you muſt alſo find. Proceed in this manner to the greateſt denomination, which add as integers.

EXAMPLES OF MONEY

EXAMPLE 1.

£.	s.	d.
41	14	$5\frac{3}{4}$
86	18	$11\frac{1}{4}$
51	19	$4\frac{1}{2}$
67	17	$7\frac{3}{4}$
12	12	$3\frac{1}{4}$
261	2	$8\frac{1}{2}$

To add up this example ſay, 1 and 3 is 4, and 2 is 6, and 1 is 7, and 3 is 10; 10 farthings are two-pence half-penny, ſet down the half-penny thus $\frac{1}{2}$, and carry the two-pence to the pence row; ſaying 2 and 3 is 5, and 7 is 12, and 4 is 16, and 11 is 27, and 5 is 32; 32 pence is 2 ſhillings and 8 pence, ſet down 8 and carry 2. Then proceed to the ſhillings, and ſay, 2 and 2 is 4, and 7 is 11, and 9 is 20, and 8 is 28, and 4 is 32, (which is 2 above 3 tens) ſet down the 2, and go on to the next row) which is compoſed of a number of ones, being ſo many ten ſhillings, as you may ſee by their being placed, or ſet in the place of tens, and carry the three tens thereto; and ſay, 3 and 1 is 4, and 1 is 5, and 1 is 6, and 1 is 7, and 1 is 8; eight ten ſhillings make 4 pounds, which carry to the place of pounds, and ſay, 4 and 2 is 6, and 7 is 13, and 1 is 14, and 6 is 20, and 1 is 21; write down 1, and carry 2, ſaying 2 and 1 is 3, and 6 is 9, and 5 is 14, and 8 is 22, and 4 is 26, which write down; and the total will be £261. 2*s*. $8\frac{1}{2}$*d*. In the ſame manner, you may proceed with any other examples of the like kind.

E. 2. £.	s.	d.	E. 3. £.	s.	d.	E. 4. £.	s.	d.	E. 5. £.	s.	d.
31	16	$8\frac{1}{4}$	21	10	11	14	13	$4\frac{1}{4}$	361	12	8
20	10	4	36	11	$2\frac{1}{2}$	16	10	8	416	18	9
68	11	$1\frac{1}{2}$	41	10	6	62	12	$4\frac{3}{4}$	618	10	$4\frac{1}{2}$
30	13	6	23	13	$4\frac{1}{4}$	71	18	$6\frac{1}{2}$	481	12	2
46	10	$2\frac{1}{2}$	34	18	8	42	16	8	310	10	$2\frac{1}{2}$
84	19	4	42	12	$2\frac{1}{2}$	81	11	$6\frac{1}{4}$	681	10	8
42	12	$8\frac{1}{4}$	32	10	4	30	10	6	322	12	$6\frac{1}{2}$
325	13	$10\frac{1}{2}$	233	7	$2\frac{1}{4}$	320	13	$7\frac{3}{4}$	3193	7	$4\frac{1}{2}$

E. 6.

E. 6. A houſekeeper had diſburſed for her lady, in marketing (per memorandum-book) for beef 10*s.* 5½*d.* Mutton 7*s.* 8*d.* Veal 6*s.* 3*d.* Chickens 3*s.* 4½*d.* and for eggs 3¼*d.* What was the ſum diſburſed?

		s.	*d.*
Beef	- - -	10	5½
Mutton	- - -	7	8
Veal	- - -	6	3
Chickens	- -	3	4½
Eggs	- - -	0	3¼
Sum	£. 1	8	0¼

E. 7. An aſſeſſment for the highway levy, in the townſhip of B——, and Pariſh of M——, in the County of Warwick, rated at 6*d.* per pound, from Michaelmas 1779, to Michaelmas 1780.

	£.	*s.*	*d.*
Sir J. Fletcher, Bart.	31	12	6½
Thomas Careleſs -	42	10	0
John Ward -	2	0	8
John Teverill -	6	12	0
Richard Moore -	12	10	2½
James Day -	0	16	0
Thomas Farrol -	21	8	2
William May -	16	10	0
Thomas Baker	8	1	4
Samuel Hodgetts	2	10	6½
William Swift -	1	0	2
Sarah Dunn -	16	4	3
John Garriſon	0	12	9½
John Howes -	0	10	4
Joſeph Rann -	21	1	0
	£. 184	0	0

E. 8. *A farmer's bill upon his labourer.*

Thomas Myatt,

1780. To Joſ. Latchford, *Dr.*

	£.	*s.*	*d.*
May 30, To a meaſure of corn } -		5	6
June 8, Ditto - -		6	4
26, Ditto - -		7	2
30, A buſhel of oats		2	3
July 1, A load of coals	1	0	4
13, Beef - -		6	4
16, Bacon - -		3	1
Aug. 4, Cheeſe -	1	0	4
8, Butter - -		2	3½
Total £.	3	13	7½

WEIGHTS and MEASURES.

TROY WEIGHT

EXAMPLES:

℔.	*oz.*	*dwts.*	*gr.*	℔.	*oz.*	*dwts.*	*gr.*
8	2	4	1	364	11	16	14
6	1	2	4	121	10	12	13
8	9	1	6	22	1	1	6
4	2	1	8	1	10	12	4
3	6	4	1	312	10	4	18
2	1	8	6	24	11	12	6
3	4	2	1	6	4	10	19
2	1	4	2	214	1	8	1
38	3	7	5	1069	1	18	9

APOTHECARIES WEIGHT.

EXAMPLES.

℥.	ʒ.	℈.	*grs.*	℔.	℥.	ʒ.	℈.	*gr.*
3	4	1	17	14	11	4	2	11
1	2	0	13	16	10	3	0	4
6	1	2	14	8	1	4	1	16
3	4	1	16	1	3	1	0	10
1	1	2	0	18	1	2	1	16
3	7	1	18	24	10	7	0	17
9	1	0	10	38	1	2	2	14
8	6	2	12	61	8	4	1	16
37	6	2	0	184	0	7	0	4

AVOIRDUPOISE WEIGHT.

EXAMPLES.

lb.	oz.	drs.		Tons	C.	qrs.	lb.
14	11	14		121	17	2	12
2	10	11		312	14	1	14
26	4	2		421	10	3	16
34	9	8		121	16	1	15
61	10	12		124	10	2	16
1	11	16		181	8	0	4
2	4	1		311	16	1	13
1	2	11		426	11	2	3
34	12	16		801	18	1	14
179	14	11		2824	4	0	23

CLOTH MEASURE.

EXAMPLES.

Yds.	q.	na.		Eng.	e.	na.		F.	e.	q.	n.
21	2	3		12	4	2		16	2	2	
2	1	1		31	3	1		41	1	3	
8	2	2		42	1	3		64	2	1	
14	3	1		12	2	1		31	1	2	
21	1	2		31	4	0		63	2	1	
4	1	3		8	1	2		5	1	2	
6	3	2		9	3	3		8	2	1	
12	1	2		14	3	1		16	1	2	
14	2	3		16	2	2		81	1	3	
107	0	3		180	1	3		327	5	1	

LONG MEASURE.

EXAMPLES.

Lea.	m.	fur.	p.		Yds.	f.	in.	b.c.
21	1	7	36		31	2	11	2
32	2	1	21		42	1	8	1
61	1	3	26		80	2	4	0
8	2	6	21		4	1	3	2
4	1	2	6		31	2	10	3
61	2	4	3		6	1	8	1
3	1	2	12		2	2	4	0
21	0	6	10		5	1	3	1
6	1	4	9		16	2	3	2
222	0	6	24		223	0	10	0

LAND MEASURE.

EXAMPLE.

A.	R.	P.
436	3	26
21	1	34
6	2	27
214	1	2
45	2	1
301	0	14
124	2	12
32	1	28
3	2	16
1184	2	0

WINE MEASURE.

EXAMPLES.

Tu.	p.	hhd.	ga.	qts.		Punch	gal.	qt.	pt.
12	1	1	14	2		14	14	2	1
4	1	1	27	3		7	32	3	1
10	1	0	61	1		24	51	2	1
6	1	1	42	2		14	14	1	1
2	0	0	26	3		49	36	3	1
13	1	1	4	2		37	17	1	1
6	0	0	36	3		8	62	3	1
12	1	0	2	1		21	2	1	1
3	0	1	15	2		24	6	0	0
73	0	0	42	3		200	71	0	0

WINCHESTER MEASURE.

EXAMPLES.

A. hhds.	gal.	qts.		B. hhds.	gal.	pts.
14	12	2		24	51	7
6	41	3		14	17	4
17	27	1		6	8	6
8	34	2		14	10	0
47	40	3		9	47	3
4	27	1		34	36	5
18	11	0		17	11	2
6	12	2		4	29	7
8	10	0		16	12	4
132	25	2		142	9	6

DRY MEASURE.

EXAMPLES:

qrs.	bu.	p.	gal.	cha.	bu.	p.
14	7	2	0	12	27	2
27	4	3	1	20	1	1
31	4	2	0	31	12	0
62	1	1	1	11	10	1
14	1	2	0	12	16	0
21	0	1	1	21	12	1
31	1	2	0	16	10	0
12	6	1	1	11	2	1
10	1	2	0	14	12	0
225	5	2	0	150	31	2

TIME.

EXAMPLES:

mo.	w.	d.	h.	d.	h.	m.	sec.
11	2	4	21	14	21	14	41
24	3	6	14	2	16	11	16
12	1	0	23	1	2	1	4
31	2	5	0	13	12	16	18
14	1	1	11	31	11	11	11
6	3	6	17	42	11	14	11
8	2	1	12	31	1	2	1
4	1	0	2	6	2	4	2
3	2	1	14	1	0	1	16
117	1	0	10	144	4	16	0

Note. You muſt write down the numbers of the ſame denomination under each other, in all the preceeding examples; and add them up as in addition of money; only take care to carry from one denomination to another, according to the table pertaining to each particular weight or meaſure.

Queſtions for exerciſe in Compound Addition.

QUESTION 1.

Frank Guzzle, Belch, and Soaking Dan,
Muſt have a bottle with Sir John,*
And, topers like, with Trot† prevail,
To fill a jug of nappy ale.
A jug! a mighty jug indeed!
A yard about, was fill'd with ſpeed;
Ten quarts it held, as neighbours tell,
Which pleas'd the landlord mighty well.

Three times b'ing fill'd, the topers they
Could ſcarce conduct themſelves away,
But paid the ſcore, which pleaſed Trot
To think what cuſtomers he'd got,
'Twas fifty-pence a-piece the ſhot.
What was the whole young Tyro tell,
Which pleas'd the landlord Trot ſo well?

By the pence table, 50 pence =4*s.* 2*d.* which ſet down three times, thus:

	s.	*d.*
Frank Guzzle -	4	2
Belch - - - - -	4	2
Soaking Dan - -	4	2
The ſum ſpent - -	12*s.*	6*d.*

Queſt. 2. How much is A (born 20 years ago) older than B, who will come into the world fourteen years hence?

To	20
Add	14
Anſwer	34 Years

Queſt. 3. A gentleman by will left the following legacies to be divided amongſt his children, viz. four ſons and three daughters: to Simon 300*l.* to Ralph 160*l.* to John 1*s.* to James 120*l.* to Suſan 93*l.* to Ruth 1*s.* and to Margery 430*l.* Query, the fortune the gentleman left?

*Sir John Barleycorn. †The Landlord.

To

To answer this Question, write down each one's fortune, and add them together thus:

		£.	s.	d.
Simon's	Share,	300	0	0
Ralph's		160	0	0
John's		0	1	0
James's		120	0	0
Susan's		93	0	0
Ruth's		0	1	0
Margery's		430	0	0
	Fortune left	1103	2	0

Quest. 4. A person was 16 years of age 29 years since; and it is said he will be drowned 21 years hence: pray in what year of his age will this happen?

His age 29 years since	=	16
His present age — 16	=	29
Years to come - - - -		21
Answer		66

Quest. 5. A gentleman dying, left his executor a sum not amounting to 2000*l.* to be so divided amongst his relations; that his father and mother, his son and his grandson, his brother and his daughter, should each receive a sum not less than 666*l.* 13*s.* 3*d.* Query, the scheme of kindred, and exact sum left?

Solution. Suppose two widows, A and B, no kin to each other, to be left each with a son, and that A's son marries B, and B's son marries A; and that A's son has a son by B, and A's son is the gentleman that leaves the money. This is the scheme of kindred, and to find the sum left, proceed thus:

	£.	s.	d.
To his father, who in this case is the same as his son,	666	13	3
To his mother, who in the same manner is his daughter,	666	13	3
To his grandson likewise, who is the same as his brother,	666	13	3
Sum left - -	1999	19	9

Quest. 6. A sheep-fold was robbed three nights successively; the first night half the sheep were stolen, and half a sheep more; the second night half the remainder were lost, and half a sheep more; the last night they took half what were left, and half a sheep more, by which time they were reduced to twenty; how many were there at first?

The number left = 20 Sheep.

21	Stolen the	3d	Night,
42		2d	
84		1st	

Answer - - 167 Sheep at first.

Quest. 7.

Quest. 7. Please to inform me, how much money I must send to my Baker; for twenty *Halfpenny* loaves, twenty *Penny* loaves; and twenty *Three - Halfpenny* loaves?

			s.	d.
20 Halfpenny loaves,	=		0	10
20 Penny ditto	=		1	8
23 Halfpenny ditto	=		0	$11\frac{1}{2}$
Answer	£.	0	3	$5\frac{1}{2}$

VIII. COMPOUND SUBTRACTION.

TEACHETH to find the difference between any two sums of divers denominations.

RULE. Subtract as in integers, only when the lower number in any denomination happens to be greater, borrow one, that is, add as many to the upper number as makes one of the next superior denomination, and then subtract the lower number, and set down the remainder; then carry 1, and add it to the lower number of the next denomination, and subtract as before.

EXAMPLES of MONEY

EXAMPLE 1.

	£.	s.	d.
From - -	19	14	$5\frac{1}{2}$
Take - -	12	16	$4\frac{1}{4}$
Remains -	6	18	$1\frac{1}{4}$
Proof - -	19	14	$5\frac{1}{2}$

To work this example, begin at the least denomination, saying 1 from 2, and there remains 1 farthing, which place under the line thus $\frac{1}{4}$; then subtract 4 from 5, and there remains 1 penny, which set down under its own denomination, and proceed to the shillings; 16 from 14 I cannot, 20 that I borrow to 14 is 34, 16 from 34, and there remains 18; then, because you borrowed 1 pound, or 20 shillings, say 1 that I borrowed and 2 is 3, 3 from 9 and there remains 6; 1 from 1 and there remains nothing; and the remainder is 6*l.* 18*s.* $1\frac{1}{4}$*d.* which add to the line above it, and the sum will be equal to the top line, which proves the work to be right.

OBSERVATION. Subtraction of all sorts and denominations, is performed after the very same manner as the preceding example, only you must borrow, and add or repay, according to each denomination; therefore to give any further explanations relating thereto would be only tautology.

E. 2.

	E. 2. £.	s.	d.	E. 3. £.	s.	d.
From -	32	16	8¼	42	10	6¼
Take -	12	14	10¾	21	19	8
Remains	20	1	9½	20	10	10¼
Proof -	32	16	8¼	42	10	6¼

E. 4.	£.	s.	d.
Borrowed - -	524	14	6
Paid at ſundry times -	12	5	1
	11	2	2
	36	1	0
	46	2	6
	31	1	8
	12	4	8
	2	1	6
	3	4	4
Paid in all - -	154	2	11
Remains unpaid -	370	11	7

E. 5.	£.	s.	d.
Lent - -	4861	14	8
Received at ſundry times - -	121	1	2
	21	2	4
	312	6	6
	2	1	2
	6	3	1
	82	4	4
	311	1	1
	8	2	1
Received in all -	864	1	9
Remains unpaid -	3997	12	11

E. 6. Suppoſe my half-year's rent is 20 guineas, and that I have laid out for the land-tax and other levies 8*l.* 18*s.* 8½*d.* and for ſeveral repairs 3*l.* 4*s.* 2*d.* what remains due to the landlord?

	£.	s.	d.	£.	s.	d.
Half-years rent - - - - -				21	0	0
Land-tax, &c. -	8	18	8½	12	2	10½
Repairs - - -	3	4	2			
Balance due to the landlord - - -				8	17	1½

E. 7. A carpenter's bill upon a farmer is 86*l.* 18*s.* 8*d.* out of which he has received in caſh 20*l.* in corn 6*l.* 12*s.* 2*d.* in coals 2*l.* 1*s.* 6*d.* and in cheeſe and bacon 12*s.* 6½*d.* what remains due to the carpenter?

		£.	s.	d.	£.	s.	d.
The carpenter's bill - - -					86	18	8
Paid in	Caſh - -	20	0	0	29	6	2½
	Corn - -	6	12	2			
	Coals -	2	1	6			
	Cheeſe, &c. -	0	12	6½			
Balance due to the carpenter - -					57	12	5½

WEIGHTS

WEIGHTS and MEASURES.

TROY WEIGHT.

	lb.	oz.	dwts.	grs.
From -	81	10	15	18
Take -	14	8	12	19
Remains	67	2	2	23

APOTHECARIES' WEIGHT.

	lb.	℥.	ʒ.	℈.	grs.
From -	38	10	1	2	4
Take -	2	8	2	1	12
Remains	36	1	7	0	12

AVOIRDUPOISE WEIGHT.

	Tons	*C.*	*qrs.*	*lb.*	*lb.*	*oz.*	*drs.*
From	36	18	2	26	26	0	8
Take	21	19	3	60	12	1	12
Rem.	14	18	3	20	13	14	12

CLOTH MEASURE.

	Yds.	*qr.*	*na.*	*Ell. Eng.*	*qrs.*	*na.*
From	326	2	3	38	4	2
Take	218	3	1	14	3	3
Rem.	107	3	2	24	0	3

LONG MEASURE.

	Lea.	*m.*	*fur.*	*p.*	*Yds.*	*f.*	*in.*	*b. c.*
From	281	1	7	26	36	2	8	2
Take	82	2	5	38	18	1	4	1
Rem.	198	2	1	28	18	1	4	1

LAND MEASURE.

	A.	*R.*	*P.*	*A.*	*R.*	*P.*
From	864	2	26	38	0	31
Take	318	1	18	21	3	24
Rem.	546	1	6	16	1	7

WINE MEASURE. A nobleman hath two cellars, the larger contains of ſeveral kinds of liquors 3 tons and 2 hogſheads, and the other 1 ton 3 hogſheads, 32 gallons, and 4 pints; how much liquor is there in the one more than the other?

	Tons.	*hhd.*	*gal.*	*pts.*
From - -	3	2	0	0
Take - -	1	3	32	4
Remains -	1	2	30	4
Proof - -	3	2	0	0

WINCHESTER MEASURE. A brewer delivers to his cuſtomers in one day 24 hogſheads and 16 gallons, in another day, 18 hogſheads and 48 gallons, what is the difference?

	hhds.	*gal.*
From -	24	16
Take -	18	48
Remains -	5	19
Proof	24	16

DRY MEASURE.

	Qu.	*qu.*	*p.*	*Cha.*	*bu.*	*p.*
From	18	14	2	22	26	0
Take	6	10	3	3	34	3
Rem.	12	3	3	18	27	2

TIME.

	Mo.	*w.*	*d.*	*h.*	*D.*	*h.*	*m.*	*ſec.*
From	18	2	6	21	8	14	46	31
Take	10	3	2	23	4	21	18	52
Remains	7	3	3	22	3	17	27	39

Questions for Exercise in Compound Subtraction.

Quest. 1. A horse in his furniture is worth 38*l.* 12*s.* out of it 16*l.* 15*s.* how much doth the price of the furniture exceed that of the horse?

In furniture - -	38	12
Out of it - -	16	15
Answer	21	17

Quest. 2. A boy was bound, by indentures, to serve his master seven years; and when he had accomplished 6 years, 6 months, 6 weeks, 6 days 6 hours, 6 minutes, and 6 seconds, pray how long had he to serve?

	Yrs.	*mo.*	*w.*	*d.*	*h.*	*m.*	*sec.*
From	7	0	0	0	0	0	0
Take	6	6	6	6	6	6	6
Answer -	0	5	1	0	17	53	54
Proof	7	0	0	0	0	0	0

Quest. 3. A snail in getting up a may-pole only 20 feet high, was observed to climb 8 feet every day; but every night it came down again four feet; in what time, by this method, did it reach the top of the pole?

Goes up the first day	8	feet
Comes down at night	4	
	4	1st day
Goes up the 2d day	8	
	12	
Comes down at night	4	
	8	2d day
Goes up the third day	8	
	16	
Comes down at night	4	
	12	3d day
Goes up the 4th day	8	
	20	feet

Answer, the fourth day at night

Quest. 4. A was born when B was 21 years of age; how old will A be, when B is 47; and what will be the age of B, when A is 60?

From	47	
Take	21	
Rem.	26	the age of A
To	60	
Add	21	
Sum	81	B's age

Quest. 5.

Quest 5.

A lady left her daughter fair
Twelve thousand pounds in gold,
To be distributed with care,
As underneath is told:
First to a niece there must be paid
Just fourteen hundred pound.
And half that sum to parson Wade,
To make his glass go round;
And to her maid miss Nancy Hare,
Three hundred pounds in cash,
Who swells with pride, and such an air!
She apes my lady Flash.
The stew'rd and butler each must have,
Just twice two hundred more,
And to a tenant, farmer Brave,
In shining pounds six score.
The greasy cook, each other maid,
B'ing three in number, they
Had twenty guineas each one paid,
To make them fine and gay;
The coachman Ralph and footman Dan,
*Ten guineas and a crown,**
Which made them toss about the can,
In ev'ry market town.
When all these legacies were paid,
What did remain behind
For miss, that blooming peerless maid,
Whose virtues made her kind?

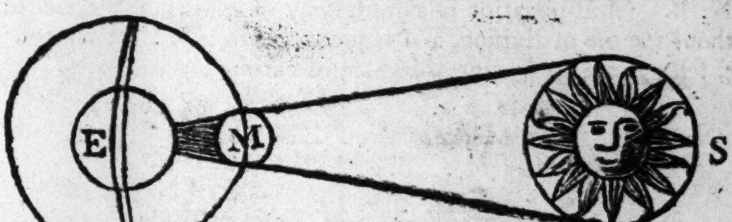
				£.	s.	d.
Sum left				12000	0	0
Niece	1400	0	0			
Parson	700	0	0			
Miss Nancy Hare	300	0	0			
Steward and butler	400	0	0			
Farmer Brave	120	0	0			
Cook, and the two other maids	63	0	0			
Coachman and footman	21	10	0			
				3004	10	0
The daughter's share				£. 8995	10	0

Quest. 6. If the mean distance between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousands; how far are those two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? And in another of the moon when the earth is in a line between her and him?

Suppose E the earth, M the moon, and S the sun; then the eclipse of the sun will be represented by fig. 1, and that of the moon by fig. 2.

Therefore 81000000—240000=80760000= S M fig. 1, or the distance these two luminaries are asunder, in an eclipse of the sun.

FIGURE 1.



*A-piece.

Likewise

Likewise $81000000+240000=81240000=$ S M fig. 2, or the diſtance theſe two luminaries are aſunder in an eclipſe of the moon.

FIGURE 2.

Queſt. 7. B, born 161 years ago, died when C was 47 years of age, who it ſeems came into the world 180 years ſince, and out-lived B 43 years; the ſum of their ages is required?

Firſt	180		Then	161
	47			133
	133	Years ſince B died	Years	28 B's age.

Add $\begin{cases} 47 \\ 43 \end{cases}$

90 C's age. Then $90+28=118$ the anſwer.

IX. COMPOUND MULTIPLICATION.

TEACHETH to multiply (by one common multiplier) any ſum or number conſiſting of divers denominations.

CASE 1. When the given quantity doth not exceed 12.

RULE 1. Write the multiplier under the loweſt denomination of the multiplicand.

2. Begin at the loweſt denomination, and multiply it by the given number, and ſee how many of the next denomination is contained in the product; ſet down the odds, and carry ſo many to the next: Then multiply the next denomination, adding what you carried, and ſet down the odds; proceed thus till all be multiplied.

N. B. Multiplication is a ſhort way of working the rule of three, without the uſe of diviſion, and is preferable to any other method in buying, ſelling and computing the value of various commodities.

EXAMPLE 1.	£.	s.
Multiply	10	10
By		3
Product	31	10

To work this example, ſay 3 times 10 is 30 ſhillings, or 1*l.* 10*s.* write down 10 and carry 1. Then ſay 3 times 10 is 30, and one is 31 pounds, which ſet down, and the anſwer is 31*l.* 10*s.* as appears by the work.

E. 2.

E. 2. What will 4 pounds of ſugar come to at $5\frac{1}{4}d.$ per pound?

	d.
	$5\frac{1}{4}$
	4
Anſwer	1 9

E. 3. What come 5 hundred of cheeſe to, at 1*l.* 12*s.* $6\frac{1}{2}d$ per Cwt,

	£.	s.	d.
	1	12	$6\frac{1}{2}$
			5
Anſwer	8	2	$8\frac{1}{2}$

Note. Cheeſe-factors, and many other dealers, who buy goods wholeſale, are allowed 120 pounds to 1 *Cwt.* but ſell them out at 112 pounds per *Cwt.*

E. 4. What will 6 ells of holland come to, at 6*s.* 10*d.* per ell?

		s.	d.
		6	10
			6
Anſwer £.	2	1	0

E. 5. What will 8 gallons of brandy come to, at 2*l.* 1*s.* 3*d.* per gallon?

	£.	s.	d.
	2	1	3
			8
Anſ.	£. 16	10	0

E. 6. What come 10 anchors of rum to, at 3*l.* 4*s.* 2*d.* per anchor?

	4	4	2
			10
Anſ. £.	32	1	8

E. 7. What will 11 barrels of ſmall beer come to, at 10*s.* 6*d.* per barrel?

		10	6
			11
Anſ.	£. 5	15	6

E. 8. What will 12 dozen of candles come to, at 6*s.* 8*d.* per dozen?

		6	8
			12
Anſwer	£. 4	0	0

CASE 2. When the given quantity exceeds 12, and is ſuch a number that any two figures (in the multiplication table) being multiplied together, will produce it;

RULE. Multiply the given price by one of thoſe numbers, and that product by the other, and if you make uſe of any more numbers, proceed in like manner, and the final product will be the anſwer.

E. 9. What will 15 buſhels of wheat come to, at 6*s.* $9\frac{1}{2}d.$ per buſhel?

		6	$9\frac{1}{2}$	
			5	$\times 3 = 15$
	1	13	$11\frac{1}{2}$	
			3	
Anſwer	£. 5	1	$10\frac{1}{2}$	

To work the preceding example, ſay 5 times 2 is 10 farthings, or $2\frac{1}{2}d.$ ſet down $\frac{1}{2}$, and carry 2; then ſay 5 times 9 is 45, and 2 is 47 pence, =3*s.* 11*d.* ſet down 11, and carry 3; then 5 times, 6 is 30 ſhillings, and 3 is 33, =1*l.* 13*s.* The firſt product being finiſhed, multiply that by the other number, ſaying, 3 times 2 is 6 farthings, or $1\frac{1}{2}d.$ ſet down $\frac{1}{2}$ and carry 1, and ſay 3 times 11 is 33, and 1 is 34 pence, = 2*s.* 10*d.* ſet down 10 and carry 2; then 3 times 3 is 9, and 2 is 11, ſet down 1 and carry 1, ſaying 3 times 1 is 3, and 1 is 4,

4 ten ſhillings or 2 pounds; then 3 times 1 is 3, and 2 is 5*l.*—and the anſwer is 5*l.* 1*s.* 10½*d.* as appears by the work.

E. 10. What will 18lb, of butter come to, at 4½*d.* per pound?

d.
4½
$3 \times 6 = 18$
1 1½
6
Anſwer 6 9

E. 11. What will 30lb. of cheeſe come to, at 3¼ per pound?

d.
3¼
$10 \times 3 = 30$
2 8½
3
Anſ. 8 1½

E. 12. What will 45lb. of bacon come to, at 5¾*d.* per pound?

5¾
$9 \times 5 = 45$
4 3¾
5
Anſ. £.1 1 6¾

E. 13. How much is the ſterling value of 50 moidores, at 27*s.* each?

1 7
$10 \times 5 = 50$
13 10
5
Anſ. £. 67 10

E. 14. What do 56 hogs come to, at 1*l.* 5*s.* 4*d.* per hog?

1 5 4
$8 \times 7 = 56$
10 2 8
7
Anſ. £. 70 18 8

E. 15. What come 64 firkins of butter to, at 1*l.* 8*s.* per firkin?

1 8
$8 \times 8 = 64$
11 4
8
Anſ. £. 89 12

E. 16. What come 72 reams of paper to, at 13*s.* 8*d.* per ream?

13 8
$9 \times 8 = 72$
6 3 0
8
Anſ. £.49 4 0

E. 17. What is the price of 80 yards of Iriſh cloth, at 10½*d.* per yard?

10½
$10 \times 8 = 80$
8 9
8
Anſ. £. 3 10 0

E. 18. What will 88 gallons of ale come to, at 1*s.* 4*d.* per gal.?

1 4
$11 \times 8 = 88$
14 8
8
Anſ. £. 5 17 4

E. 19. What come 96 buſhels of barley to, at 3*s.* 2½*d.* per buſhel?

3 2½
$12 \times 8 = 96$
1 18 6
8
Anſ. £. 15 8 0

E. 20.

E. 20. What will 144lb. of tea come to, at 4*s.* 6*d.* per pound?

	4	6	
		12×12=144	
2	14	0	
		12	
Anf. £.32	8	0	

E. 21. How much will 132 dozen feet of fawing come to, at $4\frac{1}{2}$*d.* per dozen?

	$4\frac{1}{2}$	
	12×11=132	
	4	6
		11
Anf. £. 2	9	6

Case 3. When the given quantity cannot be produced by the multiplication of two fmall numbers,

RULE. Find the neareft number to it lefs, by which multiply as before, then for what is wanting multiply the price by that number, and add it to the laft product and the total will be the anfwer required.

E. 22. What come 17 *Cwt.* of raifins to, at 1*l.* 4*s.* 2*d.* per *Cwt*?

1	4	2
		4×4+1=17
4	16	8
		4
19	6	8=16
1	4	2=1
Anf. £. 20	10	10

E. 23. What come 29lb. of fine hyfon tea to, at 19*s.* 6*d.* per lb?

	19	6
		4=7+1=29
3	18	0
		7
27	6	0=28
	19	6=1
Anf. £. 28	5	6

E. 24. What will 37 grofs of buttons come to, at 1*l.* 10*s.* $6\frac{1}{2}$*d.* per grofs?

1	10	$6\frac{1}{2}$
		6×6+1=37
9	3	3
		6
54	19	6=36
1	10	$6\frac{1}{2}$=1
Anf. £. 56	10	$0\frac{1}{2}$

E. 25. What will 42 yards of fine holland come to, at 10*s.* $2\frac{1}{2}$*d.* per yard?

	10	$2\frac{1}{2}$
		5×8+2=42
2	11	$0\frac{1}{2}$
		8
20	8	4=40
1	0	5= 2
Anf. £. 21	8	9

E. 26. Bought 65 fheep, at 1*l.* 5*s.* 4*d.* per fheep, what do they come to?

1	5	4
		8×8+1=65
10	2	8
		8
81	1	4=64
1	5	4= 1
Anf. £. 82	6	8

E. 27. What come 75 dozen of foap to, at 6*s.* $3\frac{1}{2}$*d.* per dozen?

	s.	*d.*
	6	$3\frac{1}{2}$
		9×8+3=75
2	16	$7\frac{1}{2}$
		8
22	13	0 =72
0	18	$10\frac{1}{2}$= 3
Anf. £. 23	11	$10\frac{1}{2}$

E. 28.

E. 28. What will 86 dozen of men's common hose come to, at 2*l.* 4*s.* 2*d.* per dozen?

	2	4	2
			$8 \times 11 - 2 = 86$
	17	13	4
			11
	194	6	$8 = 88$
	4	8	$4 - 2$
Ans. £.	189	18	4

E. 29. What will 104 copies of Taylor's Complete System of Arithmetic come to, at 5*s.* each?

		5	
		$10 \times 10 + 4 = 104$	
	2	10	
		10	
	25	0	$= 100$
	1	0	$= 4$
Ans. £.	26	0	

In the 28th example I have taken the two nearest numbers above the given quantity, whose product is 88; by which I found the value of that number of dozens of hose to 194*l.* 6*s.* 8*d.* from which I subtracted twice the price of one dozen to find the price of 86, the answer to the question.

CASE 4. When the given quantity consists of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

RULE. Divide the upper line (the price of one) by 4 for $\frac{1}{4}$, by 2 for $\frac{1}{2}$; and for $\frac{3}{4}$, by 2 first for $\frac{2}{4}$, then divide that quotient by 2 for $\frac{1}{4}$; add them to the product, and the sum will be the answer required.

E. 30. What will $16\frac{1}{2}$ lb. of raisins come to, at $6\frac{1}{2}$*d.* per pound?

$\frac{1}{2}$)		$6\frac{1}{2}$	
		$4 \times 4 = 16$	
	2	2	
		4	
	8	8	$= 16$
		$3\frac{1}{4}$	$= \frac{1}{2}$
Ans.	8	$11\frac{1}{4}$	

E. 31. What come $36\frac{1}{4}$ tons of hay to, at 3*l.* 4*s.* 6*d.* per ton?

4)	3	4	6	
			$6 \times 6 = 36$	
	19	7	0	
			6	
	116	2	0	$= 36$
		16	$1\frac{1}{2}$	$= \frac{1}{4}$
Ans. £.	116	18	$1\frac{1}{2}$	

E. 32. Bought $8\frac{3}{4}$ butts of strong beer, at 8*l.* 4*s.* 8*d.* per butt;

2)	8	4	8	
			8	
	65	17	4	$= 8$
2)	4	2	4	$= \frac{1}{2}$
	2	1	2	$= \frac{1}{4}$
Ans. £.	72	0	10	

When your given quantity happens to be very large, so as to consist of hundreds, thousands, &c. it may be wrought by the continual product of three or more numbers; and if your given quantity is thousands, multiply the price of 100 by 10 for 1000, and the product by the number of thousands, and for the lower quantities proceed as before. The following examples will make this sufficiently clear to be understood.

E. 33. What will 120 ounces of fine silver come to, at 5*s.* 3*d.* per ounce?

	5	3
		10×4×3=120
2	12	6
		4
10	10	0
		3
Answer £.31	10	0

E. 35. What comes 1 cwt. of hops to, at 1*s.* $2\frac{1}{2}$*d.* per pound?

	1	$2\frac{1}{2}$
		8×7×2=112
	9	8
		7
3	7	8
		2
Anf. £. 6	15	4

E. 34. What will 8462℔. of iron come to, at $2\frac{3}{4}$*d.* per pound?

		$2\frac{3}{4}$	
		10	
	2	$3\frac{1}{2}$	= the price of 10℔.
		10	
1	2	11	= 100
		10	
11	9	2	= 1000
		8	
91	13	4	= 8000
4	11	8	= 400
	13	9	= 60
		$5\frac{1}{2}$	= 2
£. 96	19	$2\frac{1}{2}$	= 8462

Answer 96*l.* 19*s.* $2\frac{1}{2}$*d.*

WEIGHTS and MEASURES.

EXAMPLE 1. A filverfmith has 4 bars of filver, each 4℔. 6*oz.* 8 *dwts.* 3 *grs.* what is the weight of the whole?

	lb.	*oz.*	*dwts.*	*grs.*
	4	6	8	3
				4
Anfwer	18	1	12	12

Note. Weights, meafures, &c. are multiplied after the fame manner as money, only you muft remember to carry according to each denomination that refpectively pertain thereto.

E. 2. An apothecary has 6 mixtures, each 3 pounds, 1 ounce, 3 drams, 1 fcruple, and 10 grains, what is the weight of the whole?

	℔.	℥.	ʒ	℈	*grs.*
	3	1	3	1	10
					6
Anfwer	18	8	5	0	0

E. 3. What is the weight of 10 cafks of raifins, when each cafk weighs 5 *cwt.* 2 *qrs.* 18 *lb.*?

	C.	*qrs.*	℔.
	5	2	18
			10
Anfwer	56	2	12

E. 4. If a perſon hath 12 bales of ſilk, each 2 pounds, 10 ounces, and 4 drams, what is the whole weight?

	℔.	oz.	drs.
	2	10	4
			12
Anſwer	31	11	0

E. 6. Multiply 30 miles 2 furlongs, and 18 poles, by 2.

	Miles	*fur.*	*p.*
	30	2	18
			2
Anſwer	60	4	36

E. 8. Multiply 68 acres, 2 roods, and 4 perches, by 9.

	A.	*r.*	*p.*
	68	2	4
			9
Anſwer	616	2	36

E. 10. Multiply 82 laſts, 6 qrs. 4 buſhels, and 1 peck, by 7?

	Laſts.	*qrs.*	*bu.*	*p.*
	82	6	4	1
				7
Anſwer	578	5	5	3

E. 11. Multiply 9 months, 2 weeks, 4 days, 12 hours, and 4 minutes, by 12?

	Mo.	*w.*	*d.*	*h.*	*m.*
	9	2	4	12	4
					12
Anſ.	115	3	5	0	48

E. 5. A ſhopkeeper bought 40 pieces of Iriſh cloth, each piece containing 42 yards, 2 quarters, 2 nails, what quantity did he buy?

	Yds.	*qrs.*	*na.*	
	42	2	2	
			4	× 10 = 40
	170	2	0	
			10	
Anſwer	1705	0	0	

E. 7. Multiply 300 yards, 1 foot, and 4 inches, by 8.

	Yds.	*ft.*	*in.*
	300	1	4
			8
Anſwer	2403	1	8

E. 9. Multiply 4 B. hogſheads, 3 gallons, and 6 pints, by 4.

	B. hhd.	*gal.*	*pts.*
	4	3	6
			4
Anſwer	16	15	0

Queſtions for exerciſe in Compound Multiplication.

Queſt. 1. If I ſpend $1\frac{1}{2}d.$ per day, how much is that per year, allowing 365 days to the year?

		$1\frac{1}{2}$	10 × 10 × 3 + 60 + 5 = 365
	1	3	
		10	
	12	6	
		3	
1	17	6	= 300
	7	6	= 60
		$7\frac{1}{2}$	= 5
2	5	$7\frac{1}{2}$	= 365 Anſwer.

Queſt. 2. The *Silk-mill* at *Derby* contains 26586 wheels, and 97746 movements, which wind off or throw 73726 yards of ſilk every time the great water wheel, which gives motion to all the reſt, turns round, which is three times a minute; the queſtion is, how many yards of ſilk may

may be thrown by this machine in a day reckoning ten hours to a day's work? And how many in the compass of a year, deducting for Sundays and holidays 63 days, provided no part of it stands still?

73726	
3	
221178	Yards in a minute
60	Minutes in an hour
13270680	
10	Hours to a day
132706800	Yards in a day
302	
265413600	
3981204000	
Answer 40077453600	Yards in a year.

Quest. 3. There are 7 chests of drawers, in each of which are 18 drawers, and in each of these are 6 divisions, in each of which there is 16*l.* 6*s.* 8*d.* how much is there in the whole?

£. s. d.	
16 6 8	
6	
98 0 0	in each drawer
18	
1764	in each chest
7	
Answer 12348	Pounds in the whole.

Quest. 4. A lady's caterer bought 10 birds of two sorts, viz. turkies and geese, for 24 shillings; the turkies cost 4 shillings, and the geese 2 shillings a-piece; how many did he buy of each sort?

2 Turkeys	8 Geese
4	2
8*s.*	16*s.* + 8 = 24*s.* the price.

Or thus; 2 × 4 = 8*s.* the price of the turkies, and 8 × 2 = 16*s.* the price of the geese; consequently 2 turkies + 8 geese = 10, the answer.

Quest. 5. Suppose a gentleman has an estate of 800*l.* per annum, and he pays land-tax 150*l.* also for repairs 38*l.* 14*s.* 2*d.* what is his neat estate per annum?

Estate per annum - - - - -		800 0 0
Land-tax - - -	150 0 0	188 14 2
Repairs, &c. - - -	38 14 2	
Neat estate per annum - - - - -		611 5 10

Quest. 6. In a company S had 3*l.* 17*s.* 2*d.* more than T, who had 6 guineas less than R, who had within 16*s.* 8*d.* as much as W, who was known to have 100 guineas, wanting 10 marks, of 13*s.* 4*d.* each; pray what money had they among them?

	£.	s.	d.		£.	s.	d.
First, 100 guineas =	105	0	0				
Ten marks - =	6	13	4				
W had - - -	98	6	8	- - - - -	98	6	8
Subtract - - -	0	16	8				
R had - - -	97	10	0	- - - - -	97	10	0
Subtract - - -	6	6	0				
T had - - -	91	4	0	- - - - -	91	4	0
Add - - - -	3	17	2				
S had - - -	95	1	2	- - - - -	95	1	2
				Answer - £.	382	1	10

X. COMPOUND DIVISION.

TEACHETH to divide by one common divisor, either a simple or compound number, into any proposed number of equal parts, whereof each shall be a compound number.

CASE. When the divisor doth not exceed 12.

RULE. Begin at the highest denomination, which divide by the given divisor, and set the answer in the quotient, which must be of the same denomination; what remains must be multiplied by the number of parts in the next inferior denomination, and added to the given number of that denomination, and then divide as before. Proceed thus through all the denominations.

EXAMPLE 1. Suppose there was 1*l.* 15*s.* $1\frac{3}{4}$*d.* to be divided amongst 7 men; what is each man's share?

	£.	s.	d.
	7)1	15	$1\frac{3}{4}$
Answer - -		5	$0\frac{1}{4}$
			7
Proof - -	1	15	$1\frac{3}{4}$

To work this example, ask how oft 7 in 1? never a time; then 1*l.*= 20*s.* added to 15*s.* is 35*s.* Then ask how oft 7 in 35? 5 times, put down 5 in the quotient, and say, 35 from 35, and there remains nothing; then ask, how oft 7 in 1? never a time, and there remains 1, one penny is 4 farthings, and 3 is 7; how often 7 in 7? once; set down 1 farthing, and the answer is 5*s.* $0\frac{1}{4}$*d.* as appears by the work.

E. 2. Bought 4 *Cwt.* of cheese, for which I gave 8*l.* 10*s.* 4*d.* at what rate did I give per *Cwt*?

	4)8	10	4
Answer £.	2	2	7

E. 3. If 10 dozen of candles cost 3*l.* 17*s.* 1*d.* what does 1 dozen cost at that rate?

	10)3	17	1
Answer £.	0	7	$8\frac{1}{2}$

CASE

CASE 2. When the diviſor exceeds 12, and is ſuch a number that any two figures (in the multiplication table) being multiplied together will produce it.

RULE. Divide by its component parts, as in diviſion of integers. See Section 5, Caſe 4.

EXAMPLE 1. Let it be required to divide 45*l.* 12*s.* 8*d.* into 16, equal parts?

$$4\times4=16\left\{\begin{array}{lll} 4)45 & 12 & 8 \\ \hline 4)11 & 8 & 2 \end{array}\right.$$

Anſwer £. 2 17 $0\frac{1}{2}$

E. 2. Divide 7*l.* 6*s.* equally amongſt 24 perſons?

$$6\times4=24\left\{\begin{array}{lll} 6)7 & 6 & 0 \\ \hline 4)1 & 4 & 4 \end{array}\right.$$

Anſwer £. 0 6 1

E. 3. If I ſell 81 buſhels of wheat for 30*l.* 7*s.* 6*d.* what is that per buſhel?

$$9\times9=81\left\{\begin{array}{lll} 9)\ 30 & 7 & 6 \\ \hline 9)\ 3 & 7 & 6 \end{array}\right.$$

Anſwer £. 0 7 6

E. 4. If I ſell 100 quarters of barley for 90*l.* what is that per quarter?

$$10\times10=100\left\{\begin{array}{lll} 10)90 & 0 & 0 \\ \hline 10)\ 9 & 0 & 0 \end{array}\right.$$

Anſwer £. 0 18 0

CASE 3. When the diviſor cannot be produced by the multiplication of two ſmall numbers, divide as in Section 4.

E. 1. Divide 21*l.* 17*s.* equally amongſt 17 perſons?

£. s. £. s. d.
17)21 17 (1 5 $8\frac{1}{4}$ Anſw.
17
4
20
17)97(5*s.*
85
12
12
17)144(8*d.*
136
8
4
17)32($\frac{1}{4}$*qrs,*
17
15 Remainder.

E. 2. Required to divide 214*l.* 17*s.* 3*d.* equally amongſt 34 perſons?

£. s. d.
34)214 17 3(6*l.*
204
10
20
34)217)6*s.*
204
13
12
34)159(4*d.*
136
23
4
34) 92($\frac{1}{2}$
68
24 Remainder.

Anſw. 6*l.* 6*s.* $4\frac{1}{2}$*d.* $\frac{24}{34}$ each.

E. 3.

E. 3. If 1 *Cwt.* of cheeſe coſt 1*l.* 15*s.* 4*d.* what is the price of 1 lb?

$$2\times7\times8=112\begin{cases}2)1 \quad 15 \quad 4 \\ 7)0 \quad 17 \quad 8 \\ 8)0 \quad 2 \quad 6\frac{1}{4}\end{cases}$$

Anſwer £. 0 0 $3\frac{3}{4}$

E. 4. If 52 tons of hay coſt 167*l.* 14*s.* what will 1 ton coſt?

```
52)167   14(3l.
   156
   ---
    11
    20
   ---
52)234(4s:
   208
   ---
    26
    12
   ---
52)312(6d.
   312
   ---
     0
```

Anſwer 3*l.* 4*s.* 6*d.* per ton.

E. 5. Let it be required to divide 200*l.* 9*s.* 1*d.* into 104 equal parts?

```
104)200   9   1(1l.
    104
    ---
     96
     20
    ----
104)1929(18s:
    104
    ---
     889
     832
     ---
      57
      12
     ---
104)685(6d.
    624
    ---
     61
      4
    ---
104)244(½
    208
    ---
     36
```

Anſwer 1*l.* 18*s.* $6\frac{1}{2}$*d.*

WEIGHTS and MEASURES.

EXAMPLE 1. Divide 16 *lb.* 1*oz.* 15*dwts.* 8*grains* by 2?

	lb.	*oz.*	*dwts.*	*grs.*
	2(16	1	15	8
Anſwer	8	0	17	16

E. 2. Divide 140 acres, 2 roods, 26 poles, by 12?

	A.	*r.*	*p.*
	12)140	2	26
Anſwer	11	2	35—6

Diviſion of weights and meaſures, &c. is performed in the ſame manner as the above (with eaſe and accuracy) paying a due regard to their ſeveral denominations.

Queſtions

Questions for exercise in Compound Division.

Quest. 1. *Says Hodge to his grandmother, Grannum, I see,*
That the money and purse you've given to me,
Is worth sixteen and eight-pence, its well its no worse,
For the cash is in value worth nine times the purse;
What sum, then, had Roger, now Tyro, pray tell,
Which tickled his fancy, and pleas'd him so well?

		s.	d.
		16	8
			9
	10)7	10	0
Answer	0	15	0

	s.	d	
Then from -	16	8	
Take - -	15	0	
Remains	1	8	Price of the purse

Quest. 2. The Spectator's club of fat people, though it consisted but of 15 persons, is said to weigh no less than 3 tons; how much on an equality was that per man?

$$\begin{array}{r} 3 \\ 20 \\ \hline 5)\ 60 \\ \hline 3)\ 12 \\ \hline \end{array}$$

Answer - 4 Cwt. each man

Quest. 3. The remainder of a division is 325, the quotient 467, the divisor is 43 more than the sum of both; what is the dividend?

$$\begin{array}{lrl} & 325 & \\ & 467 & \\ & 43 & \\ \hline \text{Multiply} & 835 & \text{Divisor} \\ \text{by -} & 467 & +325 \text{ the rem.} \\ \hline & 5850 & \\ & 5012\ & \\ & 3343\ \ & \\ \hline \text{Answer} & 390270 & = \text{Divisor.} \end{array}$$

Quest. 4. By selling 240 oranges at 5 for two-pence, half of which cost me two a penny, and the other half three a penny, I evidently lost a groat; pray how comes that about?

First, $\begin{array}{r} 240 \\ 2 \\ \hline 5)480 \\ \hline \end{array}$ Secondly, $\begin{array}{r} 2)120 \\ \hline 60d. \\ \hline \end{array}$ $\begin{array}{r} 3)120 \\ \hline 40d. \\ \hline \end{array}$

96 = what sold for. Then 60+40=100 what bought for.
And 100 — 96 = 4*d.* the money lost.

Quest. 5

Quest. 5. What difference will there be to the Proprietors of an Aqueduct, between doubling an expence, and halving a profit.

Suppose the expence or profit to be 2.
Then $2 \times 2 = 4$ double the expence.
And $2 \div 2 = 1$ half the profit.
Answer, difference 3, or as 4 to 1.

Quest. 6. *Pray can you on a Number fix,*
Which multiply'd by seven times six;
And then divide by thrice eleven,
Shall give the Quotient twelve times seven.

First $7 \times 12 = 84$, and $84 \times 33 = 2772$:
Then $2772 \div 33 = 83$; also $2772 \div 42 = 66$ the number reqd.

Quest. 7. Subtract 30079 out of fourscore and thirteen Millions, as often as it can be found, and say what the last Remainder exceeds or falls short of 21180?

Fourscore and thirteen Millions $= 93000000$.
Then $93000000 \div 30079 = 3091$, and the Remainder is 25811, from which deduct 21180, leaves 4631 excess, the answer.

Quest. 8. What number multiplied by 72084, will produce 5190048 exactly?

$5190048 \div 72084 = 72$ the Number required.

Quest. 9. Suppose a person by trading can clear 4894*l.* 2*s.* $3\frac{3}{4}$*d.* in $13\frac{1}{2}$ Years what is his yearly increase of fortune?

Years		*£.*	*s.*	*d.*
$13\frac{1}{2}$		4894	2	$3\frac{3}{4}$
2				2
27	9)	9788	4	$7\frac{1}{2}$
	3)	1087	11	7
Answer	£.	362	10	$6\frac{1}{2}$

Quest. 10. I would plant 2072 Elms, in 14 rows, twenty-five Feet asunder: how long must the Grove be?

First, $2072 \div 14 = 148$ Elms in each row; then $148 - 1 = 147$ vacancies, and $147 \times 25 = 3675$ Feet, or 1225 Yards, the length of the Grove required.

XI.

XI. REDUCTION.

TEACHETH to bring two or more numbers of different denominations, into one denomination; or it ſerveth to change or alter numbers, money, weight, meaſure, or time, from one demonation to another, and is generally performed by multiplication, or diviſion, as in Section VI.

EXAMPLES of MONEY.

E. 1. In 20 pounds, how many ſhillings, pence, and farthings?

20
20
400 Shillings
12
4800 Pence
4
19200 Farthings

E. 2. How many pounds in 19200 farthings?

4)19200
12) 4800
2|0) 40|0
Anſwer £. 20

E. 3. In 321*l.* 14*s.* 6¾*d.* how many farthings?

321 14 6¾
20
6434
12
77214
4
Anſwer 308859 Farthings

E. 4. In 298859 farthings, how many pounds?

4)298859
12) 74714 — 3
2|0) 622|6 — 2
Anſwer £. 311 6 2¾

The third example is multiplied the ſame as the firſt, and the 14*s.* 6¾*d.* are taken in, in their proper places, viz. the 14 to the product of ſhillings, the 6 in the pence, and the ¾ in the farthings.

E. 5. In 12*l.* 10*s.* 8*d.* how many four-pences?

12 10 8
20
250
12
4)3008
Anſwer 752 Four-pences

E. 6. Let it be required to reduce 752 four-pences to pounds?

752
4
12)3008
2|0) 25|0 — 8
Anſwer £. 12 10 8

E. 7. In 48 guineas, how many ſhillings, pence, and farthings?

48
21
48
96
1008 Shillings
12
12096 Pence
4
48384 Farthings

E. 9. In 4321*l.* 14*s.* 0½*d.* how many pieces of 13½*d.* per piece?

4321 14 0½
20
86434
12
1037208
4
qrs. 54 { 9)4148834
6) 460981 — 5 }
76830 — 1 } = 14

Anſwer 76830 pieces of 13½*d.* each, and 14 *qrs.* remains.

E. 11. How many half-crowns, crowns, and pounds, are there in 14400 pence?

3|0)1440|0
2) 480 Half-crowns
4) 240 Crowns
60 Pounds

E. 13. In 342*l.* 18*s.* how many ſhillings and moidores?

324 18
20
3 × 9 = 27 { 3)6858 Shillings
9)2286
254 Moidores

E. 8. In 48384 farthings, how many guineas?

4)48384
12)12096
3 × 7 = 21 { 3) 1008
7) 336
Anſwer 48 Guineas

E. 10. In 60*l.* how many crowns, half-crowns, and pence?

60
4
240 Crowns
2
480 Half-crowns
30
14400 Pence

E. 12. How many crowns, half-crowns, and ſhillings, are there in 426*l.* 15*s.* 6*d.* and of each an equal number?

s.	*d.*
5	0
2	6
1	0
8	6
12	
102	

£. *s.* *d.*
426 15 6
20
8535
12
102)102426(1004
102
426
408
18

Anſ. 1004 pieces, and 18*d.* rem.

E. 14. Reduce 480 guineas to ſhillings, crowns, and pounds?

Firſt 480 × 21 = 10080 Shillings
Then 5)10080
4) 2016 Crowns
504 Pounds

OF

OF COINS.

To reduce foreign and Engliſh coins to pounds ſterling.

RULE. Multiply the given number by the loweſt denomination of the price or value of 1; and divide the product by ſuch terms as will bring out the value in pounds.

EXAMPLE 1. In 1178 dollars, at 4*s.* 3*d.* each, how many pounds ſterling?

```
s. d.      1178 Dollars
4  3         51 Pence in 1 dollar
12         ----
--         1178
51         5890
          -----
       12)60078
          -----
       2|0)500|6 - 6
          -----
```

Anſwer £. 250 6 6

E. 2. In 340 piſtoles, each 17*s.* 6*d.* how many pounds ſterling?

```
s. d.      340
17 6       210
12        ----
---       3400
210       680
         -----
      12)71400
         -----
      2|0)595|0
         -----
```

Anſwer £. 297 10

Note. After the ſame manner may any foreign coin be brought into Engliſh ſterling.

To reduce pounds ſterling into foreign and Engliſh coin.

RULE. Reduce both the ſterling money and foreign coin into their loweſt denomination; then divide one by the other, and the quotient will be the anſwer.

E. 1. A merchant is to pay 296*l.* 12*s.* 3*d.* with dollars of 4*s.* 3*d.* each, how many will do it?

```
4  3       296  12  3
12          20
--         ----
51         5932
             12
           ----
      51)71187(1305 Anſw.
         51
         ---
         201
         153
         ---
          488
          459
          ---
           297
           255
           ---
            42
```

E. 2. In 774*l.* 18*s.* 4*d.* how many florins, at 3*s.* 2*d.* each?

```
s. d.      £.  s. d.
3  2       774 18 4
12          20
--        -----
38        15498
             12
          -----
      38)185980(4894 Anſ.
         152
         ---
         339
         304
         ---
          358
          342
          ---
           160
           152
           ---
             8
```

E. 3. How many marks, each 13*s.* 4*d.* are in 248*l.* 9*s.* 2*d.*?

```
 s. d.                              £.   s. d.
 13 4                               248  9 2
 12                                  20
 ---                               ----
 160                               4969
 ---                                 12
                                 1|0)5962|0
                                 4)5962
         10×4×4=160              4)1490—2
                                 372—2  = 10d.
              Answer  -
```

To reduce one kind of coin into another.

RULE. Divide one by the other, in their loweſt terms, and the quotient will be the anſwer.

EXAMPLE 1. How many moidores are equal to 198 guineas?

```
                      198
                       21
                      ---
                      198
                      396
                     ----
Shill. in a moidore  9)4158
     9×3=27          3) 462
                     ----
              Anſwer  154
```

E. 2. How many crowns, 5*s.* 4*d.* each, are in 474 piſtoles of 18*s.* 6*d.* each?

```
 s. d.   s. d.
 5 4     18 6      474 piſtoles
 12      12        222d. in 1 piſtole
 --      ---       ---
 64      222       948
 --      ---       948
                   948
              8)105228
     64       8)13153—4
    Crowns      1644—1  = 12d.
```

Having ſufficiently ſhewn how money is changed from one denomination to another, I ſhall now proceed to weights, meaſures, &c.

WEIGHTS and MEASURES.

EXAMPLE 1. In 12*lb.* of ſilver, how many ounces, pennyweights, and grains?

```
   12 Pounds
   12
  ---
  144 Ounces
   20
 ----
 2880 Penny-weights
   24
 ----
11523
5760
-----
69120 Grains
```

E. 2. In 69120 grains how many pounds?

```
      4)69120
 24
      6)17280
      -------
     2|0)288|0
      12)144
Anſwer   12 Pounds
```

E. 3.

E. 3. A gentleman ſent 4 lb. 2 *oz.* 8 *dwts.* of old plate, to his ſilverſmith, with orders to make it into the following articles, viz. tankards each 19 *oz.* 18 *dwts.*—cups each 14 *oz.* 10 *dwts.*—ſalts 11 *oz.* 15 *dwts.*—and ſpoons 2 *oz.* 4 *dwts.* how many of each ſort muſt he make?

		oz.	*dwt.*
The wt. of each	Tankard	19	18
	Cup -	14	10
	Salt -	11	15
	Spoon -	2	4
		48	7
		20	
		967	

lb.	*oz.*	*dwts.*
4	2	8
12		
50		
20		

967)1008(1
 967
 41

Anſwer 1 of each ſort, and 41 *dwts* over.

E. 4. In 18 pounds, 2 ounces, 4 drams, 2 ſcruples, and 12 grains, how many grains?

lb.	℥.	ʒ.	℈.	*grs.*
18	2	4	2	12
12				
218				
8				
1748				
3				
5246				
20				

104932 Grains, Anſwer.

E. 5. In 104932 grains, how many pounds?

2|0)10493|2
3)5246 - 12
8)1748 - 2
12)218 - 4

Anſwer lb. 18 - 2 4 2 12

E. 6. How many pounds of ſilver are there in one dozen of diſhes, each weighing 25 ounces, 15 dwts. and one dozen of plates, each weighing 15 ounces, 15 dwts. 22 grains?

	lb.	*oz.*	*dwts.*	*grs.*
One dozen of diſhes, each weight - -	2	1	15	0
Ditto - Plates - - - - -	1	3	15	22
One of each - - - - - -	3	5	10	22
				12
Anſwer -	41	6	11	0

E. 7. In a medicinal compoſition of 25 pounds, 7 ounces, and 6 drams, how many papers of powder may be made thereout, each weighing 2 ſcruples and 16 grains, allowing an ounce and a half to be loſt in levigating and weighing, and admitting theſe powders were to be equally divided amongſt 175 perſons, how many muſt each of them have?

From

	lb.	*oz.*	*drs.*
From the weight	25	7	6
Subtract the loss		1	4
	25	6	2

℈. *grs.*
2 16
20
66

25 6 2
12
306 Ounces
8
2450 Drams
3
7350 Scruples
20
7)147000
8) 21000
175) 2625(15 Answer
175
875
875

E. 9. In 9536646 drams, how many tons?

16 { 2)9536646
8)4768323 } = 16*drs.*
16 { 4) 596040—3
4) 149010 } = 8*oz.*
28 { 4) 37252—2
7) 9313 } = 12*lb.*
4) 1330—3
2|0) 33|2—2*qrs.*

Ans. *Tons* 16 12 2 12 8 6

E. 8. In 16 tons, 12 hundred, 2 qrs. 12 pounds, 8 ounces, and 6 drams, how many drams?

Tons.	*C.*	*q.*	*lb.*	*oz.*	*drs.*
16	12	2	12	8	6

20
332 Hundreds
4
1330 Quarters
28
10642
2661
37252 Pounds
16
596040 Ounces
16
Ans. 9536646 Drams

E. 10. In 12 hogsheads of raisins, each weighing 8 *Cwt.* 2 *qrs.* 6*lb.* how many pounds and tons?

C.	*qrs.*	*lb.*
8	2	6

4
34 Quarters
28
278
68
958 Pounds in 1 hhd.
12 No. of hogsheads
28= { 4)11496 Pounds in all
7) 2874 } = 16*lb.*
4) 410—4
2|0) 10|2—2

Ans. *Ton* 5 2*C.* 2*qrs.* 16*lb.*

E. 11.

E. 11. A grocer bought 18 hogsheads of sugar, each weighing 5*Cwt.* 3*qrs.* 14*lb.* out of which he has sold 5*Cwt.* 1*qr.* 16½*lb.* and orders the remainder to be made up into parcels of 27 pounds each; how many will there be, allowing 6 pounds to be lost in weighing them up?

	C.	*qrs.*	*lb.*		*C.*	*qrs.*	*lb*	
To	5	1	16½		5	3	14	
Add			6		4			
	5	1	22½		23			Quarters
	4				28			
	21				188			
	28				47			
	170				658			Pounds
	44				2			
	610				1316			Half-pounds in one hogshead
	2				18			Number of hogsheads
	1221				10528			
					1316			

From - - 23688 } Half-pounds { 18 hogsheads
Take - - 1221 } in the { Quantity sold

Half pounds in 1 parcel 54 { 6)22467
9) 3744—3 } =3 Half-pounds
416—0

Answer 416 parcels, and 1½*lb.* over.

E. 12. In 146 yards of cloth, how many quarters and nails?

146
4
584 Quarters
4
2336 Nails

E. 13. In 2336 nails, how many yards?

4)2336
4) 584
Answer 146 Yards

E. 14. In 864¼ yards, how many ells English?

864¼
4
5)3457
Answer *Ells* 691 2*qrs.*

E. 15. In 691 English ells, and 2 quarters, how many yards?

691 2
5
4)3457
Answer *Yds.* 864 1*qr.*

E. 16.

E. 16. In 86 pieces of cloth, each piece containing 30 yards, how many suits of clothes may be made thereout of $6\frac{3}{4}$ yards to the suit?

$6\frac{3}{4}$
4
———
27

86 Pieces
30 Yards in 1 piece
———
2580 Yards in all
4
———
3)10320 Quarters
9) 3440
———
382 - 2 } =6*qrs.*

Answer 382 suits, and $1\frac{1}{2}$*yds.* over

E. 17. In 1 mile, how many poles, yards, feet, inches and barley-corns?

1 Mile
8
———
8 Furlongs
40
———
320 Poles
$5\frac{1}{2}$
———
1600
160
———
1760 Yards
3
———
5280 Feet
12
———
63360 Inches
3
———
190080 Barley-corns

E. 18. In 190080 barley-corns how many miles?

3)190080
———
12)63360 Inches
———
3)5280 Feet
———
1760 Yards
2
———
$\frac{1}{2}$*yds.* in 1*p.* 11)3520
———
4|0)32|0 Poles
———
8) 8 Furlongs
———
Answer 1 Mile

E. 19. How many barley-corns will reach from Birmingham to London, being 109 miles?

109
1760 Yards in 1 mile
———
654
763
109
———
191840 Yards in all
3
———
575520 Feet
12
———
6906240 Inches
3
———
20718720 Barley-corns

E. 20. How many times doth the wheel which is $5\frac{1}{2}$ *yds.* in circumference, turn round between London and Liverpool, being 202 miles;

First, 202 × 1760 (yards in 1 mile) = 355520 yards.

$5\frac{1}{2}$ yards
2
———
11

Then 355520
2
———
11)711040 Half-yards in 202 miles
Answer 64640 Times

E. 21. How many Barley-corns will reach round the terreſtrial globe, which is 360 degrees, and each degree $69\frac{1}{2}$ miles?

360 Degrees
$69\frac{1}{2}$ Miles in a degree
3240
2160
24840
180
25020 Miles
190080 Barley-corns in one mile
2001600
225180
25020
Anſwer 4755801600 Barley-corns

E. 22. In 21 acres of land, how many roods and poles?

21
4
84 Roods
40
3360 Poles

E. 23. In 3360 poles, how many roods and acres?

4|0)336|0
4) 84 Roods
21 Acres

E. 24. A perſon rents a farm, which contains 400 acres of land, but he is to till no more than $196\frac{1}{2}$ acres; I deſire to know how many perches there are in the remainder?

400 Acres
4
1600
40
From 64000 Perches in the whole
Take 31440
Anſ. 32560 Per. rem.

A. *R.*
196 2
4
786
40
Perches tilled 31440

E. 25. In 12 hogſheads, 46 gal. 3 qts. of wine, how many quarts?

Hhd.	*gal.*	*qts.*
12	46	3

63
42
76
802 Gallons
4
3211 Quarts

E. 26. In 3211 quarts of wine, how many hogſheads?

4)3211
63 = { 7) 802—3 Quarts; 9) 114—4 }
12—6 } = 46 Gal.

Anſwer 12 *hhds.* 46 *gal.* 3 *qts.*

E. 27.

E. 27. A gentleman ordered his butler to bottle off 2 pipes of red port, into quart bottles, how many dozens will the two pipes fill?

```
        2 Pipes
        2
       ---
        4 Hogsheads
       63
      ----
      252 Gallons
        4
     -----
  12)1008 Quarts
Answer 84 Dozens
```

E. 28. In 8 hogsheads of beer, how many pints?

```
        8
       54
      ---
       32
       40
      ---
      432 Gallons
        8
     ----
Answer 3456 Pints
```

E. 29. In 8 lasts, 3 quarters, 2 bu. of corn, how many gallons?

```
      L.  q.  bu.
      8   3   2
     10
     --
     83 Quarters
      8
    ---
    666 Bushels
      8
   ----
Ans. 5328 Gallons
```

E. 30. In 5328 gallons of corn how many lasts?

```
     8)5328
     8) 666
    1|0) 8|3—2
Answer Last. 8  3 qrs. 2 bu.
```

E. 31. How many minutes are there in a Julian year?

```
     W.  d.  h.
     52  1   6
      7
    ---
    365
     24
   ----
   1466
   730
   ----
   8766
     60
  ------
Ans. 525960
```

E. 32. How many days, hours, minutes, and seconds, is it since the birth of our Saviour, to Christmas, 1781 (allowing Julian years?)

```
4)1781                 1781
                        365
  ----                 ----
445—1=6*               8905
                      10686
                      5343
                     ------
                     650065 h.
                        445 6
                     ------
                     650510 Days
                         24
                    -------
                    2602046
                   1301020
                   --------
                   15612246 Hours
                         60
                  ---------
                  936734760 Minutes
                         60
                -----------
                56204085600 Seconds
```

* 6 Hours being a quarter of a natural day, or 24 hours; therefore dividing the number of years by 4, the quotient will be days, as above, and $\frac{1}{4}$ or six hours over.

XII. The

XII. The RULE of THREE DIRECT.

TEACHETH, by having three numbers given, to find a fourth, in the same proportion to the third, as the second is to the first; or as the first is to the second, so is the third to the fourth, for which reason it is called the Rule of Proportion, as it is called the Rule of three, from its having three numbers given; and because of its excellent use in arithmetic, it is often named the *Golden Rule*.

To perform which observe the following

RULE. 1st. Place the three given terms in such order, that the first and third may be of one name; and the second must be of the same name with the fourth term sought.

2. If your first and third terms consist of divers denominations, reduce them into one, and the second into the lowest name mentioned.

3. Multiply the second and third terms together, and divide that product by the first; the quotient will be the answer in the same denomination you left your second term in.

4. If there happens to be a remainder, it will either make a fractional part, or it must be reduced to a lower denomination, and divided by the same divisor, the quotient will be so many of the said next name; proceed in this manner to the least name, and all the quotients together will be the answer to the question.

Before I shew how to work any questions in this rule, it will be necessary to give the learner the following instructions. First, observe that the first and fourth numbers are called extremes, and the second and third means; the product of the extremes, is equal to the product of the means.

EXAMPLE 1. As 4 is to 12, so is 36 to a certain number; what is that number?

$$\begin{array}{r} 4 \ : \ 12 \ :: \ 36 \\ 12 \\ \hline 4)432 \\ \hline \text{Answer } 108 \end{array}$$

Now it may be easily proved, that ~~the pro~~duct of the two extremes is equal to the product of the two means, for $4 \times 108 = 12 \times 36 = 432$.

CONTRACTION 1. When the second term can be divided by the first, multiply that quotient into the third term, and the product will be the answer.

Take the last example to prove this; thus, 12 divided by $4 = 3$; and 3 multiplied by $36 = 108$, the answer, as before.

E. 2. As 12 is to 18, so is 24 to a certain number, what is that number?

```
12 : 18 :: 24
           18
          ---
          192
           24
          ---
       12)432
          ---
Answer     36
          ---
```

The preceding example at full length.

```
4 : 3  17  6 :: 28
   20
   --
   77
   12
   --
  930
   28
  ---
 7440
1860
-----
4)26040
 12)6510
2|0) 54|2—6d.
-----
```

Answer £.27 2 6 as before.

Thus you may see that these contractions being considered, the work may oftentimes be performed much shorter than by the common methods.

Contraction 2. When the third term can be divided by the first, multiply that quotient by the second term, and the product will be the answer.

E. 3. If 4 yards of broad cloth cost 3*l.* 17*s.* 6*d.* what will a piece, containing 28 yards, come to, at the same rate?

```
Yds.    £.  s.  d.
 4  :   3   17  6 :: 28
                7  4)28
       ----------  ----
Answer 27   2   6    7
```

Contraction 3. When the first term can be divided by the second, and the third term by that quotient; the last quotient will be the answer.

E. 4. As 24 is to 8, so is 36 to a certain number; query that number?

```
24 : 8 :: 36
8)24     3)36
----     ----
3     Ans. 12 No. required.
```

The work at length.

```
24 : 8 :: 36
           8
          ---
   { 4)288
24 {   ---
   { 6) 72
        ---
Answer   12
```

E. 5. If 48 yards of cloth cost 6*l.* what will 64 yards cost?

Yds. £. *Yds.* £.
$48 \div 6 = 8$; and $64 \div 8 = 8$, the Answer.

Contraction 4. When the first term can be divided by the third, and the second by that quotient; the last quotient will be the answer. See the preceding example.

E. 6. If 6 yards of cloth cost 1*l.* 16*s.* what will be the value of 34 yards, at the same rate? 6 *Yds.*

```
Yds.      £. s.      Yds.
 6   :    1  16  ::  34
            20
            36
            34
           144
           108
         6)1224
       2|0)20|4
Answer £.10  4  0
```

E. 7. If 34 yards coſt 10*l*. 4*s*. what will 6 yards of the ſame coſt?

```
34  :  10  4 :: 6
       20
       204
         6
34) 1224 (36s. Anſwer.
    102
     204
     204
```

E. 8. If 1 *Cwt*. of cheeſe coſt 26*s*. what will 40 *Cwt*. of the ſame coſt?

```
Cwt.      s.    Cwt.
 1   :   26  ::  40
         40
   2|0)104|0
Anſwer  £. 52
```

E. 9. If 40 *Cwt*. of cheeſe coſt 52*l*. what will 1 *Cwt*. coſt?

```
   40  :  52 :: 1
          20
   4|0) 104|0
    2|0) 2|6
Anſwer  £.1  6  0
```

Note. To prove examples in this rule, is only varying the operation, as may be ſeen by the preceding examples.

E. 10. Suppoſe I buy 1 *oz*. of tea for 7½*d*. how much muſt I pay for 1 *Cwt*, of the ſame?

```
oz.    d.      lb.
 1  :  7½  ::  112 = 1 Cwt,
       2        16
      15       672
               112
              1792
                15
              8960
             1792
        2)26880
       12)13440
        2|0) 112|0
Anſwer  £.56
```

E. 11. Bought 36 *oz*. of ſilver, at the rate of 5*s*. 4*d*. per ounce, what does the whole come to?

```
oz.     s. d.     oz.
 1  :   5  4  ::  36
          12
          64
          36
         384
        192
     12)2304
     2|0)19|2
Anſwer £.9  12  0
```

The ſame by multiplication.

```
      5  4
         6×6=36
  1  12  0
         6
Anſwer £.9  12  0 as before.
```

This

This example plainly ſhews the extenſive uſe of multiplication, and how much preferable, in ſome caſes, it is to the rule of three, by ſolving queſtions in a more conciſe manner, and therefore it is very neceſſary for all perſons to be thoroughly acquainted with thoſe moſt uſeful rules, viz. compound multiplication and diviſion.

E. 12. If a ſoldier's pay be 6*d.* per day, how much is that per year?

```
     day.    d.     days.
      1   :  6  ::  365
                      6
               ----------
               12) 2190
               ----------
               2|0) 18|2—6
   Anſwer      £.9  2   6
```

If you would know at what rate you muſt ſell your goods by retail, ſo as to make a propoſed gain by the whole; add the money you would gain to the ſum the goods coſt you, and then ſtate your queſtion as before. Thus, if the whole be ſold for the total of the coſt and gain, at what rate muſt any part of it be ſold for?

E. 13. Bought 75 *Cwt.* 1 *qr.* 13 *lb.* of tobacco, which coſt 387*l.* 15*s.* 8*d.* and the charges upon it amounted to 6*l.* 5*s.* 8*d.* how much did it lie me in per pound?

```
                  £.   s.  d.
   Prime coſt    387  15   8
   Charges   -     6   5   8
C.  qr. lb.     -----------   lb.
75   1  13  :  394   1   4 :: 1
 4              20
---            ----
301            7881
 28              12
----           -----
2411     8441)94576(11d.
603           8441
-----         -----
8441          10166
-----          8441
               -----
               1725
               -----
```

Anſwer 11*d.* $\frac{1725}{8441}$ per pound.

E. 14. If 34 *Cwt.* 3 *qrs.* 25 *lb.* of tobacco coſt 111*l.* 15*s.* 6*d.* what will 1 pound come to at that rate?

```
Cwt. qr. lb.   £.   s.  d.   lb.
 34   3  25 :  111  15  6 :: 1
  4                20
 ---              ----
 139              2235
  28                12
 ----            ------
 1117             26826
 280                  4
 ----           --------
 3917   )       107304(27qrs.=6¾d.
                 7834
                -------
                 28964
                 27419
                -------
                  1545
                -------
```

Anſwer 6¾*d.* $\frac{1545}{3917}$ per pound.

E. 15.

E 15. If 1 *lb.* of tobacco cost $6\frac{3}{4}d.\frac{1545}{3917}$, how much may be bought for 111*l.* 15*s.* 6*d.*

d.	*qrs.*		*lb.*		*£.*	*s.*	*d.*
6	$3\frac{1545}{3917}$	:	1	::	111	15	6

4
27
3917
189
27
243
81
105759
1545
107304

20
2235
12
26826
4
107304
3917
751128
107304
965736
321912

107304)420309768(3917   28)   4)
321912   28   (139
983977   111   *Cwt.* 34   3 *qr.*   25 *lb.*
965736   84
182416   277
107304   252
751128   25 *lb.*
751128
......

Answer 34 *Cwt.* 3 *qr.* 25 *lb.*

E 16. A grocer bought 2 *Cwt.* 1 *qr.* 14 *lb.* weight of cloves, which cost him 32*l.* 4*s.* and he gained 5*l.* by the bargain, at what rate must he sell them per lb.

Cwt.	*qr.*	*lb.*		*£.*	*s.*		*lb.*
				32	4		
				5			
2	1	14	:	37	4	::	1

4
9
28
76
19
266

20
266)744(2*s.*
532
212
12
266)2544(9*d.*
2394
150
4
266)600(2 *qrs.*
532
68

Answer 2*s.* $9\frac{1}{2}d.\frac{68}{266}$ per pound.

If

If at any time damage has happened to goods, so as to make a proposed loss by the whole, then the said loss must be subtracted from the cost, and the remainder made the second term as before.

E. 17. Suppose I have by me 300 yards of holland, which cost me 80*l.* but some damage having happened to it, I am willing to lose 6*l.* 10*s.* by the whole; at what rate then must I sell it per yard?

Yds.		£. *s.*		*Yd.*
		80 0		
		6 10		
300	:	73 10	::	1

20

3|00) 14|70

s. 4 270

12

3|00) 32|40

d. 10 240

4

3|00) 9|60

3 *qrs.*

Answer 4*s.* $10\frac{3}{4}d.$ $\frac{60}{300}$ per yard

E. 18. *If ten pounds of bacon just cost me a crown,*
For a flitch of six score what must I pay down?*

lb.		*s.*		*lb.*
10	:	5	::	120

5

1|0) 60|0

2|0) 6|0

Answer £. 3

* Pounds.

E. 19. An oilman bought 3 tons of oil, which cost him 151*l.* 14*s.* and it so chanced, that it leaked out 85 gallons, but he is desirous to sell it again so that he may be no loser; how must he sell it per gallon?

First 3 tons = 756 gallons — 85 = 671; and 51*l.* 14*s.* = 3034*s.* then,

gal.		*shil.*		*gal.*
If 671	:	3034	::	1

1

671) 3034 (4*s.*
2684
350
12
671) 4200 (6*d.*
4026
174
4
671) 696 (1*qr.*
671
25

Answer 4*s.* $6\frac{1}{4}d.$ $\frac{25}{671}$ per gallon

E. 20. *If one strike of corn cost a guinea*, not more,*
Pray what must I give for one hundred and four?

bu.		*s.*		*bu.*
1	:	21	::	104

21

104
208

2|0) 218|4

Answer £. 109 4

* The price of wheat at this present time, April 15, 1800.

E. 21.

E. 21. Suppose a bankrupt owes 3000*l.* and has in money, goods, and recoverable debts 800*l.* 12*s.* $9\frac{3}{4}$*d.* now suppose these things delivered to his creditors, what do they get per pound?

```
 £.         £.  s.  d.     £.
3000   :   800  12  9¾  ::  1
            20
          ------
          16012
             12
          ------
         192153
              4
         ------
  3|000)768|615
       --------
    4)256 — 615 Remains.
      ----
   12) 64
       ----
```

Answer 5*s.* 4*d.*

In the above example, though there is a remainder of 615, yet the part of a pound each one is to receive, can be no more than 5*s.* 4*d.*

E. 22. A person owes 296*l.* 17*s.* but not being able to pay the whole, compounds with his creditors to pay them 7*s.* 6*d.* in the pound, how much money doth he pay his creditors?

```
£.      s. d.       £.   s.
1   :   7  6   ::  296  17
20      12               20
---     ---            -----
20      90             5937
---     ---              90
                      -------
                2|0)53433|0
                   ---------
                12)26716 { —10 =
                           10/20 = 1/2
                   ------
                2|0) 222|6 — 4
                    --------
          Answer £.111  6  4½
```

E. 23. Suppose a person fails in trade, and compounds with his creditors to pay them 12*s.* 6*d.* in the pound; how much doth he owe, when all his cash and effects amount only to 700*l.*

```
s.  d.      £.       £.
12  6   :   1   ::  700
12                   20
---                -----
150                14000
                      12
                   -------  £.
           15|0)16800|0(1120
                15
                ---
                 18
                 15
                 ---
                  30
                  30
                  ---
                   0
                  ---
```

Answer £.1120

E. 24. If I spend 2*s.* 6*d.* a day, and lay up 20*l.* at the year's end, how much is my yearly income?

```
Day    s.  d.      days.
1  :   2   6   ::  365
       12           30
      ----       -------
       30     12)10950
                -------
           2|0) 91|2—6
                ----
                45  12  6
                20   0  0
              -----------
   Answer £. 65  12  6
              -----------
```

E. 25. Bought coals, at $4\frac{1}{2}$*d.* per *Cwt.* how much will 30 tons come to at that rate?

Cwt.		*d.*		*Tons.*
1	:	$4\frac{1}{2}$	::	30
		2		20
		9		6009

2)5400

12)2700

2|0)22|5

Answer £. 11 5 0

E. 26. Bought a quantity of timber by the lump, for which I gave 147*l.* 16*s.* 4*d.* it is supposed to contain 70952 feet, how much did it lie me in per foot?

Ft.		*£.*	*s.*	*d.*		*ft.*
70952	:	147	16	4	::	1

20

2956
12

35476
4

70952)141904(2 *qrs.* per ft.
141904

E. 27. A woman bought 396 eggs at 2 a penny, and 294 at 3 a penny, which she sold out together at 5 for two-pence; I would know whether she gained or lost by the bargain, and how much?

First 2 : 1 :: 496
1

2)496

12)248

2|0) 2|0—8

1 0 8
0 8 2

The sum the eggs cost £. 1 8 10

Secondly 3 : 1 :: 294
1

3)294

12) 98

*s.*8—2*d.*

	e.		*d.*		*e.*
Thirdly	5	:	2	::	790

2

5)1580

12)316

2|0 2|6—4*d.*

£. 1 6 4

	£.	*s.*	*d.*	
From	1	8	10	the sum the eggs cost.
Take	1	6	4	for which they were sold.
Rem.	0	2	6	lost thereby, Answer.

E. 28. If 100*l.* principal, gain 5*l.* interest in 12 months, what will 40*l.* gain in the same time?

£.		£.		£.
100	:	5	::	40

5

1|00) 2|00

Answer £. 2

E. 29.

E. 29. What will 145 ſheep come to at 11*l.* 13*s.* 6*d.* per ſcore?

```
Sh.    £.  s.  d.     Sh.
20  :  11  13  6  ::  145
       20
       ---
       233
        12
       ---
      2802
       145
     -----
     14010
    11208
    2802
    ------
  2|0)40629|10
    ------
  12)20314—10=10/20=1/2
    ------
  2|0) 169|2—10d.
    ------
Anſ. £. 84 12 10½
```

E. 30. One who had ſold a parcel of cloaths for 2*s.* 10*d.* per yard on 3 months credit, found he gained 25*l.* per cent. by them; what did they coſt him per yard?

First 25*l.*÷by 4=6*l.* 5*s*, then 6*l.* 5*s.*+100=106*l.* 5*s.*

```
   £.  s.    s.  d.     £.
If 106 5  :  2  10  ::  100
    20       12          20
   ----     ----        ----
   2125      34         2000
   ----     ----          34
                        ----
                        8000
                       6000
                       ----12)
               2125)68000(32
                    6375 ---
                    ----     2—8
                     4250
                     4250
                     ----
                        0
                     ----
Anſwer 2s. 8d.
```

E. 31. A perſon bought 448 eggs at 3 a penny, another ſort at 2 a penny, which together were ſold out for 1*l.* 17*s.* 6*d.* how many eggs were bought at 2 a penny?

```
e.    d.     e.          £. s. d.
3  :  1  ::  448         1  17  6
               1         0  12  5
            ----        ---------
          3)448          1   5  1
            ----        20
       12)149—1         --
          ----          25
           12 5         12
          ----          --
                       301
                         2
                       ---
Anſ. eggs at 2 a penny 602
                       ---
```

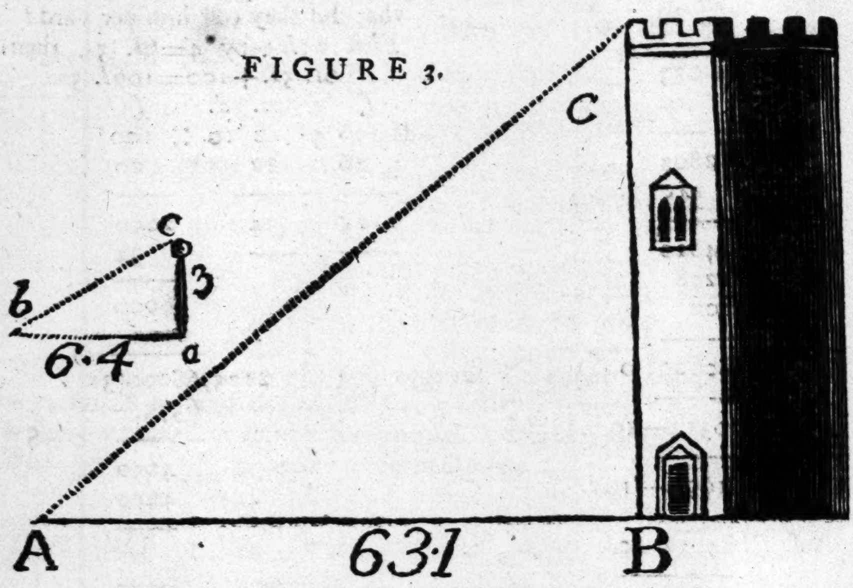
E. 32. *If 13 marks and 14 groats buy 15 loads of hay;*
How many pounds, with 16 crowns, for 90 loads will pay?

First, a mark=13*s.* 4*d.* then 13 marks+14 groats=178s. then

```
   loads.     s.       loads.
If 15   :    178   ::   90
              90
            -----
     { 5)16020
  15 {   -----
     { 3) 3204
         -----
         1068 Price of 90 loads
           80= 16 crowns
         -----
       2|0)98|8
         -----
Anſwer £.49  8
```

E. 33. A certain tower projected upon level ground a shadow, to the distance of 63 yards 1 foot, when a staff, 3 feet in length, perpendicularly erected, cast a shadow of 6 feet 4 inches, from hence the height of the tower is required?

FIGURE 3.



In the above figures, $ac=3$ feet, the length of the staff; $ab=6$ feet 4 inches, or 76 inches, length of its shadow. Also $BC=$ the height of the tower, and $AB=63$ yards 1 foot, or 2280 inches distance of its shadow; then,

	in.		*ft.*		*in.*
If	76	:	3	::	2280
					3

76)6840(90 Feet
684

0

Answer 90 feet = 30 yards, the height of the tower.

E. 34. Suppose a person travels 228 miles in 6 days, 4 hours, at what rate is that per hour, (allowing 12 hours to the day?)

D.	*h.*		*m.*		*h.*
6	4	:	228	::	1
12					

76)228(3 Miles, the answer
228

...

E. 35.

E. 35. In how long time would a million of millions of money be in counting, ſuppoſing one hundred pounds to be counted every minute without intermiſſion, and the year to conſiſt of 365 days, 5 hours, and 45 minutes?

```
  £.        M.        £.
 100   :    1    ::  1000000000000
                                 1
                     ___________________
                 1|00) 10000000000|00

Minutes in one year = 525945)10000000000(19013 Years
                             525945
                             4740550
                             4733505
                               704500
                               525945
                               1785550
                               1577835
Minutes in one hour = 6|0)     20771|5   }=55 Minutes
Hours in a day  -  24 { 4)     3461—5
                      { 6)      865—1    }=5 Hours
                                144—1
```

Anſwer 19013 years, 144 days, 5 hours, and 55 minutes.

E. 36. *If thirteen tuns of claret wine, coſt nineteen Engliſh pounds, How many pints of the ſame wine are worth a thouſand crowns?*

Firſt 13 tons reduced to pints = 26208, and 19*l.* = 380 ſhillings; and 1000 crowns = 5000 ſhillings; then,

```
    s.      pts.       s.
If 380  :  26208  ::  5000
            5000
_______________________ Pints,
38|0)13104000|0(344842 4/38, or
     114              [2/19 Anſwer
      170
      152
       184
       152
        320
        304
         160
         152
           80
           76
            4
```

E. 37. By laying out a ſum of money, one ſhilling brings $17\frac{1}{2}d.$ what is gained per cent?

```
s.      d.        £.
1  :   5½   ::   100
        2         20
       ___       ____
       11        2000
       ___         11
                 ____
              2)22000
                 ____
             12)11000
                 ____
            2|0) 91|6—8
                 ____
Anſwer      £. 45  16  8
```

E. 40. *A working alone in twelve days can compleat,*
The making a vessel of copper quite neat;
Which would take sixteen days to be made up by B,
He working more slowly than A, *you may see.*
Now working together, what time will they take,
Before the said vessel compleatly they'll make?

Suppose 192 to be the work, then A will perform $\frac{1}{12}$th part, and B $\frac{1}{16}$th, which will be 12+16=28th part of the work performed by them both together in one day, then,

w. *d.* *w.*
If 28 : 1 :: 192
28)192(6 days
168
24
12 hours in a day
28)288(10 hours
28
. . 8
60
28)480(17 minutes
28
200
196
4

Answer 6*d.* 10*h.* $17\frac{4}{28}$ *minutes.*

E. 41. How many bricks, 9 inches long and 4 inches wide, will floor a room that is 20 feet square?

First, 9×4=36 square inches in one brick, and 20×20=400 feet square in the floor, which × by 144=57600 square inches, the contents of the floor; then,

in. *b.* *in.*
If 36 : 1 :: 57600
36 { 6)57600
6) 9600

Answer 1600 bricks

E. 42. Bought a pipe of port wine, for which I gave 25*l.* 4*s.* but it leaked out 12 gallons; the remainder I sold at the rate of 18*d.* per quart; what was my gain or loss in the whole?

First by reduction, a pipe = 126 gallons, from which take 12 gallons; remains 114 gallons = 456 quarts, at 18*d.* per quart. then,

qt. *d.* *qts.*
If 1 : 18 :: 456
18
3648
456
12)8208
2|0) 68|4
£. 34 4 sold for
25 4

Answer £. 9 0 gained.

E. 43. If 2 men earn 15*s.* in 3 days, how much will 7 men earn in the same time?

M. *s.* *M.*
2 : 15 :: 7
7
2)105
2|0) 5|2—$\frac{1}{2}$=6*d.*

Answer 2*l.* 12*s.* 6*d.*

E. 44.

E. 44. A. ſets out from London to Birmingham at the very ſame time that B. at Birmingham ſets forwards for London, diſtance 109 miles; at 8 hours end they met on the road, and it then appeared that A had rode $2\frac{1}{2}$ miles an hour more than B. at what rate an hour did each of them travel?

	h.		*m.*		*h.*
Firſt, if	8	:	109	::	1
			8)109		
			m. 13 5	furlongs, what both rode per hour.	

	m.	*f.*	*p.*	
Then	13	5	0	
Leſs	2	4	0	the diſtance per hour that A. over-rode B.
	2)11	1	0	
	5	4	20	B rode per hour.
	+2	4	0	
	8	0	20	A rode per hour.

$$\therefore \left\{\begin{array}{ccc} m. & f. & p. \\ 5 & 4 & 20 \\ 8 & 0 & 20 \end{array}\right\} \times \text{ by } 8 = \left\{\begin{array}{ccc} m. & f. & \\ 44 & 4 & B. \\ 64 & 4 & A. \end{array}\right\} \text{travelled.}$$

Proof 109 miles.

E. 45.

As I was beating on the foreſt grounds,
Up ſtarts a hare before my two grey hounds:
The dogs being light of foot, did fairly run
Unto her fifteen roods, juſt twenty-one.
The diſtance that ſhe ſtarted up before,
Was fourſcore ſixteen rods, juſt and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

Firſt, from 21 take 15, remains 6 rods; the dogs gained in running 21 rods, and fourſcore $=80+16$ $=96$ rods the hare ſtarted before the dogs—then

	r.		*r.*		*r.*
If	6	:	21	::	96
					21
					96
					192
					6)2016

Diſt. the hare ſtarted before the dogs } 336 r. dogs ran. =96

240 r. hare ran.

E. 46. If the ſun moves every day one degree, and the moon thirteen, and at a certain time the ſun be at the beginning of Cancer, and in three days after, the moon in the beginning of Aries, the place of their next following conjunction is required?

First,

Firſt, $13^\circ - 1^\circ = 12^\circ$ moon gains of the ſun per day.
And $30^\circ \times 3 = 90^\circ$ from the firſt of Aries to the firſt of Cancer.
Alſo $90^\circ + 3 = 93^\circ$ ſun before the moon; then

```
                 D.
If  12°  :  1  ::  93°
          12)93
          -----
```

$7\frac{9}{12}$ days, in which time the ſun will be overtaken by the moon.

$\therefore 7\frac{3}{4} + 3 = 10\frac{3}{4}$ degrees of Cancer, the anſwer.

Note $\frac{9}{12}$ in its loweſt terms $= \frac{3}{4}$.

SOUND not interrupted, is by experiments found uniformly to move about 1150 feet in one ſecond of time.

E. 47. How long after the firing of a cannon at Birmingham may the report be heard at Worceſter, diſtance 25 miles?

```
      ft.    ſec.  miles.
If  1150  :  1  ::  25
      5280 feet in 1 mile
        25
     -----
     26400
    10560
    ------
1150)132000(114 Seconds
     1150
     ----
      1700
      1150
      ----
       5500
       4600
       ----
        900
         60
       ----
1150)54000(46 thirds
     4600
     ----
      8000
      6900
      ----
      1100
```

Anſwer 1 minute 54 ſeconds, $46\frac{1100}{1150}$ thirds.

E. 48. If I ſee a flaſh of a piece of ordinance, fired by a veſſel in diſtreſs at ſea, which happens, we will ſuppoſe, nearly at the inſtant of its going off, and hear the report a minute and two ſeconds afterwards, how far is ſhe off, reckoning for the paſſage of ſound as before?

Firſt, 1 minute 2 ſeconds = 62 ſeconds; then

```
      ''     ft.       ''
If  1  :  1150   ::   62
            62
          ----
          2300
         6900
         -----                  m.  f.  p.  y. ft.
Anſ.     71300 feet =           13   2   0   6  2
```

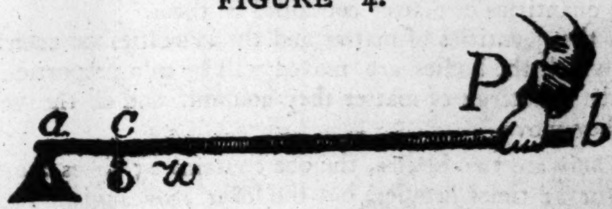
LEAVERS, of the ſecond order, are ſuch ſort where the power acts at one end, the prop fixed directly at the other, and the weight ſomewhere between them.

In

In this order of leavers, their force is in a contra-proportion to their length.

E. 49. If a leaver be 120 inches long, what weight, lying $8\frac{1}{2}$ inches from the end, resting on a pavement, may be moved with a force of 182 lb. lifting at the other end of the leaver?

FIGURE 4.



Let $ab = 120$ inches, and ac equal $8\frac{1}{2}$, P the power, or 182 lb. and w, the weight to be moved.

Inches.		*in.*		*lb.*		*in.*
120		If $8\frac{1}{2}$	:	182	::	$111\frac{1}{2}$
$8\frac{1}{2}$		2				2
$111\frac{1}{2}$ Longest end		17				223
						182
						446
						1784
						223

17)40586($2387\frac{7}{17}$ *lb.*
34
65
51
148
136
126
119
7

Answer $2387\frac{7}{17}$ *lb.* the weight.

In leavers of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

E. 50. A water-wheel turns a crank, working three pump-rods, fixed just six feet from the joint or pin; by which their several leavers, each 9 feet in length, are fastened, for the sake of the intended motion, at one end, the suckers of the pumps being worked by the other, shews them to be leavers of the third order: now I would know what the length of the stroke in each of the barrels will be, if the crank be made to play just nine inches round its centre?

First, $9 \times 2 = 18$ inches, the diameter of the crank; then

Feet.		*in.*		*ft.*		*in.*
As 6	:	18	::	9	:	27 Inches Answer.

Motion of Bodies, with their Velocities.

1. If the quantities of matter in any two or more bodies put in motion, be equal, the forces wherewith they are moved will be in proportion to their velocities.

2. If the velocities of theſe bodies be equal, their forces will be directly as the quantities of matter contained in them.

3. If both the quantities of matter and the velocities be unequal, the forces with which the bodies are moved will be in a proportion, compounded of the quantities of matter they contain, and of the velocities wherewith they move.

E. 51. There are two bodies, the one contains 25 times the matter of the other (or 25 times heavier) but the leſſer moves with 100 times the ſwiftneſs of the greater; in what proportion are the forces by which they are moved?

As 25 : 100 :: 1

$$\begin{array}{r} 1 \\ \hline 25)100(4 \\ 100 \\ \hline \end{array}$$

Anſ. 4, the leſs is moved with a force ſo much greater than the other.

E. 52. There are two bodies, the greater contains 9 times the quantity of the matter in the leſs, and is moved with a force 48 times greater; the ratio of the velocity of theſe two bodies is required?

As 9 : 48 :: 1

$$\begin{array}{r} 1 \\ \hline 9)48 \\ \hline 5\frac{3}{9} \\ \hline \end{array}$$

Anſ. Leſſer than the greater as 1 to $5\frac{1}{3}$.

Note, In comparing the motion of bodies, if their velocities be equal, the ſpaces deſcribed by them are in direct proportion of the times in which they are deſcribed.

2. If the times be equal, then the ſpaces deſcribed will be as their velocities.

3. If the times and the velocities be unequal, the ſpaces will be in a proportion compounded of the times and velocities.

E. 53. There are two bodies, one of which moves 80 times ſwifter than the other, but the ſwifter body has moved but one minute, whereas the other has been in motion two hours: the ratio of the ſpaces deſcribed by theſe two bodies is required?

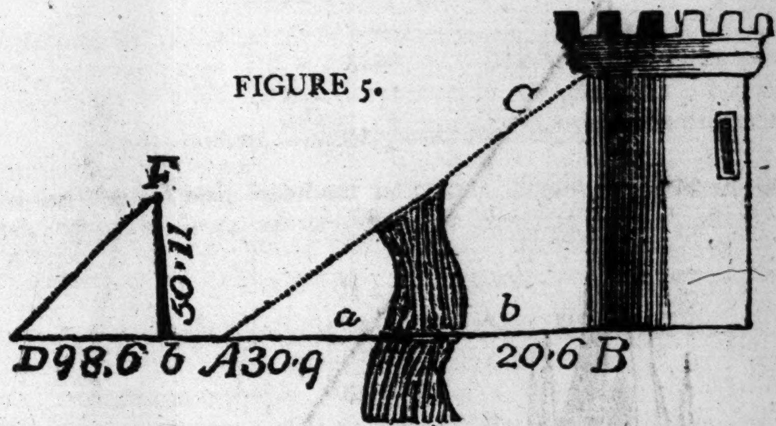
Firſt 2 hours $\times$ 60 = 120 minutes; then

As 80 : 120 :: 1 : $1\frac{1}{2}$.

Anſ. The ſwifter to the ſlower, as 1 to $1\frac{1}{2}$.

E. 54. A may-pole 50 feet 11 inches long, at a certain time of day casts a ſhadow 98 feet 6 inches long; I would thereby find the breadth of a river, that running due E. and W. within 20 feet 6 inches of the foot of a ſteeple 300 feet 8 inches high, which throws the extremity of its ſhadow 30 feet 9 inches beyond the ſtream?

FIGURE 5.



In figure 5, *Fb* = 50 feet 11 inches, = 611 inches, = the height of the may-pole; and *Db* = 98 feet 6 inches, or 1182 inches, length of its ſhadow. Alſo *BC* = 300 feet 8 inches, or 3608 inches, the height of the ſteeple; and *AB* the length of its ſhadow; then;

```
     in.     in.       in.
If 611 : 1182 :: 3608
           3608
           ----
           9456
         70920
         3546
       ---------12)
611)4264656(6979
     3666          ----
     ----          581  7 487/611 = AB
      5986          20  6 fr. ft. to r.
      5499         ----
      ----         561  1
       4875         30  9 projection
       4277        ----
       ----        530  4 487/611 = ab,
        5986       [Br. of the river
        5499
        ----
         487
```

E. 55. B and C together can build a boat in 18 days; with the aſſiſtance of A they can do it in 11 days; in what time would A do it by himſelf?

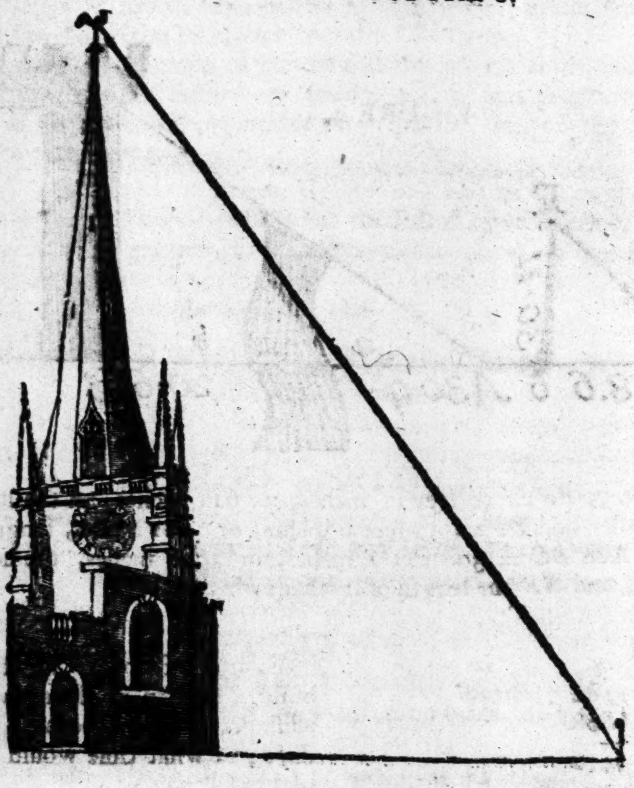
First, 18 — 11 = 7th part performed by A alone, then:

```
If 7 : 1 :: 198 the whole work
     7)198
     -----
  d. 28—2
        12 hours in a day
       ---
      7)24
       ---
    h. 3—3
         60 minutes in an h.
       7)180
        ----
         25—5/7 minutes
```

Anſwer 28 days, 3 hours, $25\frac{5}{7}$ minutes, by A himſelf.

E. 57. St. Martin's ſpire in Birmingham, at a certain time projected upon level ground, a ſhadow to the diſtance of 144 yards, 2 feet, 2 inches, when my cane, 3 feet 2 inches in length, perpendicularly erected, caſt a ſhadow of 6 feet 3 inches; from hence the height of the ſpire is required?

FIGURE 6.



SOLUTION.

f.	in.		f.	in.		yds	f.	in.
6	3	:	3	2	::	144	2	2
12			12			3		
75			38			434		
						12		
						5210		
						38		
						41680		
						15630		

in. yds. f. in.

75)197980(2639 = 73 0 11$\frac{55}{75}$ Anſwer

E. 58.

E. 58. There are two bodies, one whereof has described 50 miles, the other only 5, but the firſt has moved with 5 times the velocity of the ſecond; what is the ratio then of the times they have been deſcribing thoſe ſpaces?

First, $50 \div 5 = 10$; then:

```
As 5 : 10 :: 1
         1
        ——
      5)10
        ——
Anſwer   2
        ——
```

So that the firſt body hath been in motion double the time of the ſecond.

XIII. RECIPROCAL PROPORTION:

OR, THE

RULE OF THREE INVERSE.

RECIPROCAL PROPORTION is, when of four numbers, the third beareth the ſame proportion to the firſt, as the ſecond doth to the fourth; conſequently, the leſs the third term is in reſpect to the firſt, the greater will the fourth term be in reſpect to the ſecond.

RULE. Multiply the firſt and ſecond terms together, and divide their products by the third term, the quotient will be the anſwer required.

EXAMPLE 1. If 24 men can perform a piece of work in 12 days, how many men can do the ſame in 36 days?

```
 d.      m.     d.
 12  :   24  ::  36
         12
        ———
   ⎧ 6)288
36 ⎨    ———
   ⎩ 6) 48
        ——
Anſwer  8 Men
```

E. 2. If 36 days require 8 men to perform a piece of work in, how many men will 12 days require?

```
 d.    m.    d.
 36  :  8  ::  12
  8
 ———
12)288
 ———
Anſ. 24 Men, which proves the
     —   above work to be per-
         formed right.
```

Therefore, it is only varying the operations, and you have a proof to all queſtions of this nature.

E. 3. If 24 men can perform a piece of work in 6 days, how many men can do the fame in 36 days?

d.		*m.*		*d.*
6	:	24	::	36
		6		

36 { 6) 144
     6) 24

Anf. 4 Men

E. 4. *If a board be eight inches in breadth, pray declare,*
What lenght of the board will juft make a foot fquare?

in.		*in.*		*in.*
12	:	12	::	8
		12		

8)144

Anf. 18 Inches

E. 5. How many yards of paper that is three-quarters wide, will hang a room that is 30 yards round, and $3\frac{1}{4}$ yards high?

yds.		*yds.*		*qrs.*
$3\frac{1}{4}$	:	30	::	3
4		13		
13		3)390		

Anf. 130 Yards

E. 6. If I lend a perfon 300*l.* for a year, how long ought he to lend me 500*l.* to requite me?

£.		*d.*		£.
300	:	365	::	500
		300		

5|00)1095|00

Anf. 219 Days

E. 7. If when the price of a bufhel of wheat is 4*s.* 6*s.* the penny-loaf weighs 12 ounces, what muft the penny-loaf weigh, when the faid bufhel is worth only 4*s.*

s.	*d.*		*oz.*		*s.*
4	6	:	12	::	4
2			9		2
9			8)108		8

oz. 13 4
16
8)64
8 *drs.*

Anf. 13*oz.* 8*drs.*

E. 8. Suppofe 275 yards of cloth, which is 5 quarters wide, make coats for 130 men; how many yards of fhalloon, of 3 quarters wide, will line the faid coats?

qrs.		*yds.*		*qrs.*
5	:	275	::	3
		5		

3)1375
458—1
4
3)4
1—$\frac{1}{3}$

Anf. 458*yds.* 1*qr.* $\frac{1}{3}$*na.*

E. 9.

E. 9 A garrifon confifting of 1500 men, being befieged, have provifions only for three months, but it being neceffary they fhould ftand out five months, how many men muft depart, that the faid provifions may ferve that time?

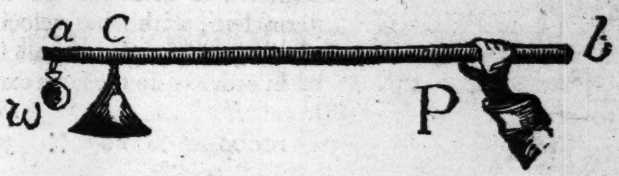
	mo.		men.		mo.
First	3	:	1500	::	5
			3		
			5)4500		
			900 Men to continue		

Then from 1500
Take 900

Remains 600 Men to depart

E, 10. What weight will a man be able to raife, who preffes with the force of a hundred and half on the end of an equipoifed hand-fpike, 100 inches long, which is to meet with a convenient prop exactly $7\frac{1}{2}$ inches above the other end of the machine?

FIGURE 7.



In Figure 7, *a b* = 100 inches, *a c* $7\frac{1}{2}$, *P* the power, or $1\frac{1}{2}$ *Cwt.* and *w* the weight.

in.		in.		lb.		in.
100	If	$92\frac{1}{2}$	:	168	::	$7\frac{1}{2}$
$7\frac{1}{2}$		2		185		2
$92\frac{1}{2}$		185		840		15
				1344		
				168		
			15 {	3)31080		
				5)10360		

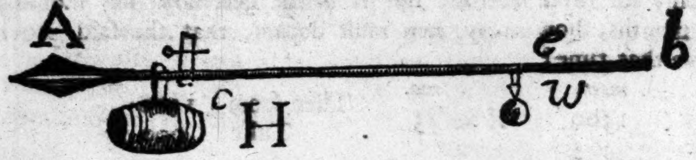
Anf. 2072lb. or $18\frac{1}{2}$ *cwt.*

A leaver of the firft order, equally divided and juftly poifed, is the balance-beam; to this, if a power be applied at one end, it will always move an equal weight at the other: in like manner, a leaver equally poifed, and unequally divided, having a power applied at one end, will move a weight at the other which will be reciprocally proportionable to the diftances of thofe ends from the fulcrum, or point fupported; of this kind is the fteelyard.

E. 11. What weight, hung at 70 inches diftance from the fulcrum of a fteel-yard, will equipoife a hogfhead of tobacco weighing $9\frac{1}{2}$*cwt.* freely fufpended at two inches diftance on the contrary fide?

FIGURE

FIGURE 8.



In the above Figure $ce = 70$ inches, $Ac = 2$, H the hogshead, and w the weight.

in. *c.* *in.*
2 : $9\frac{1}{2}$ :: 70
4
38
4 × 7=28
152
7
1064
2
7|0)212|8
30—28
16
7|0)44|8
6—28
16
7|0)44|8
6—28

Ans. 30lb. 6oz. $6\frac{28}{70}$ drs. the weight required.

E. 12. Suppose the battering ram of Vespasian weighing 100,000lb. and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time, and this was found sufficient to demolish the walls of Jerusalem; with what velocity must a bullet, that weighs but 30lb. be moved in order to do the same execution.

lb. *ft.* *lb.*
100000 : 20 :: 30
20
3|0)200000|0
66666—20
12
3|0)24|0
8

Ans. 66666 feet, 8 inches, per sec.

E. 13. A body weighing 200lb. is impelled by such a force, as to send it 100 feet in a second; with what velocity would a body of 8lb. move, if it were impelled by the same force?

	lb.		*feet*		*lb.*
As	200	:	100	::	8
			200		

8)20000

Ans. 2500 Feet per second.

In comparing the motion of bodies, the ratio, or proportion between their velocities, will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of their quantities of matter they contain.

E. 15.

E. 15. Suppose that in a room where two men, A. and B. are sitting, there is a fire, from which A is three feet and B six feet distant, it is required to find how much hotter it is at A's seat than B's?

To answer this question, it must first be philosophically considered and learnt, that the effects or degrees of light, heat, and attraction, are reciprocally proportional to the squares of their distances, from the centre whence they are propogated.

A's distance is 3 feet $\times$ 3=9; and B's distance is 6 feet, which $\times$ by 6 = 36; then

As 36 : 1 :: 9

$$9)36$$

Answer 4

So that it is evident A's place is 4 times as hot as B's.

XIV. COMPOUND PROPORTION:

OR,

The RULE of FIVE,

IS so called, from its having five numbers or terms given to find a sixth, which if the proportion is direct, the sixth term must bear such a proportion to the fourth and fifth, as the third bears to the first and second. But if the proportion is inverse, then the sixth term must bear such proportion to the fourth and fifth, as the first bears to the second and third, or as the second bears to the first and third.

The three first terms are a supposition, the two last a demand.

RULE. 1. Let the principal cause of gain, loss, or action, &c. be put in the first place.

2. Let that which denotes time, distance of place, &c. be in the second place, and the remaining one in the third place.

3. Place the other two terms which move the question, underneath those of the same name.

4. If the blank, or term sought, fall under the third term, multiply the two first terms together for a divisor, and the three last for a dividend, the quotient arising from them will be the answer, or sixth term.

5. If the blank fall under the first or second term, multiply the third and fourth terms together for a divisor, and the other three for a dividend; the quotient arising from them will be the answer.

PROOF. Is by two statings in the single rules of three.

E. 1. If 6 men can mow 72 acres of grass in 12 days, how many men can mow 120 acres in 4 days?

```
m.    d.    a.
6  :  12  :  72
       4  : 120
      72                     120
       4                      12
     ---                    ----
     288                    1440
     ---                       6
                            ----
                        288)8640(30 Men, Answer.
                            864
                            ---
                              0
```

Proof. By two statings in the single rule.

```
d.      a.       d.
12  :   72  ::   4          24  :  6  ::  120
         4                                  6
       ---                                ---
    12)288                           ( 4)720
       ---                        24{  ---
        24                           ( 6)180
       ---                            ---
```

Answer 30 Men as before.

E. 2. A usurer put out 120*l.* to receive interest for the same; but when it had continued 9 months he took it up, and received for the principal and interest 125*l.* 8*s.* I demand at what rate per cent. per annum he received?

```
 £.       s.
125   :   8
120   :   0
-------------
£.5   :   8s.=108s. Interest.

 £.       m.        s.
120   :   9   ::   108
100   :  12   ::

120                       108
  9                       100
----                    -----
1080                    10800
                           12
              108|0)12960|0( 120s.=6l. Ans.
                    108
                    ---
                     216
                     216
                     ---
                       0
```

E. 3.

E. 3. Suppose the salary of 6 persons, for 21 weeks, is 120*l.* what will be the salary of 14 persons for 46 weeks?

```
p.    w.    l.
 6 : 21 : 120        120
14 : 46 :             46
                     ---
21                   720
 6                   480
---                  ---
126                 5520
---                   14
               126)77280(613£.
                   756
                   ---
                    168
                    126
                    ---
                     420
                     378
                     ---
                      42
                      20
               126)840(6s.
                   756
                   ---
                    84
                    12
              126)1008(8d.
                  1008
                  ----
                     0
```

Answer 613*l.* 6*s.* 8*d.*

E. 4. What is the interest of 259*l.* 13*s.* 5*d.* for 20 weeks, at 5 per cent, per annum?

First 100*l.* = 24000 pence, and 259*l.* 13*s.* 5*d.* = 62321*d.* then,

```
   d.        w.    £.
 24000  :   52  :  5
 62321  :   20  :  0

      52          62321
   24000              5
   -----         ------
     208         311605
    104              20
 --------        ------
 12480|00  )  62321|00(4£.
              49920
              -----
              12401
                 20
              -----
   12480)248020(19s.
         12480
         ------
         123220
         112320
         ------
          10900
             12
   12480)130800(10d.
         12480
         -----
          6000
             4
   12480)24000(1/4
         12480
         -----
         11520
```

Answer 4*l.* 19*s.* $10\frac{1}{4}$*d.* $\frac{11520}{12480}$

E. 5. If a sack of coals be the allowance of 7 poor people for a week, how many poor belonged to that parish, which, when coals were 36*s.* per chaldron, had 41*l.* to pay in six weeks on that account?

First, 36*s.* ÷ 12 (the sacks in a chaldron) = 3*s.* what the coals cost per week, and 41*l.* = 820*s.* then,

```
p.     w.     s.
7  :   1  :   3             820
0  :   6  :  820              7
                            ----
       3               18)5740(318 8/9 Poor, Answer.
       6
      --
      18
```

 XV.

XV. COMPOUND PROPORTION:

OR, THE

RULE of THREE REPEATED.

ALL questions in the foregoing rule of five, may be resolved by two operations in the rule of three repeated; but there are some questions that cannot be solved by the rule given there, for one stating, yet may be answered by two or more statings in the rule of three repeated.

Example 1. A and B are on opposite sides of a wood, 134 toises or fathoms about; they begin to go round it both the same way at the same instant of time; A goes 11 toises in 2 minutes, and B 17 in 3; the question is, how many times will they surround this wood, before the nimbler overtakes the slower?

First, As 2 : 11 :: 3
3
2)33
$16\frac{1}{2}$ A goes while B goes 17

Then 17
$16\frac{1}{2}$
$\frac{1}{2}$ Toise B gains of A [in going 17 times round.

Again, $\frac{1}{2}$: 17 :: $\frac{1}{2}$: 17 Times round gone by A, and $16\frac{1}{2}$ by B, the Answer.

E. 2. A merchant bought hats in London, which cost him 4*l.* 16*s.* per dozen, and 6*d.* carriage; he is to gain 20 per cent. by the bargain; what must he sell them at a-piece to do it?

	hats		£. *s. d.*		*hat.*
First,	12	:	4 16 6	::	1

20
96
12
12)1158
12)96—6
8 $0\frac{1}{2}$
1 $7\frac{1}{4}$
Answer £. 0 9 $7\frac{3}{4}$ $\frac{20}{100}$

	£.		£.		*s. d.*
Again, as 100	:	20	::		8 $0\frac{1}{2}$

12
96
4
386
20
1\|00)77\|20
4)77
12)19--$\frac{1}{4}$
1 $7\frac{1}{4}$

E. 3. If a leaver, 40 effective inches long, will, by a certain power thrown successively thereon, in 13 hours raise a weight 104 feet, in what

what time will two other leavers, each 18 effective inches long, raiſe an equal weight 73 feet; the force of ſtrait leavers being in direct proportion of their lengths?

Firſt, $18 \times 2 = 36$ inches, length of the leaver; then

in.		ft.		in.
As 40	:	104	::	36
		36		
		624		
		312		
		4|0)374|4		
		$93 - \frac{24}{40} = \frac{3}{5}$		

Again, $93\frac{3}{5}$ ft. : 13 h. :: 73 ft. : 10 hours, 8 minutes, 20 ſeconds, Anſ.

E. 4. A weight of $1\frac{1}{2}$ lb laid on the ſhoulder of a man, is no greater burthen to him than its abſolute weight, or 24 ounces; what difference will he feel between the ſaid weight applied near his elbow, at 12 inches from the ſhoulder, and in the palm of his hand, 28 inches therefrom; and how much more muſt his muſcels then draw to ſupport it at right angles; that is, have his arm extended right out?

	in.		lb.		in.
Firſt, as	1	:	$1\frac{1}{2}$	::	12
			2		3
			3		2)36
					18 *lb. wt.* 12

inches from the ſhoulder

	in.		lb.		in.
Then, as	1	:	$1\frac{1}{2}$	:	28
			2		3
			3		2)84
					42*lb. wt.* 28

[inches from the ſhoulder.

Conſequently, $42 - 18 = 24$*lb.* the Anſwer.

E. 5. If when port wine is 17 guineas the hogſhead, a company of 45 people will ſpend 20*l.* therein, in a certain time; what is wine a pipe, when 13 perſons more will ſpend 63*l.* in twice the time, drinking with equal moderation?

Firſt, $45 + 13 = 58$ perſons, and 17 guineas $= 357$ ſhillings; then

p.		£.		p.
As 45	:	20	::	58
				20
				45 { 5)1160
				9) 232
				£. $25 - \frac{7}{9}$ in the ſame time

And in twice that time the 58 perſons will ſpend $25\frac{7}{9}$, which multiplied by $2 = 51\frac{5}{9}$ pound's worth, at 17 guineas per head.

Then, As $51\frac{5}{9}$ £. : 17 G. :: 63 £. : 436 s. $2\frac{3}{4}\frac{416}{464}$ d. $= 21$ £. 16 s. $2\frac{3}{4}\frac{416}{464}$ d. per hogſhead, which $\times$ by $2 = 43$*l.* 12*s.* $5\frac{3}{4}$*d.* $\frac{368}{464}$ per pipe the anſwer.

E. 6.

E. 6. Suppose a person to travel 152 miles in 7 days, when the days are 12 hours long; how many days will he be in travelling 576 miles, when the days are 16 hours long?

h.		d.		h.
As 12	:	7	::	16
		12		

$$16\left\{\begin{array}{l} 4)84 \\ \overline{} \\ 4)21 \end{array}\right.$$

$5\frac{1}{4}$

	m.		d.		m.
Then, as	152	:	$5\frac{1}{4}$	::	576
			4		21
			21		576
					1152

152)12096(79
1064
1456
1368
88

4)79
$19-\frac{3}{4}$

Answer, 19 days $\frac{3}{4}\ \frac{88}{152}$

E. 7. *If twenty dogs for 30 groats,*
Go forty weeks to grass;
How many hounds, for sixty crowns
May winter in that place?

First, from 52 (the weeks in a year) subtract 40, and there remains 12; then 30 groats=2 crowns ∴.

	c.		dogs.		c.
As	2	:	20	::	60
			60		

2)1200
600

	w.		dogs.		w.
Again, as	40	:	600	::	12
			40		

12)24000

Answer 2000 dogs

E. 8. My water-tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges, at a medium, 40 gallons in 31 minutes. Supposing these both carelessly to be left open, and the water to be turned at 2 in the morning; the servant at 5, finding the water running, shuts the tap, and is solicitous to know in what time the tub will be filled after this accident, in case the water continues flowing from the main?

m.		gal.		m.
As 9	:	14	::	31
		31		
		14		
		42		
		9)434		

$48\frac{2}{9}$ Gallons, fills in 31 minutes

Then, $48\frac{2}{9}-40=8\frac{2}{9}$ gallons in the tub at the end of 31 minutes.

And $5-2=3$ hours, or 180 minutes.

	m.		gal.		m.		gal.	
Again as	31	;	$8\frac{2}{9}$	::	180	:	$47\frac{23}{31}$	fills in three hours.

And $147-47\frac{23}{31}=99\frac{8}{31}$ gallons wants of being full.

	gal.		m.		gal.		m. sec.	
Also, as	14	:	9	::	$99\frac{8}{31}$	:	63 $48\frac{228}{434}$,	the tub will be full; which

added to 5 o'clock, will give 3 minutes $48\frac{228}{434}$ seconds after 6, the tub will be full.

E. 9.

E. 9. In giving directions for making an Italian chair, the shafts whereof where settled at 11 feet between the axle-tree, whereon the principal bearing is, and the back-band, by means of which the weight is partly thrown upon the horse; a dispute arose where-about on the shafts the centre of the body of this machine should be fixed. The coach-maker advised this to be done at 30 inches from the axle: others were of opinion, that at 24, it would be a sufficient incumberance to the horse. Now, admitting the two passengers, with their baggage, ordinarily to weigh 2 *Cwt.* a-piece, and the body of the vehicle to be about 70 pounds more; pray what will the beast, in both those cases, be made to bear more than his harness?

First, 30 inches $= 2\frac{1}{2}$ feet, $24 = 2$ ditto, and 4 *Cwt.* 70 *lb.* $= 518$ *lb.* then $11 - 2\frac{1}{2} = 8\frac{1}{2}$ feet; also $11 - 2 = 9$.

Then, as $\overset{ft.}{11} : \overset{lb.}{518} :: \overset{ft.}{8\frac{1}{2}} : \overset{lb.}{400\frac{3}{11}}$ force in the former case.
And contra, as $8\frac{1}{2} : 400\frac{3}{11} :: 2\frac{1}{2} : 117\frac{2}{11}$ pressure.
Again, as $11 : 518 :: 9 : 423\frac{9}{11}$ force in the latter case.
Also, as $9 : 423\frac{9}{11} :: 2 : 94\frac{2}{11}$ pressure.

E. 10. There is an island 73 miles round, and three footmen all start together, to travel the same way about it; A travels 5 miles a day, B 8, and C 10; when will they all come together again?

$\left.\begin{array}{l}\text{First, } 8-5=3 \text{ miles B} \\ \text{And } 10-5=5 \text{ ——C}\end{array}\right\}$ gained of A in one day

Then, as $\overset{m.}{3} : \overset{d.}{1} :: \overset{m.}{73}$ Again, as $\overset{m.}{5} : \overset{d}{1} :: \overset{m.}{73}$

$3)73$ $5)73$

$24\frac{1}{3}$ when A and B meet $14\frac{3}{5}$ when A and C meet

So that B nor C can never meet with A, but at the end of these periods, when A and C. will have travelled 219 miles.

Therefore, as $\overset{d.}{14\frac{3}{5}} : \overset{d.}{219} :: \overset{d.}{24\frac{1}{3}} : 365$

$\left.\begin{array}{l}219 \\ 365\end{array}\right\} \times \text{by} \left\{\begin{array}{l}24\frac{1}{3} \\ 14\frac{3}{5}\end{array}\right. = 5329$ days, the 73d time of their general meeting

$\therefore 5329 \div 73 = 73$ days, their first general meeting.

For, as $73 : \left\{\begin{array}{l}24\frac{1}{3} \\ 14\frac{3}{5}\end{array}\right| :: \left|\begin{array}{l}219 \\ 365\end{array}\right\} : 73$ days, the Answer.

E. 11. A certain man hires a labourer on this condition, that for every day he worked he should receive 1*s.* but for every day he was idle, he should be mulcted 8*d.* When 390 days were past, neither of them were indebted to one another; how many days did he work, and how many days was he idle?

First, for every day he worked he received 12 pence,
And for every day he played, he paid - 8 pence.

Sum 20

Likewise,

Likewise, as his idle days came to the same money as those he worked, therefore the proportion will be as follows:

	days.		*days.*		*d.*		*days.*		*days.*		*d.*
As	20	:	390	::	8	Again, as	20	:	390	::	12
			8						12		
			2|0)312|0						2|0)468|0		

156 Days he worked   Days he played 234

For 156 days, at 12*d.* per day, comes to the same money as 234 at 8*d.* per day, viz. 7*l.* 16*s.* proof.

E. 12. A man hired a labourer for 40 days, on condition that he should have 20*d.* for every day he worked, and forfeit 10*d.* for every day he idled; at last he received 2*l.* 1*s.* 8*d.* for his labour; how many days did he work, and how many was he idle?

First 2*l.* 1*s.* 8*d.* $=$ 500 pence, and $500 \div 20 = 25$ days wages; then $40 - 25 = 15$ days more.

For every day he worked he had - - - 20 Pence
And for every day he played he forfeited 10

Sum 30 Pence

$$\text{Then, as } \overset{d.}{30} : \overset{days.}{15} :: \begin{cases} 10 : 5 \text{ worked} \\ 20 : 10 \text{ idle} \end{cases}$$

Therefore he was idle 10 days, and worked $(5 + 25) = 30$ days.

E. 13. If 4 compositors, in 16 days, of 12 hours long each, can compose 14 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line; how many days of 10 hours long each, will it take 9 compositors (all working together, at the same rate with the former, and on the same sized letter) to compose a volume, or book, to be printed, consisting of 30 sheets, 16 pages in a sheet, 50 lines in a page, and 45 letters in a line?

Thus stated,

C.		D.		C.
4	——	16	——	9*
12 *h.*	——	——	——	10* *h.*
*14 *s.*	——	——	——	30 *s.*
*24 *p.*	——	——	——	16 *p.*
44 *l.*	——	——	——	50 *l.*
*40 *l.*	——	——	——	45 *l.*

First, $14 \times 24 \times 40 \times 9 \times 10 = 5322200$ divisor
And, $4 \times 12 \times 44 \times 30 \times 16 \times 50 \times 45 \times 16 = 829440000$ dividend
Then, $829440000 \div 53222400 = 15\frac{45}{77}$ days, Answer.

N. B. Those marked thus* are multiplied together for the divisor and those not marked for the dividend.

XVI.

XIV. PRACTICE.

SO called from the general uſe it is of to all perſons concerned in trade and buſineſs.

When a queſtion in the rule of three has 1 for the firſt term, it is more expeditiouſly reſolved, by taking ſome aliquot part, or parts of the thing propoſed; by which means many tedious reductions may be avoided.

In order to perform this rule expeditouſly, let the learner get by heart the following

TABLES of ALIQUOT PARTS.

The even Parts of Money.

Of a Pound			Of a Shilling		Of Two Shillings		Of a Penny	
s.	*d.*		*d.*		*d.*		*qr.*	
1	8	= $\frac{1}{12}$	1	= $\frac{1}{12}$	$1\frac{1}{2}$	= $\frac{1}{16}$	1	= $\frac{1}{4}$
2	0	$\frac{1}{10}$	$1\frac{1}{2}$	$\frac{1}{8}$	2	$\frac{1}{12}$	2	$\frac{1}{2}$
2	6	$\frac{1}{8}$	2	$\frac{1}{6}$	3	$\frac{1}{8}$	3	$\frac{3}{4}$
3	4	$\frac{1}{6}$	3	$\frac{1}{4}$	4	$\frac{1}{6}$		
4	0	$\frac{1}{5}$	4	$\frac{1}{3}$	6	$\frac{1}{4}$		
5	0	$\frac{1}{4}$	6	$\frac{1}{2}$	8	$\frac{1}{3}$		
6	8	$\frac{1}{3}$			12	$\frac{1}{2}$		
10	0	$\frac{1}{2}$						

The even Parts of Weight.

Of a Ton		Of a Hundred				Of a Quarter of a Hundred	
Cwt.		*qr.*		*lb.*		*lb.*	
2	= $\frac{1}{10}$	1	or	28	= $\frac{1}{4}$	$3\frac{1}{2}$	= $\frac{1}{8}$
$2\frac{1}{2}$	$\frac{1}{8}$	2	or	56	$\frac{1}{2}$	4	$\frac{1}{7}$
4	$\frac{1}{5}$	0		16	$\frac{1}{7}$	7	$\frac{1}{4}$
5	$\frac{1}{4}$	0		14	$\frac{1}{8}$	14	$\frac{1}{2}$
10	$\frac{1}{2}$						

Case 1: When the price is leſs than a penny,

RULE. Divide by the aliquot parts that are in a penny, then by 12 and 20, which will give the anſwer.

EXAMPLES.

$\frac{1}{4}$	$\frac{1}{4}$	2067 Yards of tape, at one farthing per yard?
	12	$516\frac{3}{4}$
	2|0	4|3
		£. 2 3 $0\frac{3}{4}$ Anſwer

O

The

The price being a farthing a yard, the given quantity is conſequently ſo many farthings, and a farthing being $\frac{1}{4}$ of a penny, and a penny the $\frac{1}{12}$ of a ſhilling; and a ſhilling the $\frac{1}{20}$ of a pound; therefore the diviſors are 4, 12, and 20.

$\frac{1}{2}$ | $\frac{1}{2}$ | 416 lb. at $\frac{1}{2}d.$ per lb ?

12)208

Anſwer £. 17*s.* 4*d.*

$\frac{1}{2}$ | $\frac{1}{2}$ | 2067 at $\frac{3}{4}d.$ per lb. ?

$\frac{1}{4}$ | $\frac{1}{2}$ | 1033—$\frac{1}{2}$

516—$\frac{1}{2}$

12)1550

2|0) 12|9$\frac{2}{12}$

£.6 9 2$\frac{1}{4}$ Anſwer

The laſt example may be done by multiplying the given quantity by the farthings in the price, and reducing the product into ſhillings, pounds, &c. thus:

2067

3

4)6201

12)1550$\frac{1}{4}$

2|0) 12|9$\frac{2}{12}$

Anſ. £. 6 9 2$\frac{1}{4}$ ſame as above

CASE 2. When the given price is a fraction,

RULE. Multiply the given number or quantity by the numerator, and divide the product by the denominator; the quotient will be the anſwer in the ſame denomination with the whole number, of what the price is a part.

EXAMPLES.

What will the carriage of 8372 lb. come to, at $\frac{7}{8}$ of a penny per lb ?

8372

7

8)58604

12) 7325$\frac{4}{8}$

2|0) 61|0$\frac{5}{12}$

Anſwer £. 30 10 6$\frac{1}{4}$

CASE

CASE 3. When the price is leſs than a ſhilling,

RULE. Take the aliquot part or parts that are in a ſhilling, and add them together, and the ſum will be the anſwer in ſhillings, &c. which, divided by 20, will give pounds, &c.

EXAMPLES.

12)24501 Pieces, at 1*d.* per piece?
2|0)204|1—9
£.102 1 9 Anſwer

$\frac{1}{4}$ | $\frac{1}{4}$ | 1400 Yards, at 1$\frac{1}{4}$*d.* each?
350
12 | 1750
2|0 | 14|5—10
£.7 5 10 Anſwer

1$\frac{1}{2}$ | $\frac{1}{8}$ | 1231 lb. at 1$\frac{1}{2}$*d.* per lb?
2|0 | 15|3—10$\frac{1}{2}$
£.7 13 10$\frac{1}{2}$ Anſ.

1$\frac{1}{2}$ | $\frac{1}{8}$ | 1041 lb. at 1$\frac{3}{4}$*d.* per lb?
$\frac{1}{4}$ | $\frac{1}{6}$ | 130—1$\frac{1}{2}$
21—8$\frac{1}{4}$
2|0 | 15|1 9$\frac{3}{4}$
£.7 11 9$\frac{3}{4}$ Anſ.

2 | $\frac{1}{6}$ | 736 lb. at 2*d.* per lb.?
2|0 | 12|2—4=8
£.6 2 8 Anſ.

2 | $\frac{1}{6}$ | 2408, at 2$\frac{1}{4}$*d.*?
$\frac{1}{4}$ | $\frac{1}{8}$ | 401—4
50—2
2|0 | 45|1 6
£.22 11 6 Anſ.

2 | $\frac{1}{6}$ | 604, at 2$\frac{1}{2}$*d.*?
$\frac{1}{2}$ | $\frac{1}{4}$ | 100—8
25—0
2|0 | 12|5 8
£.6 5 8 Anſ.

2 | $\frac{1}{6}$ | 1740, at 2$\frac{3}{4}$*d.*?
$\frac{1}{2}$ | $\frac{1}{4}$ | 290
$\frac{1}{4}$ | $\frac{1}{2}$ | 72—4
36
2|0 | 39|8 4
£.19 18 4 Anſ.

3	$\frac{1}{4}$	746 *lb.* at 3*d.* per *lb.*?
	2|0	18|6—6
		£. 9 6 6 Anſwer

4	$\frac{1}{3}$	961, at 4*d.*?
	2|0	32|0—4
		£. 16 0 4 Anſwer

3	$\frac{1}{4}$	1417, at $3\frac{1}{4}$*d.*
$\frac{1}{4}$	$\frac{1}{12}$	354—3
		29—$6\frac{1}{4}$
	2|0	38|3 $9\frac{1}{4}$
		£. 19 3 $9\frac{1}{4}$ Anſwer

3	$\frac{1}{4}$	569, at $4\frac{1}{4}$*d.*?
1	$\frac{1}{3}$	142—3
$\frac{1}{4}$	$\frac{1}{4}$	47—5
		11—$10\frac{1}{4}$
	2|0	20|1 $6\frac{1}{4}$
		£. 10 1 $6\frac{1}{4}$ Anſwer

4	$\frac{1}{3}$	5674, at $4\frac{3}{4}$*d.*?
$\frac{1}{2}$	$\frac{1}{8}$	1891—4
$\frac{1}{4}$	$\frac{1}{2}$	236—5
		118
	2|0	224|5 9
		£. 112 5 9 Anſwer

4	$\frac{1}{3}$	814 Ells, at 5*d.* per ell?
1	$\frac{1}{4}$	271—4
		67—10
	2|0	33|9 2
	£.	16 19 2 Anſwer

4	$\frac{1}{3}$	2147, at $5\frac{1}{4}$*d.*?
1	$\frac{1}{4}$	715—8
$\frac{1}{4}$	$\frac{1}{4}$	178—11
		44—$8\frac{3}{4}$
	2|0	93|9 $3\frac{3}{4}$
		£. 46 19 $3\frac{3}{4}$ Anſwer

4	$\frac{1}{3}$	674, at $5\frac{1}{2}$*d.*?
		224—8
$1\frac{1}{2}$	$\frac{1}{8}$	84—3
	2|0	30|8 11
	£.	15 8 11 Anſwer

3	$\frac{1}{4}$	1746, at $5\frac{3}{4}$*d.*?
2	$\frac{1}{6}$	436—6
$\frac{3}{4}$	$\frac{1}{4}$	291 0
		109 0
		2|0)83|6 6
		£. 41 16 6 Anſwer

6	$\frac{1}{2}$	1741, at 6*d.*?
		2|0)87|0—6
		£. 43 10 6

4	$\frac{1}{3}$	2142, at $6\frac{1}{4}$*d.*?
2	$\frac{1}{2}$	714
$\frac{1}{4}$	$\frac{1}{8}$	357
		44—$7\frac{1}{2}$
		2|0)111|5 $7\frac{1}{2}$
		Anſwer £. 55 15 $7\frac{1}{2}$

1401,

6 | $\frac{1}{2}$ | 1401, at $6\frac{1}{2}d$?		
$\frac{1}{2}$ | $\frac{1}{12}$ | 700—6		
58—$4\frac{1}{2}$		
2|0)75|8 $10\frac{1}{2}$		
£. 37 18 $10\frac{1}{2}$ Anſ.		

6 | $\frac{1}{2}$ | 1631, at 7*d*?		
1 | $\frac{1}{6}$ | 815— 7		
135—10		
2|0)95|1 5		
£. 47 11 5 Anſ.		

6 | $\frac{1}{2}$ | 112 *lb.* at 11*d*?		
3 | $\frac{1}{2}$ | 56		
2 | $\frac{1}{3}$ | 28		
18-8		
2|0)10|2—8		
£. 5 2 8 Anſ.		

Sometimes the value may be eaſily found by reckoning the price of ſome even number above what is given, which done, take ſome aliquot part for what it is above, and ſubtract it from the former.

Take the laſt example thus:

	£.	s.	d.
112 lb. (at 1*s.*) = - -	5	12	0
112 lb. (at 1*d.* is $\frac{1}{12}$) = Subtract	0	9	4
Anſwer -	£. 5	2	8 the ſame as above

When the price is 2, 3, 4, 6, or 8 pence, you may make uſe of this method; thus, for 2*d.* divide the given quantity by 120; for 3*d.* by 80; for 4*d.* by 60; for 6*d.* by 40, and for 8*d.* by 30, which will give the anſwer in pounds, &c.

CASE 4. When the given price is more than a ſhilling but leſs than two,

RULE. Leave the top line, or given quantity, for ſhillings, and take your parts as before for the remaining pence and farthings, which add to the given quantity, and the ſum will be the anſwer in ſhillings, &c. which divided by 20, will give pounds.

EXAMPLES.

1 \| $\frac{1}{12}$ \| 261 Ells, at 1*s.* 1*d.* per ell?
21—9
2\|0)28\|2 9
£.14 2 9 Anſwer

3 \| $\frac{1}{4}$ \| 578 Yards, at 1*s.* 3*d.*?
144—6
2\|0)72\|2 6
£.36 2 6 Anſwer

4 \| $\frac{1}{3}$ \| 2140, at 1*s.* 5*d.*?
1 \| $\frac{1}{4}$ \| 713—4
178—4
2\|0)303\|1 8
£. 151 11 8 Anſwer

6 \| $\frac{1}{2}$ \| 1453, at 1*s.* $7\frac{1}{2}d.$?
$1\frac{1}{2}$ \| $\frac{1}{4}$ \| 726—6
181—$7\frac{1}{2}$
2\|0)236\|1 $1\frac{1}{2}$
£. 118 1 $1\frac{1}{2}$ Anſwer

CASE 5. If at ſuch a profit in the ſhilling, you would know what is gained per Cent,

RULE.

RULE. Divide 100 by the parts that the propoſed profit is of a ſhilling, the quotient or total of the quotient is the anſwer.

EXAMPLES. At $1\frac{1}{2}d.$ profit in the ſhilling, what is gained per cent.?

$1\frac{1}{2} | \frac{1}{8} |$ 100

£. 12 10 Anſ.

At $1\frac{3}{4}d.$ profit in the ſhilling, what is gained per cent.?

$1\frac{1}{2} | \frac{1}{8} |$ 100

$\frac{1}{4} | \frac{1}{6} |$ 12—10
2— 1—8

£. 14 11 8 Anſ.

CASE 6. When the given price is two ſhillings,

RULE. Double the unit figure for ſhillings, the reſt are pounds.

EXAMPLES.

5168 Ells, at 2*s.* per ell?

£.516 16*s.* Anſ.

8429 Yards, at 2*s.* per yard?

£.842 18 Anſ.

8164 *lb.* at 2*s.*?

£.816 8 Anſ.

4213 *lb.* at 2*s.*

£.421 6 Anſ.

CASE 7. When the given price is ſuch pence as want an aliquot part of two ſhillings,

RULE. Work for two ſhillings, as taught before, and then take for that part, and ſubtract it from what it comes to at two ſhillings.

EXAMPLES.

532 *lb.* at 16*d.* per *lb.*

8 | $\frac{1}{3}$ | 53 —4
17—14—8

£. 35 9 4 Anſ.

465, at 22*d.*

2 | $\frac{1}{12}$ | 46—10
3—17—6

£. 42 12 6 Anſ.

CASE 8. When the given price is any number of pence above 12, and under 20,

RULE. Multiply by the ſaid pence at once, as taught in the multiplication table, and divide that product by 12 and 20 for the anſwer.

EXAMPLES.

541 *lb.* at 13*d.*?
13

12)7033

2|0)58|6—1

£.29 6 1 Anſ.

743 *lb.* at 14*d.*?
14

12)10402

2|0) 86|6—10

£. 43 6 10 Anſ.

CASE

CASE 9. When the price confifts of any even number of fhillings under 20,

RULE. Multiply the given quantity by half the price, doubling the firft figure of the product for fhillings, and the reft of the product will be pounds.

EXAMPLES.

182 Yards, at 4*s*. per yard?
2
36*l*. 8*s*. Anfwer

642, at 6*s*?
3
192*l*. 12*s*. Anf.

536, at 10*s*?
5
268*l*. 0*s*. Anfwer

1267, at 18*s*?
9
1140*l*. 6*s*. Anfwer

CASE 10. When the price is any odd number of fhillings under 20,

RULE. Multiply the given quantity by the price, and the product will be the anfwer in fhillings, which divided by 20 will give pounds.

EXAMPLES.

648 lb. at 7*s*.?
7
2|0)453|6
226*l*. 16*s*. Anf.

312, at 12*s*.?
12
2|0)374|4
187*l*. 4*s*. Anf.

662, at 17*s*.?
17
2|0)1125|4
562*l*. 14*s*. Anf.

764, at 19*s*.?
19
2|0)1451|6
725*l*. 16*s*. Anf.

CASE 11. When the price is fhillings, or fhillings and pence, or fhillings, pence, and farthings, and no even part of a pound,

RULE. Multiply the given quantity for the fhillings, and take parts for the pence, &c. and add them together, the fame will be the anfwer in fhillings, which divided by 20, will give pounds.

EXAMPLE.

6	$\frac{1}{2}$	2470 lb. at 11*s*. 8$\frac{1}{2}$*d*. per lb?
		11
		27170
2	$\frac{1}{3}$	1235
	$\frac{1}{2}$	411 — 8
	$\frac{1}{4}$	102 — 11
		2|0)2891|9 7
		£. 1445 19 7 Anf.

CASE

CASE 12. When the price is shillings, or shillings and pence, and they an aliquot part of a pound.

RULE. Divide by the aliquot part, and the quotient will be the answer.

EXAMPLES.

1s. 8d. is $\frac{1}{12}$)132 Yards, at 1s. 8d. per yd? 6s. 8d. is $\frac{1}{3}$)831 at 6s. 8d.?

11l. Answer. 277l. Ans.

3s. 4d. is $\frac{1}{6}$)736, at 3s. 4d.? 5s. is $\frac{1}{4}$)736, at 5s.?

122l. 13s. 4d. Ans. 184l. Ans.

CASE 13. When the price is shillings and pence, and such shillings and pence as are the same figure.

RULE. Multiply the given quantity by the shillings, and take $\frac{1}{12}$ of the product for the pence; the total divided by 20, gives the answer in pounds.

EXAMPLES.

144 lb. at 6s. 6d. per lb?	784, at 11s. 11d.?
6	11
12)864	12)8624
72	718—8
2|0)93|6	2|0)934|2 8
£.46 16 Answer.	£.467 2 8 Answer.

CASE 14. When the given price is any even number of shillings, and you would know what quantity of any thing may be bought for any even number of pounds.

RULE. Add a cypher to the given pounds, and divide that sum by half the proposed price, and the quotient is pounds.

EXAMPLES. How many pounds of tea may be bought for 86l. at 4s. per pound?

2)860

Answer 430 lb.

How many yards, at 14s. per yard, may be bought for 50l.?

7)500

Answer 71 yds. 1 qr. 3 na.

CASE 15. When the price is pounds only.

RULE. Multiply the given quantity by the price, and the product will be the answer.

EXAMPLES. 260 Tons, at 7l. per ton?

7

Answer 1820l.

364, at 4l.?

4

Answer 1456l.

405

405, at 6*l.*?
6

2430*l.* Anſ.

96 *Cwt.* at 26*l.* per *Cwt.*?
26

576
192

2496*l.* Anſ.

CASE 16. When the price is pounds and ſhillings.

RULE. Multiply the given quantity by the number of pounds; and for the ſhillings take aliquot parts, and add them together; the ſum will be the anſwer. Or reduce the given price to ſhillings, by which multiply the given quantity, and divide by 20 will give the anſwer.

EXAMPLES.

10	½	164, at 4*l.* 17*s.*?
		4
		656
5	½	82
2	⅕	41
		16—8
		£.795 8 0 Anſ.

Or thus, 164, at 4*l.* 17*s.*?

164	20
97	97
1148	
1476	
2|0)1590|8	

Anſ. £.795 8 as before

CASE 17. When the given price be ſuch a fractional part of a pound, ſhilling, &c. that the numerator is more than a unit,

RULE. Multiply the given quantity by ſuch numerator, or top figure, and divide the product by the denominator, or lower figure, the quotient is the anſwer in pounds.

EXAMP. 15*s.*=¾)516, at 15*s.*?
3

4)1548

£. 387 Anſ.

12*s.* 6*d*=⅝)876, at 12*s.* 6*d.*
5

8)4380

£.547 10 Anſ.

CASE 18. When the price is pounds, ſhillings, and pence, and the ſhillings and pence be an aliquot part of a pound,

RULE. Multiply the given quantity by the pounds, as in Caſe 16, and take parts for the ſhillings and pence as in Caſe 12; add them together, and the ſum will be the anſwer.

EXAMPLES.

2*s.* 6*d.*=⅛)247, at 3*l.* 2*s.* 6*d.*
3

741
30—17—6

£.771 17 6 Anſ.

6*s.* 8*d.*=⅓)274, at 7*l.* 6*s.* 8*d.*?
7

1918
91—6—8

£.2009 6 8 Anſ.

CASE 19. When the price is pounds, shillings, pence and farthings, and the shillings and pence be not an aliquot part of a pound,

EXAMPLE.

RULE. Reduce the pounds and shillings into shillings, multiply the given quantity by the shillings, and take parts for the pence and farthings as before.

```
6 | 1/2  |  267 Cwt. at 2l. 12s. 6¾d.?
  |      |   52        20
  |      |  ----       --
  |      |  534        52s.
  |      | 1335        --
¾ | 1/8  |  133—6
  |      |   16—8½
  |      |  ---------
  2|0)1403|4  2½
  ---------------
  £.701  14  2½ Ans.
```

Note. When the given quantity doth not exceed 100, proceed as in Section IX.

CASE 20. When the price hath a fraction annexed,

RULE. Work for the pounds, shillings or pence, by the shortest of the foregoing rules, and value the fraction as directed in Case 2; or if an aliquot part of the money foregoing, take such part for it.

EXAMPLES.

```
454 lb. at 19⅝d. per lb.          1496 French crowns, at 5 3/9d. per
 19                                   3                   [crown?
----                               -----                1496
8626            454              9)4488                    5
 283¾             5              ------                -----
-----           ----             498 6/9 = 1/3          7480
12)8909¾       8)2270                                    498⅓
-------        ------                                 -------
2|0)74|2—5     283¾ or 6/8                            12)7978⅓
-------        ----------                             -------
£.37 2 5¾ Ans.                                        2|0)66|4—10
-------                                               ----------
                                              Ans.    £.33 4 10⅓
```

CASE 21. When the price and quantity given are of several denominations,

RULE. Multiply the price of one by the quantity given, and take parts for quarters, pounds, &c. add them together, and the sum will be the answer.

EXAMPLES. Sold 8 *Cwt.* of raisins, at 1*l.* 16*s.* per *Cwt.*

```
1 qr.=¼)  1  16
              8
         ------
         14   8
          0   9
         ------
Ans.  £.14  17
```

In this example the price is multiplied by 8, and divided by 4, the aliquot part for one quarter.

Bought

Bought 7 *Cwt.* 3 *qrs.* 18 *lb.* of ſugar, at 17*s.* 6*d.* per *Cwt.* what does it come to?

		£.	s.	d.
2 *qrs.*	½		17	6
				7
		6	2	6
1 *qr.*	½		8	9
16 *lb.*	$\frac{1}{7}$		4	4½
2 *lb.*	⅛		2	6
			0	3¾
		£.6	18	5¼ Anſ.

What is the value of 24 *lb.* of double-refined ſugar, at 4*l.* 17*s.* per *Cwt.*?

lb.		£.	s.	d.
16	$\frac{1}{7}$	4	17	
8	½	0	13	10¼
		0	6	11
		£. 1	0	9¼ Anſ.

Bought tobacco at 3*l.* 17*s.* 4½*d.* per hundred weight, what is the worth of 72 *Cwt.* 3 *qrs.* 19*lb.*?

qrs.	*lb.*		£.	s.	d.
2	0	½	3	17	4½
					9×8=72
			34	16	4½
					8
			278	11	0
1	0	½	1	18	8¼
	16	$\frac{1}{7}$		19	4
	2	⅛		11	0½
	1	½		1	4½
				0	8¼
			£. 282	2	1½ Anſ.

CASE 22. When the given quantity hath a fraction annexed,

RULE. Value the whole number as before, and for the fraction multiply the price by the numerator, and divide that product by the denominator; the quotient is the value of the fraction, and muſt be added to the value of the whole number.

EXAMPLE. What will 358⅜ ells of holland come to, at 6*s.* 11*d.* per ell?

6	½			358	
				6	
				2148	
3		½		179	
1½		½		89	—6
½		$\frac{1}{3}$		44	9
				14	11
			2|0)	247|6	2
				123 16	2
				2	7⅛
			£.	123 18	9⅛ Anſ.

	£.	s.	d.
		6	11
			3
8)	1	6	8
	0	2	7⅛

CASE 23. When then given number is not of the ſame name with that on which the price is ſet.

 RULE

RULE. Reduce it into the ſame as taught in reduction, and then find the amount by the ſhorteſt of the foregoing rules.

EXAMPLES.

48 *Thouſ.* at 17*s.* 6*d.* per *Cwt.*

```
       10
    ------
6 | ½ | 480
    |    17
    | -----
    | 8160
    |  240
    ------
2|0)840|0
    ------
£. 420  0 Anſwer
```

1811 *lb.* 11 *oz.* at 2*s.* 10*d.* [per *oz.*

```
          16
       -------
        28987
       -------
8 | ⅓ | 2898—14 at 2s.
2 | ¼ |  966   4  8
          241  11  2
       -------------
      £.4106   9 10 Anſ.
```

CASE 24. When the given quantity is of leſs denomination than that on which the price is ſet,

RULE. Divide the price by the part or parts the quantity given is of; the quotient, or ſum of the quotient, is the anſwer.

EXAMPLES. At 14*l.* 14*s.* per hogſhead, what will one gallon of wine come to?

```
          £.  s.
     ⎧ 7) 14  14
  64 ⎨    -------
     ⎩ 9)  2   2
       ----------
      £.0   4  8 Anſwer
```

At 8*l.* 12*s.* per *Cwt.* what will 42 *lb.* come to?

```
lb.      £.  s.
28 | ¼ |  8  12
   |     -------
14 | ½ |  2   3
          1   1  6
        ----------
       £.3  4  6 Anſ.
```

At 42*s.* per *Cwt.* what will 35 *lb.* come to?

```
lb.      s.
28 | ¼ | 42
   |    -----
 7 | ¼ | 10—6
          2  7½
       -------
         13  1½ Anſwer
```

Note. It often happens in buſineſs, that by inverting a queſtion; that is, by calling your price the quantity, and the quantity your price; you may find the anſwer much eaſier, and ſooner than by the common method.

EXAMPLES. What will 29 yards come to, at 1*s.* 6*d.* or 18 yards at 2*s.* 5*d.*?

```
           s.  d.
           2   5
               2
         -------
           4  10
               9
      ----------
   £. 2    3   6 Anſ.
```

What will 26*lb.* come to at 6½*d.* per *lb.* or 6½ *lb.* at 2*s.* 2*d.*?

```
       s.  d.
   ½)  2   2
           6
     -------
      13   9
       1   1
     -------
      14   1 Anſ.
```

Practical Methods, for casting up particular goods and quantities: some in the wholesale way.

METHOD 1. In goods sold by six-score to the hundred,

RULE. Half the pence in the price of one, is the value of the hundred in pounds.

EXAMPLES.

What will 120 deal boards come to, at $22\frac{1}{2}d.$ per dozen?

$2)22\frac{1}{2}$

£. 11 5 Ans.

What will 120 yards of cloth come to, at $9\frac{1}{2}d.$ per yard?

$2)9\frac{1}{2}$

£. 4 15 Ans.

METHOD 2. If the quantity given happens to be 240, the pence in the price of one is the value of the whole in pounds.

Note. A farthing must be reckoned as 5*s.*—a half-penny 10*s.*—and three-farthings 15*s.*

EXAMPLES.

What will 240 *lb.* come to, at $10\frac{1}{2}d.$ per *lb.*

£. 10 10 Ans.

What will 240 *lb.* of double-refined sugar come to, at $13\frac{1}{4}d.$ per *lb.*

£. 13 5 Ans.

METHOD 3. If the given quantity be 160, take $\frac{1}{3}$, and multiply that product by 2 for the answer.

EXAMPLES. What will 160 yards come to, at $10\frac{1}{2}d.$ per yard?

```
3)10  10
 -------
   3  10
       2
 -------
£. 7   0 Ans.
```

What will 160 ells of cloth come to, at $14\frac{3}{4}d.$ per ell?

```
3)14  15
 ----------
   4  18  4
          2
 ----------
£. 9  16  8 Ans.
```

METHOD 4. If the given number be 96, multiply $\frac{1}{5}$ of the price by 2.

EXAMPLES. What will 96 yards come to, at $15\frac{1}{2}d.$ per yard?

```
5)15  10
 -------
   3   2
       2
 -------
£. 6   4 Ans.
```

What will 96 *lb.* come to, at $13\frac{1}{4}d.$ per *lb.*?

```
5)13   5
 -------
   2  13
       2
 -------
£. 5   6 Ans.
```

If the given number be 80, the $\frac{1}{3}$ is the answer.

EXAMPLE. What will 80 yards come to, at $19\frac{3}{4}d.$ per yard?

```
3)19  15
 ----------
£. 6  11  8 Ans.
```

If

If the given number be 60, the $\frac{1}{4}$ is the anſwer.

EXAMPLE. What will 60 yards come to, at $22\frac{1}{2}d.$ per yard?

	£.	s.	
4)	22	10	
£.	5	12	6 Anſ.

Note. If the price be given in ſhillings and pence, bring them into pence.

If the given number be 48, take $\frac{1}{5}$ for the anſwer.

EXAMPLE. What will 48 yards come to, at 3*s.* 6*d.* per yard?

s.	d.
3	6
12	
5)42	
£. 8	8 Anſ.

If the given number be 40, take $\frac{1}{6}$ for the anſwer.

EXAMPLE. What will 40 ells of holland come to, at 44*d.* per ell?

£.	s.	d.	
6)44			
7	6	8	Anſ.

If 30 is your quantity take $\frac{1}{8}$ for the anſwer.

EXAMPLE.

s.	d.
8	6
12	
102	

£.	s.	
8)102	0	
12	15	Anſ.

GOODS *ſold by the* THOUSAND.

RULE. Multiply the pence that one coſt by 50, and divide that product by 12; the quotient is the value of a thouſand in pounds.

Note: The reaſon of the above rule is $\frac{1000}{240}=\frac{50}{12}l.$

EXAMPLES. What will a thouſand Dutch tiles come to, at 2*d.* each?

d.		
2		
50		
12)100		
£. 8	6	8 Anſ.

What will 1000 ells of cloth come to, at 8*s.* 6*d.* per ell?

8	6
12	
102	
50	
12)5100	
£. 425	Anſ.

GOODS ſold by the great groſs of 144 dozen, are caſt up by the following

RULE. Multiply the price that one coſt by 3, and divide the product by 5, the quotient is the value of the great groſs in pounds.

EXAMPLES

EXAMPLES. What will a great grofs of buttons coft, at $7\frac{1}{2}d.$ per dozen?

£.	s.
7	10
	3
5)22	10
4	10 Anf.

What will a great grofs come to, at $14\frac{1}{2}d.$ per dozen?

£.	s.
14	10
	3
5)43	10
8	14 Anf.

GOODS fold by the fmall grofs of twelve dozen may be caft up by the preceding rule; feeing there are as many particulars in a fmall grofs as dozens in a great one.

EXAMPLES. What will a fmall grofs of tobacco boxes come to, at $6\frac{1}{2}d.$ per box?

£.	s.
6	10
	3
5)19	10
3	18 Anf.

What will a fmall grofs of buttons come to, at $7\frac{1}{2}d.$ per pair?

£.	s.
7	10
	3
5)22	10
4	10 Anf.

To value the common HUNDRED WEIGHT, of 112 *lb.*

RULE. Multiply the price that 1 *lb.* coft by 14, and divide the product by 30, or multiply by 7 and divide by 15; in both cafes the quotient is the anfwer in pounds: $\frac{112}{240}=\frac{14}{30}=\frac{7}{15}$ of a pound.

EXAMPLE. What will 1 hundred weight come to, at $6\frac{1}{2}d.$ per pound?

	£.	s.	
	6	10	
15 { 5)	45	10	
3)	9	2	
	3	0	8 Anf.

Note. There are feveral other methods and contractions which might have been added, but as they are more curious than ufeful, I fhall here conclude this rule, and proceed to tare and trett.

XVII. TARE and TRETT.

TARE and TRETT, are allowances made in buying and felling commodities that are liable to lofs or wafte.

In this rule there are fix things to be obferved, viz. 1, The Grofs weight. 2, Tare. 3, Trett. 4, Suttle. 5, Cloff. 6, Net weight.

1. The grofs weight, is the whole weight of any commodity, be what it will, and that which it is packed up in.

2. Tare is an allowance made for the weight of any box, cask, &c. that contains any commodity.

3. Trett is an abatement of 4 *lb.* per 104 *lb.* and is the twenty-sixth part allowed for waste, dust, &c. made by the merchant to the buyer.

4. Suttle is the weight of the goods, when only the tare is taken out, and not the trett.

5. Cloff is an allowance of 2 lb. to the citizens of London, on every draught above 3 *Cwt.* on some sorts of goods, as beaver, galls, madder, &c.

6. Net weight is the weight of any goods, when all allowances are deducted from the gross.

CASE 1. When the net weight of any goods is required, and only tare allowed.

RULE. Subtract the tare from the gross, and the remainder is the net weight.

EXAMPLE 1. Suppose I buy 194 *Cwt.* 2 *qrs.* 18 *lb.* of tobacco, and am allowed 13 *Cwt.* 1 *qr.* 12 *lb.* tare, what is the net weight?

	Cwt.	*qrs.*	*lb.*
Gross -	194	2	18
Tare -	13	1	12
Answer Net	181	1	6

E. 2. In 8 bags of hops, each weighing gross 3 *Cwt.* 2 *qrs.* 15 *lb.* tare 12 *lb.* per bag, what is the net weight?

	lb.
Tare per bag -	12
Number of bags	8
Tare -	3 12

	Cwt.	*qrs.*	*lb.*	
	3	2	15	
			8	
	29	0	8	Gross
		3	12	Tare
Answer Net	28	0	24	

CASE 2. When tare is at so much per *Cwt.* to find the net weight,

RULE. Divide the whole gross, by the said part or parts, that there are of a hundred weight, and the quotient thence arising will be the tare, which subtracted from the gross, will give the net weight.

E. 3. What is the net weight of 57 *Cwt.* 3 *qrs.* 14 *lb.* gross, tare at 16 *lb.* per *Cwt.*?

	Cwt.	*qrs.*	*lb.*	
16 *lb.* $=\frac{1}{7}$)	57	3	14	Gross
	8	1	2	Tare
Answer	49	2	12	Net

CASE 3. When tret is allowed with the tare to find the net weight,

RULE. Find the tare as before, and subtract it from the gross, the remainder will be the suttle, which divide by 26, and the quotient will be the trett, which subtract from the suttle, the remainder will be the net weight.

E 4. In 12 *Cwt.* 1 *qr.* 18 *lb.* gross, tare 40 *lb.* trett 4 *lb.* per 104, what is the net weight?

From

		Cwt.	qrs.	lb.	
From the grofs	- - -	12	1	18	
Deduct 40*lb.* tare =	. .	0	1	12	
	26)	12	0	6	Suttle
	-		1	$23\frac{3}{4}$	Trett
Anfwer	- -	11	2	$10\frac{1}{4}$	Net wt.

CASE 4. When cloff is allowed with tare, to find the net weight,

RULE. Divide the whole grofs by 168, 2 pounds being the 168th part of 3 hundred weight, or 336 pounds; or divide the number of hundreds by 3, which brings them into 3 hundreds; then 2 pounds being allowed for every three hundred, fo as many as it produces, fo many 2 pounds it will allow, which divided by 56 (the double pounds in a hundred weight) the quotient will be the hundreds, and the remainder will be fo many 2 pounds, to which adding what may be allowed for the odd hundreds, quarters, and pounds of the given weight, will make the whole cloff, which fubtract from the grofs, will be the net weight.

E. 5. What will be the net weight of 5647 *Cwt.* 3 *qrs.* 13 *lb.* grofs, allowing for cloff 2*lb.* for every hundred weight?

	Cwt.	qr.	lb.	
	168)5647	3	13	Grofs
	33	2	13	Cloff
Anfwer	5614	1	0	Net

CASE 5. When tare, trett, and cloff are allowed with any quantity grofs, to find the net weight,

RULE. For the tare and trett, proceed as in Cafe 3, and the remainder, which was called the net there, will be the futtle here, and to find the cloff, proceed as in the laft cafe.

EXAMPLE. What is the net weight of tobacco, weighing

	Cwt.	qr.	lb.	
No 1,	5	3	10	Grofs { Tare 7*lb.* per *Cwt.* trett 4*lb.* per 104, and cloff 2*lb.* per *Cwt.*
2,	4	1	12	
Wt. grofs	10	0	22	

7*lb.* is $\frac{1}{16}$)	10	0	22	Grofs	Cwt.
	0	2	$15\frac{1}{4}$	Tare	9
					2
26)	9	2	$6\frac{3}{4}$		
Deduct	0	1	13	Trett	3)18
	9	0	$21\frac{3}{4}$	2d Suttle	6 Cloff
Deduct	0	0	6	Cloff	
Net -	9	0	$15\frac{3}{4}$	Anfwer	

Note. What odd weight remains in finding cloff is inconfiderable, and need not be noticed.

E. 2. The net proceeds of a hogshead of sugar, were 4*l.* 14*s.* 6*d.* the custom and fees 2*l.* 8*s.* 6*d.* freight 22*s.* 8*d.* factorage 4*s.* 9*d.* the gross weight was 9 *Cwt.* 3 *qrs.* 10 *lb.* tare 1 *lb.* in ten; pray then, how was the sugar rated in the bill of parcels?

	£.	s.	d.
Net proceeds	4	14	6
Custom, &c.	2	8	6
Freight	1	2	8
Factorage	0	4	9
	8	10	5=2045 *d.*

	C.	qrs.	lb.	
$\frac{1}{10}$)	9	3	10	Gross
	0	3	$26\frac{1}{5}$	Tare
	8	3	$11\frac{4}{5}$	Net

8
4
——
35
28
——
281
71
——
991
5
——
4959 Fifths

Cwt.	qr.	lb.
1	0	0

4
——
4
28
——
112
5
——
560 Fifths

Then, if 4959 : 2045 :: 560
560
——
122700
10225
—— 12)
4959)1145200(230 —— 19*s.* 2*d.*
9918
——
15340
14877
——
4630
4
——
4959)18520($\frac{3}{4}$ *qrs.*
14877
——
3643 Remainr.

Answer 19*s.* $2\frac{3}{4}$*d.* $\frac{3643}{4959}$

XVIII. SIMPLE INTEREST.

IS the profit allowed for the use of any sum of money for a certain time; the money so lent upon interest is called the principal; the rate per cent. per annum is the sum allowed for the use of 100*l.* a year, which according to the law must not exceed 5*l.* the amount is the principal and interest added together.

CASE 1. To find the interest of any sum of money for any number of years,

RULE. Multiply the principal by the rate per cent. and cut off two figures towards the right hand (which is the same as dividing by 100) and the figures towards the left are pounds. Then multiply the figures thus cut off to the right hand by 20, and take in the odd shillings (if any) and cut off two figures as before, and the figures on the left hand are shillings; then multiply the remainder by 12, and cut off two figures, and the figures on the left hand are pence. Again, multiply by 4, and cut off as in the others, and you have the farthings.

Note. The rules for simple interest serve also for calcuating factorage, brokerage, insurance, purchasing of stocks, or any thing else, that is rated at so much per cent.

EXAMPLE

EXAMPLE 1.

What is the intereſt of 465*l.* for a year, at 5*l.* per cent. per annum?

l.
465 Principal
5 Rate per cent.

23|25
20

5|00

Anſwer 23*l.* 5*s.*

The ſame by practice,

l. *l.*
$5 = \frac{1}{20}$) 465

Anſwer £. 23 5

The above example is worked by two different methods, to ſhew the conciſeneſs of each.

E. 2. What is the intereſt of 212*l.* 9*s.* 1*d.* for a year, at 4 per cent. per annum?

£. *s.* *d.*
212 9 1
4

8|48
20

9|69
12

8|29
4

1|16

Anſwer 8*l.* 9*s.* $8\frac{1}{4}$*d.* $\frac{16}{100}$

E. 3. What is the amount of 526*l.* 18*s.* 8*d.* for 6 years, at 4 per cent. per annum?

£.	*s.*	*d.*	£.	*s.*	*d.*	
526	18	8				
4						
21|04			21	0	$11\frac{3}{4}$	Int. for 1 yr.
20					6	No. of years
|98			126	5	$10\frac{1}{2}$	Int. for 6 yrs.
12			526	18	8	Principal
11,84			653	4	$6\frac{1}{2}$	Anſwer
4						
3|36						

CASE 2. When the rate per cent. is $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, more than the pounds given in the ſaid rate,

RULE. Multiply the principal by the pounds in the rate per cent. then take parts for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, from the principal, which add to the product, and the ſum divide by 100 as before.

E. 4. *At ſimple intereſt tell me plain*
What fourteen thouſand pounds will gain,
At three pound ten per cent. per annum,
For ſeven years, to pleaſe a grannum?

£.
$\frac{1}{2}$) 14000
$3\frac{1}{2}$

42000 = 3
7000 = $\frac{1}{2}$ } $3\frac{1}{2}$ Rate per cent.

490|00 Intereſt for 1 year
7 Number of years

3430£. Anſwer

E. 5. What is the amount of 320*l.* for 4 years, at $4\frac{3}{4}$ per cent. per annum?

£.
$\frac{1}{2}$ | $\frac{1}{2}$ | 320
$4\frac{3}{4}$

1280
$\frac{1}{4}$ | $\frac{1}{2}$ | 160
80

15|20
20

4|00

£.	*s.*	
15	4	
	4	
60	16	Intereſt
320	0	Principal
£. 380	16	Amount

Commission is an allowance generally made from a merchant to his agents or factors abroad, for buying and selling of goods, and is at a certain rate per cent. according to the custom of the country, where the factor or merchant resides,

E. 6. My factor writes me word, that he has bought goods upon my account, to the value of 649*l.* 10*s.* I desire to know what his commission comes to, at $3\frac{1}{4}$ per cent?

	£.	s.	d.
$\frac{1}{4}$)	649	10	
		$3\frac{1}{4}$	
	1948	10	
	162	7	6
	21|10	17	6
		20	
		2|17	
		12	
		2|10	

Answer 21*l.* 2*s.* 2*d.* $\frac{10}{100}$

Brokerage is an allowance or fee paid to brokers, for assisting others in buying, or disposing of their goods; and in the City of London they are not to act without a licence from the lord-mayor.

Case 3. To find the brokage for any sum of money, at any rate under one pound per cent,

RULE. Divide the given sum by 100, and it will give the interest at one pound per cent, which interest you must take parts from, with the rate per cent. and add them together, the sum will be the brokage required.

E. 7. What is the brokage of 682*l.* 10*s.* 6*d.* at 5*s.* 10*d.* per cent.?

£.	s.	d.
6|82	10	6
20		
16|50		
12		
6|06		

		£.	s.	d.
5	$\frac{1}{4}$	6	16	6
10	$\frac{1}{6}$	1	14	$1\frac{1}{2}$
			5	$8\frac{1}{4}$
Answer		1	19	$9\frac{3}{4}$

E. 8. Suppose a broker disposes of goods for me to the amount of 864*l.* 12*s.* 4*d.* what does the brokage come to, at 12*s.* 6*d.* per cent.?

£.	s.	d.
8|64	12	4
20		
12|92		
12		
11|08		
4		
|36		

		£.	s.	d.
10	$\frac{1}{2}$	8	12	11
2*s.* 6*d.*	$\frac{1}{4}$	4	6	$5\frac{1}{2}$
		1	1	$7\frac{1}{4}$
Answer		5	8	$0\frac{3}{4}$

Insurance is security given by persons who oblige themselves to answer for the loss or damage of ships, houses, goods, &c. by storms, pirates, fire, &c. in consideration of a premium paid by the proprietors of the thing injured.

E. 9.

E. 9. Suppoſe I make an inſurance of goods to the value of 6840*l.* at 2*s.* 6*d.* per cent. per annum, what doth the inſurance come to?

```
      £.
    68|40
       20
    -----
     8|00
    -----

s. d.      £.  s.
2  6 = 1/8)68  8
           ------
Anſwer £. 8 11
```

E. 10. Shipped at Jamaica goods to the value of 2500*l.* upon which I made an inſurance of $6\frac{1}{8}$*l.* per cent. what does it come to?

```
  2500            2500
     6               7
 ------          ------
 15000          8)17500
  2187 10       ------
 ---------        2187—4/8=10s.
 171|87 10       -----------
      20
 -------
   17|50
      12
 -------
    6|00          Anſ. 171l. 17s. 6d.
```

PURCHASING OF STOCKS.

Stocks are the public funds of the nation, the ſhares of which being transferable from one perſon to another, occaſion that extenſive buſineſs called ſtock-jobbing.

RULE, Multiply the ſum to be purchaſed, by the exceſs of the rate per cent. above 100; the product divide by 100, as before, and the quotient added to the given ſum, will give the purchaſe required.

Note. If under 100 per cent. proceed as in Caſe 2.

E. 11. What is the purchaſe of 460*l.* South Sea ſtock, at 116*l.* 4*s.* per cent?

```
s.      £.     £.   s. d.
4=1/5)460     460   0 0    Principal
       16      74  10 4 3/4 Intereſt for
      ----    -----------   the exceſs
     7360     534  10 4 3/4 Anſwer
       92     -----------
     -----
    74|52
       20
    -----
    10|40
       12
    -----
     4|80
        4
    -----
     3|20
```

E. 12. What is the purchaſe of 320*l.* bank ſtock, at $87\frac{3}{4}$ per cent?

```
              £.
1/2|1/2|     320
   |   |      87
   |   |     ----
   |   |    2240
   |   |   2560
   |   |   -----
   |   |   27840
1/4|1/2|     160
              80
          ------
          280|80
              20
          ------
           16|00
Anſwer   280l. 16s.
```

Case 4. When the intereſt is for $\frac{3}{4}$, $\frac{1}{2}$, of $\frac{1}{4}$ of a year, or any number of years beſides,

RULE. Find the intereſt for the years, as in Caſe 1; then for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, take parts from the intereſt of 1 year, i. e. for $\frac{1}{4}$, take one-fourth part of the ſaid intereſt, for $\frac{1}{2}$ take one-half, &c. which, added to the intereſt for years (if any) the ſum will be the required intereſt.

E. 13.

E. 13. What is the intereſt of 462*l.* for 3 months, at 4 per cent, per annum?

	£.		
	462		
	4		
	18|48		
	20		
	9|60		
	12		
	7|20		
mo.	£.	*s.*	*d.*
3|¼|	18	9	7 Int. for 1 year
	4	12	4¼ Anſwer

E. 14. A gentleman dying, left his daughter 604*l.* 17*s.* 6*d.* for her fortune, to be paid her when at age, with intereſt, at 5*l.* per cent. per annum. Now ſhe came of age in 3 years 9 months, after her father's death, what is the amount of her fortune?

5	$\frac{1}{20}$	604	17	6	
6 *mo.*	½	30	4	10½	
				3	
		90	14	7½	=3 years int.
3 *mo.*	½	15	2	5¼	=6 months
		7	11	2½	=3 months
		604	17	6	Principal
Anſ. £.		718	5	9¼	Amount

CASE 5. When the intereſt required is for any number of weeks,

RULE. Find the intereſt of the given ſum for a year, and then ſay as 52 weeks are to that intereſt, ſo are the weeks given to the intereſt required.

E. 15. What is the amount of 800*l.* for 13 weeks, at 4¾ per cent. per annum?

	£.
½=½	800
	4¾
	3200
¼=½	400
	200
£.	38|00

w.	£.	*s.*	
13=¼)	38	0	
	9	10	Intereſt for 13 weeks
	800	0	Principal
Anſwer	809	10	Amount

CASE 6. To find the intereſt of any ſum for any number of days,

RULE. Find the whole year's amount, then ſay as 365 days are to the year's intereſt, :: ſo are the number of days given : to the intereſt required.

A TABLE

A TABLE of DAYS for any given time less than a Year.

Days	January	February	March	April	May	June	July	August	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	60	88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The USE of the TABLE.

First, to know the number of days from the beginning of the year to any given day of any month.

This is obtained by inspection only; thus, from January the 1st to July the 14th, is 195 days; to September the 26th is 269 days, &c.

Secondly, to know what is the number of days from any given day of any month, to the end of the year;

Suppose June the 4th; then from	- - -	365 days
Subtract the number answering to June 4th	-	155
There remains the number of days sought, viz.	-	210

Thirdly,

Thirdly, to find the number of days between the given day of any one month, and any given day of any other month, in the same year.

For instance, to know how many days there are between May the 8th, and September the 4th;

Thus, from the number to September the 4th -	247
Subtract that answering to May 8th - -	128
The remainder is the number of days sought -	119

Fourthly, to find the number of days from any given day of any month in one year, to any given day of any month in the next year.

How many days is it from September the 7th in one year, to April the 19th in the next?

From the days of a whole year - -	365
Subtract the number to September the 7th - -	250
Remains the number to the end of the year -	115
To which add the number to April 19th -	109
Ths sum is the number of days required, viz.	224

And thus is the number of days readily found for any interval of time given, in the same year, compleatly; or which is part of one, or part of another year.

E. 16. What is the interest of 399*l.* 13*s.* 4*d.* for 4 days, at 5 per cent. per annum?

£. s. *d.*
399 13 4
5
19|98 6 8
20
19|66
12
8|00

Days. £. *s.* *d.* *days.*
If 365 : 19 19 8 :: 4
20
399
12
4796
4
12)
365)19184(52
1825
4*s.* 4½*d.* Ans.
934
730
204
4
365)816(2 *qrs.*

E. 17. What is the amount of 340*l.* 10*s.* from Jan. 1st, 1781, to July 18th following, at 5 per cent?

First, by the table, from Jan. 1st, to July 18th, there are 199 days.

£. *s.* *Days.* £. *s.* *d.* *days.*
340 10 If 365 : 17 0 6 :: 199
5 20
17|02 10 340
20 12
|50 4086×199=
12 813114
6|00

12)
365)813114(2227
730
2|0)18|5 7
831
730 9 5 7½ Interest
340 10 0 Principal
1011
730 £. 349 15 7½ Amount
2814
2555
259
4
365)1036(2 *qrs.*

CASE

CASE 7. When the amount, time, and rate per cent. are given to find the principal,

RULES. 1. Say, as the amount of 100*l.* at the rate and time given is to 100*l.* ſo is the amount given to the principal required.

Note. The examples in this, and the two following caſes, may be ſolved by the rule in compound proportion.

E. 18 What principal being put out to intereſt for 8 years, at 5 per cent. per annum will amount to 429*l.*?

5	Rate per cent.
8	Time
40	Intereſt
100	Principal
140	Amount

```
  l.      l.      l.
If 140 : 100 :: 429
                100
          140)42900(306
              420
              ---
                900
                840
                ---
                 60
                 20
          140)1200(8
              112
              ---
               80
               12
          140)960(6
              840
              ---
              120
                4
          140)480(3
              420
              ---
               60
```

Anſwer 306*l.* 8*s.* 6$\frac{3}{4}$*d.* $\frac{60}{140}$

E. 19. What principal, being put to intereſt for 9$\frac{1}{2}$ years, at 4$\frac{1}{2}$ per cent. per annum, will amount to 856*l.* 10*s.*?

	£.	s.	
$\frac{1}{2}=\frac{1}{2}$)	4	10	Rate per cent.
		9$\frac{1}{2}$	
	40	10	
	2	5	
	42	15	Intereſt
	100	0	
	142	15	Amount

```
   £.  s.     £.     £.  s.
If 142 15 : 100 :: 856 10
    20                 20
  2855               17130
                       100
          2855)1713000(600l.
               17130
               -----
                 000
```

Anſwer 600*l.*

CASE 8. When the principal, rate per cent. and the amount are given, to find the time,

RULE. Say, as the intereſt of the principal for a year is to one year, ſo is the whole intereſt to the time required.

E. 20. In what time will 132*l.* amount to 171*l.* 12*s.* at 5 per cent. per annum?

132 Principal	£.	s.	
5 Rate per cent.	171	12	Amount
	132	0	Principal
6|60			
20	39	12	Interest
12|00			

	£.	s.		Year.		£.	s.
If	6	12	:	1	::	39	12
	20					20	
	132					132)792(6 Years, the Answer	
						792	

Case 9. When the principal, amount, and time are given, to find the rate per cent.

RULE. 1. Say, as the principal is to the interest for the whole time, so is 100*l.* to the interest for the same time.

2. Divide that interest by the given time, and the quotient will be the rate per cent. required.

E. 21. At what rate per cent. per annum, will 528*l.* amount to 686*l.* 8*s.* in 6 years?

£.	s.			£.		£.	s.		£.
686	8	Amount	If	528	:	158	8	::	100
528	0	Principal				20			
158	8	Interest				3168			
						100			

2|0)60|0	528)316800(60|0 Shillings
Time 6)30	3168
Answer 5 Rate per cent.	000

PROMISCUOUS QUESTIONS.

Quest. 1. Lent at Christmas 1771, the sum of 5000*l.* at $4\frac{1}{2}$ per cent. after which time I lent several sums at the same rate, and drew upon the borrower as business required; viz. on Lady-day 1772, I drew for 185 guineas; on Midsumer-day following I lent 500 moidores, and drew for 700*l.* and on Michaelmas-day in the same year, I lent 569*l.* 17*s.* I demand what cash the borrower owed me at that time?

First,

First, 5000
$4\frac{1}{2}$

20000
2500

4)225|00 Intereſt for 1 year.

£. 56 5s. Intereſt due to Lady-day, which is a quarter and by proceeding in this manner with each new principal, you will gain the reſpective intereſts; as in the work following:

		£.	s.	d.	
1771.	Lent at Chriſtmas -	5000	0	0	at $4\frac{1}{2}$ per cent.
1772.	Intereſt due at Lady-day	56	5	0	
	Amount - -	5056	5	0	
	Drew out -	194	5	0	= 185 Guineas
	Remains -	4862	0	0	New principal
	Intereſt of the ſame till Midſ.	54	13	$11\frac{1}{4}$	
	Amount -	4916	13	$11\frac{1}{4}$	
	Paid 500 moidores =	675	0	0	
	Sum - -	5591	13	$11\frac{1}{4}$	New principal
	Drew out -	700	0	0	
	Remains -	4891	13	$11\frac{1}{4}$	
	Intereſt to Michaelmas	55	0	$7\frac{1}{2}$	
	Amount -	4946	14	$6\frac{3}{4}$	
	Paid in part -	569	17	0	
	Anſwer £.	5516	11	$6\frac{3}{4}$	Caſh due to me.

Queſt. 2. Lent to John Jemeſon, per bill, dated 18th of Jan. 1771. payable one day after date, 878*l.* 19*s.* 10*d.* which I received back in the following partial payments, viz. on the 27th of Feb. 57*l.* 15*s.* 7*d.* on the 18th of March 37*l.* 14*s.* on the 29th of April 34*l.* 11*s.* on the 12th of May 136*l.* 15*s.* 7*d.* on the 19th of June 67*l.* 13*s.* 4*d.* on the 15th of July 15 guineas and 6*d.* on the 25th ditto 111*l.* 11*s.* 11*d.* on the 3d of October 78*l.* 7*s.* 4*d.* on the 19th of November 100*l.* on the 23d ditto 100*l.* and on the 30th of Dec. received the balance of the principal; how much intereſt ought I to claim, at 5 per cent.?

Note. The reſpective products are found by multiplying each principal by the number of days it was employed; ſee the following operation.

	£.	s.	d.	days	Products £.	s.	d.
To a bill payable 1 day after date	878	19	10	40	35159	13	4
Feb. 27, Received in part	57	15	7				
Balance	821	4	3	19	15603	0	9
Mar. 18, Received	37	14	0				
Balance	783	10	3	42	32907	10	6
Apr. 29, Received	34	11	0				
Balance	748	19	3	13	9736	10	3
May 12, Received	136	15	7				
Balance	612	3	8	38	23262	19	4
June 19, Received	67	13	4				
Balance	544	10	4	26	14157	8	8
July 15, Received	15	15	6				
Balance	528	14	10	10	5287	8	4
25, Received	111	11	11				
Balance	417	2	11	70	29200	4	2
Oct. 3, Received	78	7	4				
Balance	338	15	7	47	15922	12	5
Nov. 19, Received	100	0	0				
Balance	238	15	7	4	955	2	4
23, Received	100	0	0				
Balance	138	15	7	37	5134	16	7
Dec. 30, Rd. in full of the prin.	138	15	7				

Then, £. s. d. l.
73|00)1873|27 6 8)25
146
413
365
4827
20
73|00)965|46(13*s.*
73
235
219
1646
12
73|00)197|60(2*d.*
146
5160
4
73|00)206|40(2 *qrs.*
146
604

Answer, 25*l.* 13*s.* $2\frac{1}{2}d\frac{604}{730}$ the interest required.

The total sum of the products 187327 6 8

Note. The reason of dividing by 73 in the above operation, is this; as 100 : 365 :: 5, or any other rate to the fourth term. Or, as 100 : 73 :: 1, that is $\frac{365}{5} = 73$, and $\frac{5}{5} = 1$. Hence the second and third terms will allways admit of the same abbreviations.

Quest. 3. June 23d, 1745, bought 900*l.* of New South Sea annuities at $111\frac{3}{8}$ per cent. viz. the day before the closing the books, the brokerage whereof is always 2*s.* 6*d.* per cent, on the capital, whether you buy or sell; the Midsummer dividend 2 per cent. became due and payable on the 10th of August following, by which time the rebellion growing considerable in the North, the said annuities where down at $92\frac{1}{2}$ per cent. In the general alarm, sold 400*l.* capital at that price; but continued the remainder, till a second, third, fourth, and fifth dividend,

dividend, as before, came due; and on opening the books on the 10th of August, 1747, sold out at $102\frac{5}{8}$ per cent. Now reckoning I might have made 5 per cent. of my money, had I kept it out of the stocks, how stood this article in point of profit and loss?

	£.	s.	d.
First 900*l*. at $111\frac{3}{8}$ per cent. = –	1002	7	6
Brokerage of ditto, at 2*s*. 6*d*. per cent. = –	1	2	6
£.	1003	10	0
Midsummer dividend, at 2 per cent. –	18	0	0
	985	10	0
Interest of 1003*l*. 10*s*. for 49 days. at 5 per cent.	6	14	8
Brokerage of 400*l*. at 2*s*. 6*d*. per cent. –	0	10	0
	992	14	8
Sold 400*l*. at $92\frac{1}{2}$ per cent. – –	370	0	0
	622*l*	14	8
Interest for half a year, due Feb. 10, 1746	15	11	$4\frac{1}{4}$
	638	6	$0\frac{1}{4}$
Dividend received at that time –	10	0	0
	628	6	$0\frac{1}{4}$
Interest due to August 10th – –	15	14	$1\frac{3}{4}$
	644	0	2
Dividend received at that time –	10	0	0
	634	0	2
Interest due to February, 1747 –	15	17	0
	649	17	2
Dividend received then – –	10	0	0
	639	17	2
Interest due to the 10th of August –	15	19	11
	655	17	1
Midsummer dividend received August 10th	10	0	0
	645	17	1
Sold off 500*l*. at $102\frac{5}{8}$ per cent. –	512	2	6
	133	14	7
Brokerage – – – –	0	12	6
Answer, Lost in the whole £.	133	2	1

XIX. COMPOUND INTEREST.

IS that which arises from any principal, and its interest put together, as that interest becomes due but not paid, the same interest is allowed upon that interest unpaid, so it becomes part of the principal, for which reason it is called interest upon interest, or compound interest.

RULE. 1. Find the amount of the given sum by simple interest for the first year, which is the principal for the second year; then find the amount of that principal for the second year, and that is the principal for the third year; and so on for any number of years.

2. Subtract

2. Subtract the given principal from the last amount, and the remainder is the compound interest required.

EXAMPLES. What is the compound interest of 900*l.* forborne 3 years, at 5 per cent. per annum?

£.	
900	= 1st year's principal
5	
45|00	

£.	
900	Principal
45	Interest
945	= 2d year's principal
5	
47|25	
20	
5|00	

£.	s.	
945	0	Principal
47	5	
992	5	= 3d year's principal
	5	
49|61	5	
20		
12|25		
12		
3|00		

	£.	s.	d.	
	992	5	0	
	49	12	3	
	1041	17	3	Amount
	900	0	0	Principal
Answer	141	17	3	Comp. Int.

The preceeding example performed otherwise, thus:

	£.	s.	d.	
$5=\frac{1}{20}$)	900	0	0	= 1st year's pr.
	45	0	0	= Interest
$5=\frac{1}{20}$)	945	0	0	= 2d year's pr.
	47	5	0	Interest
$5=\frac{1}{20}$)	992	5	0	= 3d year's pr.
	49	12	3	= Interest
	1041	17	3	= Amount
	900	0	0	Principal
Anſ.	141	17	3	Comp. intereſt, as before.

The foregoing methods being rather tedious (though generally taught in ſchools) I have thought proper to omit giving any more examples, till I come to treat on decimals, where the ſame may be more conveniently and expeditouſly performed.

XX. REBATE OR DISCOUNT.

IS the ſatisfying any ſum of money due at ſome time to come, by paying ſo much preſent money as being put to intereſt would amount to the given ſum, in the ſame ſpace of time.

RULE. 1. Find the intereſt for 100*l.* for the time given, and rate per cent. which intereſt add to 100*l.*

2. Then ſay, as that ſum is to the intereſt of 100*l.* ſo is the debt, or ſum propoſed, to the rebate, or preſent worth required. Or when the preſent worth is ſubtracted from the given ſum, the remainder is the rebate required.

EXAMPLE.

EXAMPLE 1. What is the rebate of 210*l.* for 7 months 6 days, at 5 per cent. per annum?

mo.	*da.*	£.	*s.*	
6	$0 = \frac{1}{2})$	5	0	Per cent.
1	$6\frac{1}{5} =)$	2	10	
		0	10	
		3	0	Intereſt
		100	0	Principal
		103	0	

If 103 : 3 :: 210
210
103)630(6
618
12
20
103)240(2
206
34
12
103)408(3
309
99
4
103)396(3
309
87

Anſwer 6*l.* 2*s.* $3\frac{3}{4}$*d.* $\frac{87}{103}$.

E. 2. What is the preſent worth of 100*l.* for 12 months, at 6 per cent? First, 100+6=106*l.* amount of 100*l.* for a year.

£. £. £.
Then if 106 : 100 :: 100
×100
106)10000(94*l.* 6*s.* $9\frac{1}{2}$*d.* $\frac{2}{53}$ the Anſwer

E. 3. Sold goods to the value of 73*l.* 5*s.* to be paid in a years time; what muſt be diſcounted for preſent payment, if rebate be allowed at $4\frac{1}{2}$ per cent?

£.		£.	*s.*	
$\frac{1}{2})$100		100	0	Prin.
$4\frac{1}{2}$		4	10	Int.
400				
50				
4|50				
20				
10|00				

	£.	*s.*		£.	*s.*		£.	*s.*
If	104	10	:	4	10	::	73	5
	20			20			20	
	2090			90			1465	
							90	2|0

209|0)13185|0(6|3
1254 £.3 3 1*d.* $\frac{7}{209}$ Anſ.
645
627
18
12
209)216(1*d.*

The

The common method of discounting bills is done by the intereſt of the whole ſum for ſo long a time: but ſuch ſum cannot be eſteemed a principal, nor is it in full value, till the time of payment is expired; therefore, leſs intereſt muſt be required according to the true rules of diſcount.

E. 4. What difference is there between the intereſt of 500*l.* at 5 per cent. per annum, for 12 years, and the diſcount of the ſame ſum, at the ſame rate, and for the ſame time?

£. *l.*
$5 = \frac{1}{20}$)500 Principal
25 Int. of 500*l.* for 1 year
12 Number of years
300 Intereſt for 12 years

Then 5 × 12 = 60*l.* int. of 100*l.* for 12 yrs.
If 160 : 60 :: 500
500
16|0)3000|0(187*l.* 10*s.* Diſ.
16
140
128
120
112
8
20
16)160(10
16
0

	£.	*s.*	
	300	0	Intereſt
	187	10	Diſcount
Anſwer	112	10	Difference

Note. By the preceeding examples it is evident, he who allows intereſt for diſcount wrongs himſelf conſiderably; for ſo much money ought to be paid, as at intereſt would amount to the ſum due in the time propoſed.

E. 5. What ready money will diſcharge a debt of 134*l.* due two years, three quarters, diſcount at $4\frac{3}{8}$ per cent. per annum?

20
3
8)60
7 6 = $\frac{3}{8}$*l.*

mo.	£.	*s.*	*d.*
6 | $\frac{1}{2}$	4	7	6 = $4\frac{3}{8}$*l.*
			2
	8	15	0
3 | $\frac{1}{2}$	2	3	9
	1	1	$10\frac{1}{2}$
Intereſt	12	0	$7\frac{1}{2}$
Princip.	100	0	0
Amount	112	0	$7\frac{1}{2}$

£. *s.* *d.* £. £.
If 112 0 $7\frac{1}{2}$: 100 :: 134
20 20
2240 2680
12 12
26887 32160
4 4
107550 128640
100
l.
10755|0)1286400|0)119
12*s.* $2\frac{1}{4}$

Anſwer 119*l.* 12*s.* $2\frac{1}{4}$*d.* $\frac{1525}{10755}$

E. 6.

E. 6. What is the present value of a 10*l.* bill due 4 months hence, discounted at 4 per cent?

mo.	*l.*	*s.*	*d.*	
$4 = \frac{1}{3}$)	4	0	0	
	1	6	8	Interest
	100	0	0	Principal
	101	6	8	

l. *s.* *d.* : *l.* :: *l.*

If 101 6 8 : 100 :: 10

20 | 20
2026 | 200
12 | 12
24320 | 2400
 | 100

2432|0)24000|0(9
21888
2112
20
2432)42240(17
2432
17920
17024
896
12
2432)10752(4
9728
1024
4
2432)4096(1

Answer 9*l.* 17*s.* $4\frac{1}{4}$*d.* $\frac{1664}{2432}$

E. 7. What is the present money and discount of one pound for one year, at 5 per cent. per annum?

£.
100
5
500

l. : *l.* :: *l.*

If 105 : 100 :: 1
20
105)2000(19
105
950
945
5
12
60
4
105)240(2
210
30

	l.	*s.*	*d.*	
From	1	0	0	
Take	0	19	$0\frac{1}{2}$ $\frac{30}{105}$	present money
	0	0	$11\frac{1}{4}$ $\frac{75}{105}$	reb. or discount

By the last example it appears that one pound in a year is decreased to 19*s.* $0\frac{1}{2}$*d.* $\frac{30}{105}$ at 5*l.* per cent.

XXI. EQUATION of PAYMENTS.

IS when several debts are payable at different times, but is mutually agreed upon between debtor and creditor, that all those several sums be paid at once, without loss to debtor or creditor.

RULE. Multiply the sum of each particular payment by the time it is to continue in the hands of the debtor; add these products together, and divide the sum by the whole debt, the quotient is the equated time for the payment of the whole debt.

REMARK. The above rule is not exactly true, though it may serve in common business; but to find the just mean or equated time of payment, you

you muſt firſt find out the preſent payment of every particular ſum in the queſtion, payable at a time to come, by rebating at the rate of intereſt agreed on; then find in what time the ſum of thoſe preſent worths will be augmented to the total of all the particular ſums payable at times to come, according to the firſt agreement; ſo ſhall the time found out be the mean for paying the whole debt.

EXAMPLE 1. A owes B 140*l.* which by agreement was to be paid as follows, viz. 50*l.* at 2 months, and 90*l.* at 6 months; but they agree that the whole ſhould be paid at once; required the equated time of payment?

l.		*mo.*		*l.*
50	×	2	=	100
90	×	6	=	540
140				14|0)64|0(4 Mo. 2 weeks, 2 days, Anſwer.

E. 2. James owes Thomas 80*l.* which is to be paid as follows, viz. 40*l.* at 3 months, and 40*l.* at 7 months, but they agree to reduce the whole to one payment; query, the equated time?

l.		*mo.*		*l.*
40	×	3	=	120
40	×	7	=	280
				8|0)40|0
Anſwer				5 Months

E. 3. C owes D 600*l.* whereof 200*l.* is to be paid at 3 months, 150*l.* at 4 months, and the reſt at 6 months; but they agree the whole ſhould be paid at once, required the time?

l.		*mo.*		*prod.*
200	×	3	=	600
150	×	4	=	600
250	×	6	=	1500
6|00				6|00)27|00
				Anſwer 4 Mo. 15 days.

E. 4. B owes C a certain ſum, which is to be diſcharged thus, viz. $\frac{1}{4}$ preſent, $\frac{1}{4}$ at 4 months, $\frac{1}{4}$ at 5 months and the reſt at 6 months; what is the equated time for the whole?

In this example the debt is to be paid at 4 equal payments, and $\frac{1}{4}$ being paid down, there remains $\frac{3}{4}$ to be paid at 3 equal payments; conſequently, the ſum of the different times that each payment is to be made, being divided by 3, will give the anſwer, thus:

Firſt, $4+5+6=15$, $15\div3=5$ Months, the Anſwer.

E. 5. A debt is to be diſcharged in the following manner, viz. $\frac{1}{2}$ at 3 months, $\frac{1}{3}$ in 4 months, and $\frac{1}{6}$ in 9 months; but they afterwards agree to have but one payment of the whole; the equated time is required?

Suppoſe 120*l.* to be the ſum owed.

$$\text{Then, } \left.\begin{matrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{6}\end{matrix}\right\} \begin{matrix}\pounds.\\ 120 =\end{matrix} \left\}\begin{matrix}\pounds.\ mo.\quad pro.\\ 60\times3=180\\ 40\times4=160\\ 20\times9=180\end{matrix}\right.$$

12|0)52|0

mo. 4—4

30 Days in one month

Anſwer, 4 months, 10 days. 12)120

10 Days

In

In examples of the above nature, any number may be taken at pleaſure, that is diviſable into the propoſed parts, without a remainder.

Scholium. I might introduce various other rules by different authors who have endevoured to make improvements on this common method, but room will not permit them; and the common method being more adapted to practice, and is near enough the truth in common affairs.

Mr. Malcolm's rule is the only true one, which is as follows,

Put d for the firſt payment, t the diſtance of its term of payment; D the laſt payable debt, and T the diſtance of its term, and r the rate of one year's intereſt for 1*l.* and $x =$ the diſtance of the equated time, Then by proceeding according to the principles of ſimple intereſt, we have $T+t+\frac{D+d}{dr}$ the firſt number found,

And $\frac{DT+dt}{ar}+Tt$ the ſecond number found, which two numbers are called a and s, then $ax-x^2=s$, whence $x=\frac{a\pm\sqrt{a^2-4s}}{2}$ the preſent rule, or equated time for any two payments.

XXII. SINGLE FELLOWSHIP:

OR,

FELLOWSHIP WITHOUT TIME.

IS when two or more perſons join their ſtocks and trade together: to determine how much gain or loſs is due to every partner concerned, by having the whole gain or loſs, and their particular ſtocks given.

RULE. As the ſum of their ſeveral ſtocks to the gain or loſs, ſo is each perſon's ſhare in the ſtock, to his ſhare in the gain or loſs.

Proof. Add all the ſhares together, and that ſum (if right) will be equal to the whole gain or loſs.

Example 1. Two perſons, A and B, join in partnerſhip; A lays in 40*l.* B 80. and they gain 50*l.* what is each man's ſhare of the ſaid gain?

	l.		*l.*		*l.*		*l.*		*l.*	*s.*	*d.*		
A's ſtock	40	If	120	:	50	::	40	:	16	13	4	A's	Gain
B's ——	80	If	120	:	50	::	80	:	33	6	8	B's	
	120						Proof	£.	50	0	0		

E. 2. Three perſons, C, D, and E, trade together, and make a joint ſtock of 824*l.* and in three years time they gained as much, and 70*l.* over; C's ſtock was 320*l.* D's 340*l.* I demand E's ſtock, and what each perſon gained by trading?

First, 320+340=660*l.* C and D put in; then from 824*l.* take 660*l.* remains 164*l.* E's stock; and 824+70=894*l.* their whole gain; then,

	£.		£.		£.		£.	s.	d.	Rem.		
	£.		£.		320	:	347	3	8¼	72	C's	Gain
As	824	:	894	::	340	:	368	17	8	128	D's	
					164	:	177	18	7½	624	E's	
						Proof	894	0	0			

E. 3. *Some time ago, as people say,*
A debt four men agreed to pay,
Of just one pound each share was fix'd,
One-third, one-fourth, one-fifth, one-sixth.

Then, Tyro, what was each man's due
Of cash to pay? Pray tell me true.

First, the fractions in this example, viz. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, parts of 20 shillings, when added together make just 19*s.* =228*d.* and each respective part, viz. $\frac{1}{3}$ = 80*d.* $\frac{1}{4}$ = 60*d.* $\frac{1}{5}$ =48*d.* $\frac{1}{6}$ = 40*d.* then say,

d. s. d.
As 228 : 20 :: 80
80
228)1600(7*s.*
1596
4
12
48
4
192 *Rem.*

d. s. d.
As 228 : 20 :: 60
60
228)1200(5*s.*
1140
60
12
228)720(3*d.*
684
36
4
144 *Rem.*

d. s. d.
As 228 : 20 :: 48
20
228)960(4*s.*
912
48
12
228)576(2*d.*
456
120
4
228)480($\frac{1}{2}$*qr.*
456
24 *Rem.*

d. s. d.
As 228 : 20 :: 40
40
228)800(3*s.*
684
116
12
228)1392(6*d.*
1368
24
4
96 *Rem.*

The remainders in the above operations being 192, 144, 24, and 96, which added together, thus, 192+144+24+96=456, which, divided by

by the ſum of their money, paid the quotient = 2, which is added to the farthings for the proof, thus:

				l.	*s.*	*d.*	
Anſwer	1ſt	Man's ſhare		0	7	0	$\frac{192}{228}$
	2d			0	5	3	$\frac{144}{228}$
	3d			0	4	$2\frac{1}{2}$	$\frac{24}{228}$
	4th			0	3	6	$\frac{96}{228}$
			Proof	1	0	0	

Note. The late Mr. Sadler has expeditiouſly ſolved this queſtion by vulgar fractions; but to ſhew my readers that queſtions of this kind are very well adapted to this rule, was the reaſon of my inſerting it in this place.

E. 4. Four merchants, A, B, C, and D, join their ſtocks and trade together, of which A put in one-half, B one-third, C one-fourth, and D one-fifth; but at the expiration of twelve months, they had the misfortune to loſe 120*l.* what muſt each perſon ſuffer of the ſaid loſs?

Note. You may ſuppoſe any ſum at pleaſure to be their ſtock; as, ſuppoſe 600*l.*

$\frac{1}{2}$	of 600*l.* =	300 A's	ſtock
$\frac{1}{3}$		200 B's	
$\frac{1}{4}$		150 C's	
$\frac{1}{5}$		120 D's	
		770 Sum, then	

			£.		£.	*s.*	*d.*	*Rem.*		
As 770	:	120 ::	300	:	46	15	$0\frac{3}{4}$	9	A's	Loſs
			200	:	31	3	$4\frac{1}{2}$	6	B's	
			150	:	23	7	$6\frac{1}{4}$	43	C's	
			120	:	18	14	$0\frac{1}{4}$	19	D's	
			Proof		120	0	0			

E. 5. Four merchants, A, B, C, D, gain 2000*l.* by trade, whereof $\frac{1}{2}$ of A's ſhare is equal to $\frac{3}{4}$ of B's, $\frac{4}{5}$ of C's, and $\frac{5}{6}$ of D's; what ſhare had each?

Take any number at pleaſure, and divide in proportion to their ſhares, thus:

A's ſhare 120, then
B's —— 80
C's —— 75
D's —— 72

				£.		£.	*s.*	*d.*	*Rem.*	
Sum	347	:	£. 2000 ::	120	:	691	12	10	328	For A
				80	:	461	1	$10\frac{3}{4}$	103	—— B
				75	:	432	5	$6\frac{1}{4}$	205	—— C
				72	:	414	19	$8\frac{1}{2}$	78	—— D
				Proof		2000	0	0		

E. 6. A and B venturing equal ſums of money, clear by joint trade 308*l.*—By agreement, A was to have 8 per cent. becauſe he ſpent time in

in execution of the project, and B was to have only 5; the question is, what was allotted A for his trouble?

8
5
—
13 = their gain per cent.

$$\text{Then, as } \overset{\pounds.}{13} : \overset{\pounds.}{308} :: \left\{\begin{array}{l} \overset{\pounds.}{8} \\ 5 \end{array}\right. \begin{array}{l} : \\ : \end{array} \left.\begin{array}{rrrl} \overset{\pounds.}{189} & \overset{s.}{10} & \overset{d.}{9} & \frac{12}{13} \text{ A's} \\ 118 & 9 & 2\frac{3}{4} & \frac{1}{13} \text{ B's} \end{array}\right\} \text{Gain}$$

Answer, A had for his trouble 71 1 $6\frac{1}{4}$ $\frac{11}{13}$

XXIII. DOUBLE FELLOWSHIP:

OR

FELLOWSHIP WITH TIME.

IS when each person's stock continues unequal time in company; so that a consideration must be made of the time of continuance, as well as of the stock.

RULE. Multiply the particular stocks of each person by the time of continuance, and the sum of the several products, make the first term in the single rule of three direct; the whole gain or loss the second, and every man's particular stock, multiplied by its time, the third.

PROOF. Add all the parts of the gain or loss together, which must be equal to the whole.

EXAMPLE 1. Two persons, A and B, enter into partnership thus; A puts in 40*l.* for 18 months, and B 40*l.* for 12 months; they gain 120*l.* what is each man's share of the gain?

$$\left.\begin{array}{l} 40 \times 18 \\ 40 \times 12 \end{array}\right\} = \left\{\begin{array}{l} 720 \\ 480 \end{array}\right\} = \left\{\begin{array}{l} \text{A's} \\ \text{B's} \end{array}\right\} \text{Stock multiplied into his time.}$$

1200 The sum of the products

$$\text{Then, as } 1200 : 120 :: \left\{\begin{array}{l} 720 \\ 480 \end{array}\right. \begin{array}{l} : \\ : \end{array} \left.\begin{array}{l} 72 \text{ A's} \\ 48 \text{ B's} \end{array}\right\} \text{Share of the gain}$$

£. 120 Proof

E. 2. Three merchants, A, B, and C, enter into partnership; A puts in 65*l.* for 8 months; B 78*l.* for 12 months; and C 84*l.* for 4 months, and 90*l.* for 2 months; they gain 166*l.* 12*s.* what is each man's share of the gain?

$$\left.\begin{array}{l} \begin{array}{rcrcrl} 65 & \times & 8 & = & 520 & \text{A's} \\ 78 & \times & 12 & = & 936 & \text{B's} \end{array} \\ \text{Add} \left\{\begin{array}{rcrcr} 84 & \times & 4 & = & 336 \\ 90 & \times & 2 & = & 180 \end{array}\right\} \text{C's} \end{array}\right\} \text{Stock and time}$$

1972 Sum

As

$$\text{As } 1972 : \overset{\pounds.}{166}\ \overset{s.}{12} :: \left\{\begin{array}{rcl} 520 & : & 43\ 18\ 7\tfrac{1}{2}\text{ A's} \\ 936 & : & 79\ 1\ 6\tfrac{1}{4}\text{ B's} \\ 516 & : & 43\ 11\ 10\tfrac{1}{4}\text{ C's} \\ & & \overline{166\ 12\ 0}\text{ Proof} \end{array}\right\} \text{Gain}$$

E. 2. Two merchants together make up a ſtock of 600*l.* A's ſtock continued in company 9 months, and B's 11; they gain 200*l.* which they divide equally; how much did each put in?

Firſt, since the gains are equal, A's ſtock multiplied by his time 9, is equal to B's ſtock multiplied by his time 11, ∴ A's ſtock is to B's ſtock as 11 to 9.

$$\begin{array}{l} 11 \\ \underline{9} \\ 20 : 600 :: \left\{\begin{array}{rcl} 11 & : & \overset{\pounds.}{330}\text{ A's ſtock} \\ 9 & : & 270\text{ B's ſtock} \\ & & \pounds.\ \overline{600}\text{ Proof} \end{array}\right. \end{array}$$

E. 4. A ſhip's company take a prize, value 4000*l.* which they agree to divide amongſt them according to their pay and time, they have been on board; now the officers and midſhipmen have been on board 4 months, and the ſailors 3; the officers have 50*s.* a month, the midſhipmen 40*s.* and the ſailors 28*s.*—moreover, there are 4 officers, 8 midſhipmen, and 120 ſailors; I deſire to know what each perſon's ſhare is of the ſaid prize?

First $4 \times 4 \times 50 = 800$ Officers pay and time
$8 \times 4 \times 40 = 1280$ Midſhipmen's ditto
$120 \times 3 \times 28 = 10080$ Sailor's ditto

£. 12160 Sum

$$\text{As } 12160 : 4000 :: \left\{\begin{array}{rcllll} & & \pounds. & s. & d. & \\ 800 & : & 263 & 3 & 1\tfrac{3}{4} & 704\text{ Officers} \\ 1280 & : & 421 & 1 & 0\tfrac{1}{2} & 640\text{ Midſhipmen} \\ 10080 & : & 3315 & 15 & 9\tfrac{1}{4} & 1088\text{ Sailors} \\ & & \overline{4000} & 0 & 0 & \end{array}\right.$$

Note. The above being the ſhare of each company, each perſon's ſhare is found as follows, thus:

$$\left.\begin{array}{rrlcrcrrl} \pounds. & s. & d. & & \text{Number} & & \pounds. & s. & d. \\ 263 & 3 & 1\tfrac{3}{4} & \div & 4 & = & 65 & 15 & 9\tfrac{1}{4} \\ 421 & 1 & 0\tfrac{1}{2} & \div & 8 & = & 52 & 12 & 7\tfrac{1}{2} \\ 3315 & 15 & 9\tfrac{1}{4} & \div & 128 & = & 27 & 12 & 7\tfrac{1}{2} \end{array}\right\} = \text{Each perſon's ſhare}$$

The fractions, or remainders, are omitted, as inconſiderable.

E. 5. A and B paid equally for a horſe, February 7, 1781; A on the 10th took him a journey into the Weſt, and returned on the 10th of June following; B on the 2d of Auguſt took him into Scotland, and ſtayed till November 13, and this concluded his ſervice for this year. From January 17 following A uſed him 10 Days, and in ſix weeks after his return, employed him till April 30th; B then rode him

him from May-day to Midsummer; A had him from the 14th of July to 14 days after St. James's tide; B, on September 30th, took him into Norfolk, and came back October 19th; he then was sold for 7*l.* 10*s.* and they would have the money parted equitably between them, viz. in proportion to the use each made of their steed?

First, from February 10, to June 10	= 122		Days
January 17, to April 30	61	= 208 A's time	
July 14, to 14 after St. James's	25		
August 2, to November 13	104		
May 1, to July 24	55	= 179 B's time	
September 30, to October 19	20		
		387 = the time	

the horse was in use. Then,

$$\text{As } 387 : \overset{£.}{7}\ \overset{s.}{10} :: \left\{\begin{array}{l} 208 \\ 179 \end{array}\right. \begin{array}{l} : \overset{£.}{4}\ \overset{s.}{0}\ \overset{d.}{7\tfrac{1}{4}}\tfrac{237}{387} \text{ A's} \\ : 3\ \ 9\ \ 4\tfrac{1}{2}\tfrac{90}{387} \text{ B's} \end{array}\left.\right\}\text{Share}$$

7 10 0 Proof

XXIV. BARTER.

IS the exchanging wares for wares, or one commodity for another; and informs merchants so to proportion their goods, that neither may sustain loss or disadvantage by such a barter or exchange.

If the commodities exchanged are not of equal value, the defect is supplied with money.

RULE. 1. Find the value of that commodity, whose quantity is given; then find what quantity of the other, at the given rate, you can have for the aforesaid value, which quantity will be the answer.

2. When one has goods at a certain price ready money, but in barterage advances it to something more, say, as the ready money price of the one is to its bartering price, so is the ready money price of the other, to its bartering price; then the quantity of the latter commodity may be found either from the ready money or bartering price.

EXAMPLE 1. How much sugar, at 1*l.* 10*s.* per hundred weight must be given in barter for 4 hundred weight of tea, at 12*s.* per pound?

Cwt.			*s.*	*Cwt.*		*s.*
4		If	30	: 1	::	5376
112				3|0)537|6		
448 *lb.* at 12*s.* per *lb.*				*C.*179—6 *Rem.*		
12				112		
5376*s.* The value of the tea				3|0)67|2		
				*lb.*22—12		

Answer 179 *Cwt.* 22 $\frac{12}{30}$ *lb.*

E. 2.

E. 2. How much coffee, at 5*s.* per pound, must be given for 367 pounds of tea, at 8*s.* per pound?

lb. *s.* *lb.* *s.* *lb.* *s.*

First, as 1 : 8 :: 367 If 5 : 1 :: 2936

367
8
2|0)293|6

Value of the tea £. 146 16

5)2936
587—1
16
5)16
3—1
16
5)16
3—1 Rem.

Answer 587 *lb.* 3 *oz.* 3 *drs* $\frac{1}{5}$

E. 3. A hath tea, at 8*s.* 6*d.* per pound ready money, but in barter will have 10*s.* per pound; B hath tobacco worth 18*d.* per pound ready money; how must B rate his tobacco per pound, that his profit may be equivalent with A's tea?

s. *d.* *s.* *s.* *d.*

As 8 6 : 10 :: 1 6
12 12
102 18
10
s. *d.*
102)180(1 9$\frac{3}{17}$ Ans.
102
78
12
102)936(9*d.*
918
18

E. 4. A hath 14 *Cwt.* of raisins at 6*d.* per pound, for which B gives him 1 *Cwt.* 3 *qrs.* of cinnamon; I demand how B rated the cinnamon per pound?

C. qrs. *d.* *Cwt.*

If 1 3 : 6 :: 14
4 4
7 56
6
7)336
12)48

Answer 4*s.* per *lb.*

E. 5. A, with an intention of clearing 30 guineas on a bargain with B, rates hops at 16*d.* per pound, that stood him in 10*d.*—B, apprized of that, set down malt, which cost 20*s.* a quarter, at an adequate price; how much malt did they contract for?

d. *d.* *d.* *s.*

If 10 : 16 :: 240 = 20
16
1|0)384|0
12)384
32*s.* The advanced value of the malt
20*s.* real value
12*s.* B gains per quarter

30 Guineas
21
12)630
Answer 52$\frac{1}{2}$ Quarters

T E. 6.

E. 6. A, in order to put off to B 720 ells of damaged holland, worth 5*s.* an ell, at 6*s.* 8*d.* propoſes, in caſe he has half the value in money, to give B thereon a diſcount of 10 per cent. the reſt A is to take out in ſaffron, which B, apprized of the whole management, rates in juſtice at 36*s.* the pound; pray what was it really worth in ready money, and what quantity of ſaffron was he to deliver on the change?

Firſt, 5*s.* $=\frac{1}{4}$)720 Ells
180 Real value of the holland
6*s.* 8*d.* $=\frac{1}{3}$)720
240 Adv. val. of ditto
$\frac{1}{10}$)240
£. 24 = Diſcount at 10*l.* per cent.

Then, £. 240 Adv. value of the hol.
24 Diſcount
2)216
£. 108 = the ready money pd.

Again, as £. 216 : £. 180 :: *s.* 36
20 · 180
4320 · 4320)6480(1*l.* 10*s.* = 30*s.* Real value of the ſaffron per pound.
432
2160
20
4320)43200(10*s.*
432

£. 108
20
30)2160
72 *lb.* Quant. of ſaff. deliv.

E. 7. A has 100 reams of paper, at 8*s.* ready money, which in barter he ſets down at 10*s.* B, ſenſible of this, has pamphlets at 6*d.* a-piece ready money, which he adequately charges, and inſiſts, beſides, on $\frac{1}{4}$ of the price of thoſe he parts with in ſpecie; what number of the books is he to deliver in lieu of A's paper, what caſh will make good the difference, and how much is B the gainer by this affair?

If *s.* 8 = *d.* 96 : *s.* 10 :: *d.* 6
6
60
12
96)720($7\frac{1}{2}$*d.* Barter price of the pamphlets

100 Reams at 8*s.* = 40*l.* real } Value of the paper
Ditto at 10*s* = 50*l.* advanced }
$\frac{1}{4}$ = 50*l.* ÷ 4 = 12*l.* 10*s.* B to have in caſh
40*l.* value of B's pamphlets
× 40 Six-pences in a pound
1600 Pamphlets to be delivered

From 40*l.* take 12*l.* 10*s.* remains 27*l.* 10*s.* what they ſtood him in; ſo B, in this tranſaction, gains 12*l.* 10*s.*

E. 8.

E. 8. A and B truck; A has 14 *Cwt.* 2 *qrs.* 25 *lb.* of Farnham hops, at 2*l.* 19*s.* per hundred weight, but in barter insists on three guineas; B has wine worth 6*s.* per gallon, which he raises in proportion to A's demand on the ballance; A received but a hogshead and a half of wine: pray what had he in ready money?

First, 14 *Cwt.* 2 *qrs.* 25 *lb.* at 3*l.* 3*s.* per hundred weight, = 46*l.* 7*s.* $6\frac{3}{4}$*d.* the advanced value of A's hops.

$$\begin{array}{llllll} & £. & s. & s. & s. & s. \\ \text{If} & 2 & 19 = 59 & : 63 & :: & 6 \\ & & & \underline{6} & & \end{array}$$

$59)378(6s.\ 4\frac{3}{4}d.\ \frac{31}{59}$ the advanced price of B's wine per gallon. Now $1\frac{1}{2}$ *Hhd.* = 94 gallons, at 6*s.* $4\frac{3}{4}$*d.* $\frac{31}{59}$

$$\begin{array}{rrrrl} & 6 & 4\frac{3}{4} & \frac{31}{59} & \\ & & 94\frac{1}{2} & = 10 \times 9 + 4\frac{1}{2} & \\ \hline £.\ 30 & 5 & 5\frac{1}{4} & \frac{9}{59} & \text{Value of B's wine} \\ 46 & 7 & 6\frac{3}{4} & & \text{Value of A's hops} \\ \hline 16 & 2 & 1\frac{1}{4} & \frac{50}{59} & \text{In ready money, Answer} \\ \hline \end{array}$$

XXV. LOSS AND GAIN.

IS a rule by which men of trade and business know what they get by retailing goods; and in case of damage, what they lose by selling it at any given rate; and whether they gain or lose, to know at what rate per cent.

In this rule there are four varieties.

1. To know what is gained or lost per cent.
2. To know what it should be sold for to gain or lose so much per cent.
3. Having gained or lost so much per cent. to know what it cost.
4. There being so much gained per cent. when sold at such a price, to know what is gained per cent. when sold for more, or what is lost per cent. when sold for less;

RULE. When there is gain per cent. add the gain per cent. to 100*l.* but when there is loss per cent. subtract as much as you lose per cent. from 100*l.* the sum or difference is the third number in the rule of three.

E. 1. Bought 240 yards of cloth, at 14*s.* 6*d.* per yard, and sold it again at 18*s.* per yard; what did I gain by the whole?

$$\begin{array}{rl} 6d. = \frac{1}{2})\ 240 & \text{yards, at } 14s.\ 6d. \\ \underline{14} & \\ 3360 & \\ \underline{120} & \\ 2|0)348|0 & \\ £.\ 174 & \text{Cost} \end{array} \qquad \begin{array}{rl} 240 & \text{yards, at } 18s. \\ \underline{18} & \\ 2|0)432|0 & \\ \hline 216 & \text{Sold for} \\ \underline{174} & \text{Cost.} \\ \text{Ans. } 42l. & \text{Gained} \end{array}$$

 Again,

Again, anſwered by a practical method at the end of practice; ſee Section XVI.

Thus, 240 yards at 18*s*. 0*d*. $=$216*l*. Sold for
240 ditto at 14*s*. 6*d*. $=$174*l*. What coſt

Anſwer £.42 Gained thereby, as above

Mr. Vyſe's anſwer, in his Key to the Tutor's Guide, is 102*l*.

E. 2. If 276 fodders of lead, each $19\frac{1}{2}$ hundred weight, be ſold for 256*l*. at 5 months credit, and I gain 11*l*. per cent. per annum; the queſtion is how much the whole coſt ready money?

£. 100
+11
111 Amount

	£.		£.		£.		£.	s.	d.	
Then, If	111	:	256	::	100	:	230	12	$7\frac{1}{4}\ \frac{45}{111}$	Anſ.

E. 3. If by ſelling cloth, at 5*s*. per ell, I gain 8*l*. per cent. what ſhall I gain per cent. if I ſell the ell at 6*s*. 3*d*.?

First, 100+8=108*l*. amount; then

	s.		£.		s.	d.		£.	
As	5	:	108	::	6	3	:	135	From which
					Subtract			100	
					Remains			35	Anſwer

Mr. Webſter, in his Arithmetic, makes the anſwer only 10*l*.

E. 4. At 5*s*. per dozen I gain 7*l*. 10*s*. per cent. how much ſhall I gain per cent. if I ſell the dozen at 5*s*. 9*d*.?

Firſt, 100*l*. + 7*l*. 10*s*. = 107*l*. 10*s*. amount

	s.		l.	s.		s.	d.		l.	s.	d.	
Then, If	5	:	107	10	::	5	9	:	123	12	6	Amount
				From which deduct					100	0	0	
						Anſwer			23	12	6	

Mr. Stonehouſe's anſwer, in his Arithmetic, is only 8*l*, 12*s*. 6*d*.

E. 5. Suppoſe I ſell 500 deals, at 15*d*. per piece, and 9*l*. per cent. loſs, what do I loſs by the whole quantity?

Firſt, from 100
Take 9

		l.		l.	s.		l.	s.	d.	
Then, as 91	:	100	::	31	5	:	34	6	$9\frac{3}{4}$	= the price

of the deals, at 15*d*. each. Subtract 31 5 0

Anſwer 3 1 $9\frac{3}{4}$

Mr. Dilworth's anſwer to this queſtion in the ſecond edition of his Arithmetic, is only 2*l*. 16*s*. 3*d*.

E. 6. A Mancheſter tradeſman going to a fair, ſold fuſtins for 11*s*. 6*d*. the end, wherein was gained 15*l*. per cent. but ſeeing no other tradeſman had ſo good, raiſed them, at the latter end of the fair, to 12*s*. the end; I demand what he gained per cent. by this laſt ſale?

First,

First, 100+15=115*l.* the amount; then,

	s.	*d.*		*l.*		*s.*		*d.*	
As	11	6	:	115	::	12	:	120	The amount per cent.

∴ 120—100=20*l.* per cent. Answer.

Mr. Hill's answer, in his Arithm. page 289 is only 15*l.* 13*s.* 0½*d.* $\frac{2}{23}$

E. 7. Suppose I sell 1 hundred weight of hops, for 6*l.* 15*s* and gain 25*l.* per cent, what would have been the gain per cent. if I had sold them for 8*l.* per hundred weight?

First, 100+25=125*l.* amount; then,

	l.	*s.*		*l.*		*l.*		*l.*	*s.*	*d.*	
As	6	15	:	125	::	8	:	148	2	11½	Amount per cent.

Then, 148*l.* 2*s.* 11½*d.*—100=48*l.* 2*s.* 11½*d.* Answer.

Mr. Walkingame's answer to this question, in his arithmetic, page 70 3d edit. is only 29*l.* 12*s.* 7*d.* $\frac{1}{9}$.

Note. The reason of these errors in the above authors, in questions of this sort, is by making the gain or loss of 100*l.* the second term in the stating, instead of its amount (in case of gain) or deduction in case of loss. Some of these questions have been remarked by other authors; but as my readers should not be at a loss to solve questions of this sort, I thought it necessary to give them a place in this treatise.

E. 8. If by sending pewter to Turkey, and parting with it at 25⅔*d* per pound, the merchant clears cent. per cent. what does he gain in Holland, where he disposes of the hundred weight for 8*l.*?

	s.	*d.*	
	2	1⅔	
		8×7×2=112	
	16	8	
		7	
5	16	8	
		2	
11	13	4	=112
	6	2⅔	=⅔
2)11	19	6⅔	=112⅔ Sold for at Turkey
£. 5	19	9⅓	What cost him

112
2
3)224
74⅔*d.* = 6*s.* 2⅔*d.*

Then from	8	0	0	
Take	5	19	9⅓	
				Ans.
£.	2	0	2⅔	*Rem.* his loss

E. 9. Sold a repeating-watch for 50 guineas, and by so doing lost 17 per cent. whereas I ought in dealing to have gained 20 per cent. then how much was it sold under the just value?

First, 100
−17
83

And, 100
+20
120

	l.		*l.*		*l.*	*s.*			*l.*	*s.*		
Then, if	83	:	100	::	52	10 (=50 guineas)	:		63	$5\frac{5}{83}$		
	l.		*l.*		*l.*	*s.*			*l.*	*s.*	*d*	
Again as	100	:	120	::	63	$5\frac{5}{83}$	:		75	18	0¾	Worth
									52	10	0	Sold for
								Answer	23	8	0¾	Under val.

E. 10.

E. 10. Bought hose in London, at 4*s.* 3*d.* the pair; and sold them afterwards in Dublin at 6*s.* the pair, now taking the charges at an average to be 2*d.* the pair, and considering that I must lose 12 per cent. by remitting the money home again, what do I gain per cent. by this article of trade?

First, 4*s.* 3*d.* + 2*d.* = 4*s.* 5*d.* prime cost and charges.

	s.	d.		s.		l.		l.	s.	d.
Then, as	4	5	:	6	::	100	:	135	16	$11\frac{3}{4}\ \frac{5}{53}$

Again, 100—12=88*l.*

	l.		l.		l.	s.	d.		l.	s.	d.	
Also, as	100	:	88	::	135	16	$11\frac{3}{4}\ \frac{5}{53}$	:	119	10	$11\frac{1}{4}$	The amt.
							Subtract		100	0	0	
			Answer,		Gained per cent.				19	10	$11\frac{1}{4}$	

XXVI. ALLIGATION MEDIAL.

TEACHETH how to mix or unite many simples, or particulars, into one mass or sum, according to any price or sum required.

RULE. Multiply each quantity by its price; then say, as the sum of all the quantities, is to the sum of the said products, so is any part of the mixture, to the mean price of that part.

PROOF. Find the value of the whole mixture, at the mean price or rate, and if it agrees with the total value of the several quantities at their respective prices, the work is right.

EXAMPLE 1. A farmer would mix 24 bushels of wheat at 6*s.* per bushel, with 40 bushels of rye at 3*s.* per bushel, what will a bushel of this mixture be worth?

	Bush.			s.		s.	
First,	24	- multiplied by its price		- 6	=	144	Value of the wheat
And,	40	- ———— by ————		- 3	=	120	Ditto of rye
	64			Sum		264	

Then if 64 : 264 :: 1

```
      1
64)264(4s. 1½d. Answer.
   256
   ---
     8
    12
64)96(1d.
   64
   --
   32
    4
64)128(2qrs.
   128
```

4*s.* $1\frac{1}{2}$*d.* = 198 *qrs.* which multiplied by 64, the number of bushels = 12672 ÷ 4 = 3168*d.* ÷ 12 = 264*s.* proof.

E. 2.

E. 2. An hoftler mixed provender for his horfes, viz. 18 bufhels of oats at 2*s.* 1*d.* per bufhel, with 16 bufhels of beans at 4*s.* 9*d.* per bufhel, and 13 bufhels of malt at 3*s.* 10*d.* per bufhel; I demand what a bufhel of this mixture is worth?

$$\text{First, } \begin{array}{ccccc} bu. & & d. & & d. \\ 18 & \times & 25 & = & 450 \\ 16 & \times & 57 & = & 912 \\ 13 & \times & 46 & = & 598 \\ \hline 47 & & & & 1960 \end{array}$$

As 47 : 1960 :: 1

$$47)1960(41\tfrac{1}{2}d.\ \tfrac{38}{47} = 3s.\ 5\tfrac{1}{2}d\ \tfrac{38}{47} \text{ Anfwer}$$

XXVII. ALLIGATION ALTERNATE.

IS that by which the particular quantities of every ingrediant in any mixture are found; when the particular rates of every one of the ingrediants, and the mean rates, are given.

RULE. 1. Place the rates of the feveral things one over another, and the propofed price of the compofition againft them; then link the feveral rates fo together, as that one greater than the mean rate, or price of the compofition, may be coupled to a lefs; then take the differences between the mean rate and the feveral prices, and place each of them againft its yoke-fellow; this being the reverfe of alligation medial, may be proved thereby.

EXAMPLE 1. A grocer would mix fugar of 10*d.* 5*d.* and 4*d.* per pound, fo that the compofition may be worth 6*d.* per pound, what quantity muft he take?

$$6\left\{\begin{array}{c} d. \\ 10 \\ 5 \\ 4 \\ \hline 11 \end{array}\right\} \begin{array}{l} \\ 2+1 = \\ \\ \\ \end{array} \begin{array}{ccc} lb. & & d. \\ 3 & \text{at} & 10 \\ 4 & \text{at} & 5 \\ 4 & \text{at} & 4 \end{array} \Bigg\} = \left\{\begin{array}{c} d. \\ 30 \\ 20 \\ 16 \\ \hline 11)66 \\ \text{Proof } 6 \end{array}\right\} \text{Anfwer}$$

Having linked the feveral rates, agreeable to the rule (whereby it is plain that thefe rates will admit but of one way of linking) then the difference between 6, the mean price, and 4, viz. 2, is placed againft 10, its yoke-fellow: the difference between 6 and 5 is 1, which is alfo placed againft 10 its yoke-fellow; and the difference between 6 and 10 is 4, which, becaufe it has two yoke-fellows, is placed againft them both, viz. againft 5 and 4; fo that as oft as the grocer takes 3*lb.* at 10*d.* he muft take 4 *lb.* of each of the other two forts to make up the mixture.

Note

Note. The differences are not only the quantities, which anſwer the queſtion; but any other numbers, in the ſame proportion as they are, will anſwer the queſtion as well.

For	3,	4,	4
All multiplied by			3
Produce the proportionals	9,	12,	12
Theſe multiplied by			4
Produce theſe numbers in the ſame ratio,	36,	48,	48

and ſo on, in infinitum.

E. 2. A miller hath four ſorts of meal, viz. one ſort at 6*s*. 8*d*. another at 5*s*. 6*d*. the third at 4*s*. 4*d*. and the fourth at 3*s*. 8*d*. per buſhel; but he is deſirous of mixing ſo much of each ſort together, that he may ſell it at 5*s*. per buſhel; how much of each ſort muſt he take?

d.					
60	80	16	at	80 = 1280	Anſwer.
	66	8		66 = 528	
	52	6		52 = 312	
	44	20		44 = 880	
		50		5|0) 300|0	
				Proof 60	

The ſeveral rates being linked together, and their reſpective differences placed againſt their yoke-fellows, as before, you will find 16 buſhels at 80*d*. 8 at 66*d*. 6 at 52*d*. and 20 at 44*d*. will compoſe the mixture required.

Note. Examples of this nature will admit of as many anſwers as there are different ways of linking together a larger price and a leſſer than the mean rate propoſed.

XXVIII. ALLIGATION PARTIAL.

IS when the particular rates, the mean rate, and the quantity of one ingredient, is given, to find the quantity of all the reſt of the ingredients. This is called alligation partial, becauſe a part of the mixed ingredients only are given.

RULE. 1. Take the difference between each price and the mean rate, as in the laſt rule.

2. As the difference oppoſite to the known quantity, is to the known given quantity; ſo is any other difference, to the quantity of its oppoſite name.

Example 1. A farmer being determined to mix 12 buſhels of wheat at 6*s*. per buſhel, with rye at 4*s*. barley at 3*s*. and oats at 2*s*. 6*d*. per buſhel; I demand how much rye, barley and oats, muſt be mixed with the ſaid 12 buſhels of wheat, ſo that the whole may be ſold for 3*s*. 6*d*. per buſhel?

42*d*.

	d.		
d.	72	6	
42	48	12	Difference
	36	30	
	30	6	

Diff.		bu.		Diff.		bu.	
As 6	:	12	::	12	:	24 of rye	To be mixed with
				30	:	60 of barley	the 12 buſhels
				6	:	12 of oats	of wheat.

All examples belonging to this and the following rule, may be proved by the rule in alligation medial.

Note. A compoſition made of 6 buſhels of wheat at 72*d.* per buſhel, 12 of rye at 48*d.*—30 of barley at 36*d.* and 6 of oats at 30*d.* per buſhel, will bear the mean price of 42*d.* or 3*s.* 6*d.* per buſhel; you muſt obſerve, that in this compoſition there are only 6 buſhels of wheat, but the demand is 12 buſhels; therefore the proportion above is found thus:

As the difference annexed to the branch, is to the other particular differences, ſo is the given quantity to the ſeveral quantities required.

To find how much rye, barley and oats muſt be mixed with the 12 buſhels of wheat, ſay, if 6 buſhels of wheat require 12 buſhels of rye, what will 12 buſhels of wheat require? Anſwer, 24 buſhels of rye. And by proceeding in like manner with the other mixtures, you will find their reſpective proportions as in the preceding work.

E. 2. A tobacconiſt has by him 120 *lb.* of Oroonoko tobacco, worth 2*s.* 6*d.* a pound; to this he would mix York-River ditto at 20*d.* and other inferior tobacco at 18*d.* and 15*d.* a pound, as will make up a mixture anſwerable to 2*s.* a pound; what will this parcel weigh?

24	30	4+6+9=19	Differ.
	20	-- -- 6	
	18	-- -- 6	
	15	-- -- 6	

Diff.		lb.		diff.
If 19	:	120	::	6
		6		

19)720(37$\frac{17}{19}$ *lb.* of each of the other ſorts muſt be mixed with 120*lb.* of the quant. given, Anſwer.
57
150
133
17

E. 3. What quantity of gold, at 15, 16, and 18 caracts fine, muſt be mixed with 80 ounces of pure gold, viz. ſuch as is 24 caracts fine, ſo that the compoſition may be 20 caracts fine?

20	24	4+5+2=11	Differ.
	18	-- -- 4	
	15	-- -- 4	
	16	-- -- 4	

oz.		oz.		oz.
As 11	:	80	::	4
		4		

11)320(29$\frac{1}{11}$ *oz.* of 18, 15, and 16 caracts fine, Anſ.

XXIX. ALLIGATION TOTAL.

IS when the price of each simple is given, also the mean rate and quantity of the compound, to find how much of each sort will make that quantity.

RULE. Say, as the sum of the differences, to the quantity given, so is every particular difference, to its respective quantity.

EXAMPLE 1. A brewer hath three sorts of beer, viz. at 9*d.* 13*d.* and 18*d.* per gallon, which he would mix together, and the whole mixture to contain 60 gallons; how much of each sort must be taken that the mixture may be worth 10*d.* per gallon?

$$\begin{array}{l} d. \\ 10 \end{array}\left\{\begin{array}{c} 9 \\ 13 \\ 18 \end{array}\right. \quad \begin{array}{r} 8+3=11 \\ \text{--}\quad 1 \\ \text{--}\quad 1 \end{array}\Bigg\}\text{Differences}$$

$$\text{Sum}\quad 13$$

$$\begin{array}{cccc} \textit{Sum} & & \textit{gall.} & \\ \text{As } 13 & : & 60 & :: \end{array}\left\{\begin{array}{ccc} 11 & : & 50\frac{10}{13} \\ 1 & : & 4\frac{8}{13} \\ 1 & : & 4\frac{8}{13} \end{array}\right\}\text{at}\left\{\begin{array}{c} 9 \\ 13 \\ 18 \end{array}\right\}\text{Per gallon, Answer.}$$

PROOF. As 60 *gall.* : 614 *d.* the value of the whole mixture :: 1 *gall.* : 10 *d.* the mean price given.

E. 2. A mixture of wine is to be made up, consisting of 130 quarts, from these five sorts, whose prices are 7*d.* 8*d.* 10*d.* 14*d.* and 15*d.* a quart; and the whole is to be sold at 12*d.* per quart; how much of each sort must be taken?

First way.

$$\begin{array}{l} d. \\ 12 \end{array}\left\{\begin{array}{c} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right. \quad \begin{array}{r} \textit{Differences.} \\ \text{---- --}\ 5 \\ 4+2=6 \\ \text{-- --}\ 2 \\ \text{-- --}\ 2 \\ \text{-- --}\ 3 \\ \hline \text{Sum}\ \ 18 \end{array}$$

Second way.

$$\begin{array}{l} d. \\ 12 \end{array}\left\{\begin{array}{c} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right. \quad \begin{array}{r} \textit{Differences.} \\ 4+2=6 \\ \text{-- --}\ 5 \\ \text{-- --}\ 3 \\ \text{-- --}\ 3 \\ \text{-- --}\ 2 \\ \hline \text{Sum}\ \ 19 \end{array}$$

$$\text{Third way}\quad\begin{array}{l} d. \\ 12 \end{array}\left\{\begin{array}{c} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right. \quad \begin{array}{r} 2+4+5=11 \\ 2+4+5=11 \\ \text{-- --}\ 5 \\ \text{-- --}\ 5 \\ \text{-- --}\ 5 \\ \hline \text{Sum}\ \ 37 \end{array}$$

Operation

OPERATION by the laſt way thus:

$$\text{As } 37 : 130 :: \left\{ \begin{array}{l} 11 : 38\tfrac{24}{37} \text{ quarts, at } 15d. \text{ and } 14d. \text{ per qt.} \\ 5 : 17\tfrac{21}{37} \text{ quarts, at } 10d.\ 8d. \text{ and } 7d. \text{ – –} \end{array} \right\} \text{Anſ.}$$

Now as alligation anſwers not queſtions compleatly, that is, does not give all the anſwers ſuch queſtions are capable of; and, perhaps, not always thoſe that ſuit the occaſion; I ſhall ſhew, for the ſatisfaction of my ingenious readers, how this inperfection of common arithmetic is ſupplied by *Algebra*, and all the poſſible anſwers to any queſtion may be clearly and eaſily diſcovered.

E. 3. A tobacconiſt hath three ſorts of tobacco, viz. one at 2*s.* 8*d.* per pound, another at 20*d.* per pound, and a third ſort at 16*d.* per pound; of theſe he would make a mixture to contain 56 pounds, that may be ſold for 22*d.* per pound; how much of each ſort muſt he take?

$$\text{Let} \left\{ \begin{array}{l} a = \text{the quantity of that worth } 2s.\ 8d. = 32d. \\ e = \text{that at } 20d. \text{ per pound} \\ y = \text{that at } 16d. \text{ per pound} \end{array} \right.$$

Then	1	$a+e+y=56$
And	2	$32a+20e+16y=1232$
$1-a$	3	$e+y=56-a$
$2-32a$	4	$20e+16y=1232-32a$
3×16	5	$16e+16y=896-16a$
$4-5$	6	$4e=336-16a$
$6\div4$	7	$e=84-4a$
$3-7$	8	$y=3a-28$

Hence it is evident from the 7th ſtep, that the quantity ſignified by *a* muſt be leſs than 21, and (by the 8th ſtep) greater than $9\frac{1}{8}$; that is, *a* may be any number between 21 and $9\frac{1}{3}$; whence 12 anſwers flow from the limits of *a* only, and by preceeding with each ſingle value of *a*, all the anſwers in whole numbers may be obtained.

If there be more than three quantities concerned in the queſtion, the work will be more large; becauſe the limits of all the quantities above two muſt be found.

E. 4. A vintner would mix four ſorts of wine together, viz. one worth 7*s.* 4*d.* a ſecond worth 4*s.* 7*d.* a third worth 3*s.* 8*d.* and a fourth worth 2*s.* 9*d.* per gallon; how much of each ſort muſt be taken to make a mixture of 63 gallons, to be ſold for 5*s.* 6*d.* per gallon without loſs?

$$\text{Firſt, let} \left\{ \begin{array}{c} a \\ e \\ y \\ u \end{array} \right\} = \text{that quantity worth} \left\{ \begin{array}{ccccc} s. & d. & & d. \\ 7 & 4 & = & 88 \\ 4 & 7 & & 55 \\ 3 & 8 & & 44 \\ 2 & 9 & & 33 \end{array} \right.$$

$$\text{The mean rate} \ldots\ldots \quad 5 \quad 6 \quad 66$$

 Then

Then	1	$a+e+y+u=63$
And	2	$88a+55e+44y+33u=4158$
$1-a$	3	$e+y+u=63-a$
$2-88a$	4	$55e+44y+33u=4158-88a$
3×33	5	$33e+33y+33y=2079-33a$
$4-5$	6	$22e+11y=2079-55a$
$6\div 11$	7	$2e+y=189-5a$
3×55	8	$55e+55y+55u=3465-55a$
$8-4$	9	$11y+22u=33a-693$
$9\div 11$	10	$y+2u=3a-63$
Suppose	11	$a=22$. Then $5a=110$, and $3a=66$
per 7th	12	$2e+y=189-5a=79$
$12-2e$	13	$y=79-2e$
per 3d	14	$e+y+u=63-a=41$
$14-e$	15	$y+u=41-e$
$15-13$	16	$u=e-38$

From the seventh and tenth steps it appears that the quantity denoted by a, must be less than $37\frac{4}{5}$, and greater than 21; whence 16 answers flow from the limits of a. Then if a be put $=22$, by the 13th and 16th steps it appears $e=39$, $y=1$, and $u=1$; and thus proceeding with each single value of a, above 120 answers may be found to this question in whole numbers; in fractions, infinite.

XXX. EXCHANGE.

CONSISTS in finding the true sum or value of one country coin, &c. equivalent to any given sum or value of that of another country.

The par of exchange is fixed, and standard value of foreign coins, &c. expressed in sterling money of our own; it is so called, because in exchange, one equal value for another is given.

The course of exchange is the current price, and is always unsettled, being sometimes above, and sometimes below the par; according to the various circumstances and accidents of trade, and nations.

Money in the bank of other kingdoms, is finer or purer than that which is current, the difference of value in each is called Agio

As it would be endless to treat of every kind of exchange, I shall only give a few examples of the exchange of England, with a few of the chief countries in Europe.

First, with FRANCE.

At France, accounts are kept in livres, sols and deniers, exchange being made by the French crown, whose par is 4*s*, 6*d*, sterling.

12 Deniers	make one	Sol
20 Sols	make one	Livre
3 Livres	make one	Crown

First,

Firſt, to change French money into ſterling,

RULE. As 1 crown is to the given rate, ſo is the given French ſum, to the ſterling required.

Second, to change ſterling money into French,

RULE. As the rate of exchange is to one crown, ſo is the ſterling ſum, to the French required,

Note. The ſame rule muſt be obſerved with moſt of the following countries.

EXAMPLE 1. What ſterling money muſt a merchant pay in London, to receive in Paris 2000 crowns, exchange at 54*d*. per crown?

```
As 1 : 54 :: 2000
                2000
          ----------
          12)108000
          ----------
          2|0)900|0
          ----------
Anſwer £. 450
```

E. 2. What number of crowns muſt be paid in Paris, to receive in London 450*l*. exchange 54*d*, per crown?

```
    d.     cr.     £.
As 54  :   1   ::  450
                    20
                  ----
                  9000
                    12
                ------
54 { 9)108000
   {   ------
   { 6) 12000
       ------
Anſwer 2000 Crowns
       ----
```

E. 3. Change 640 crowns, 12 ſols, 8 deniers, at 56*d*. per crown, into ſterling?

```
   c.   d.      c.  ſols. den.
If 1 : 56 :: 640   12    8
   3              3
   -        ------
   3           1920
  20             20
  --         ------
  60          38412
  12             12
 ---         ------
 720         460952
                 56
            -------
            2765712
           2304760
         -----------
     72|0)2581331|2(35851d.= 149l. 7s.
          216              7¾d. 20/72
          ---
           421
           360
           ---
           613
           576
           ---
            373
            360
            ---
            131
             72
            ---
            592
              4
          -----
     72|0)236|8(3
          216
          ---
           20
```

E. 4. Change 149*l*. 7*s*. 7¾*d*, ſterling, into French crowns, exchange at 56*d*. per crown?

```
    d.    c.      £.   s.  d.      c.  ſols. den.
As 56  :  1  ::  149   7   7¾  :  640   12    8 Anſwer.
```

Second,

Second, with SPAIN.

They keep their accounts in piasters, reals, and maravedis, and exchange by the piaster, whose par is 4*s.* 6*d.* sterling

4 Maravedis vellon, or $2\frac{1}{8}$ maravedis plate ---- ----	make one	Quartas
$8\frac{1}{2}$ Quartas, or 34 mar. vellon	make one	Rial vellon
16 Quartas, or 34 maravedis plate	make one	Rial of plate
8 Rials of plate ---- ----	make one	Piece of eight, or dollar

N. B. A Rial vellon is $\frac{17}{32}$ of a rial of plate, and $\frac{17}{250}$ of a piaster.

E. 5. Change 630*l.* into Spanish money, exchange at 50*d.* per piece of $\frac{8}{8}$?

	d.		*piece.*		*l.*		*pieces.*	
As	50	:	1	::	630	:	3024	the Answer

E. 6. Suppose Spain draws upon London for 3024 pieces of $\frac{8}{8}$, what sterling money will this draft amount to, exchange at 50*d.* per piece of eight.

	piece.		*d.*		*pieces*		£.	
As	1	:	50	::	3024	:	630	the Answer

E. 7. If I pay in Seville 1426 pieces of $\frac{8}{8}$, 4 rials, 26 maravedis, what may I draw my bill for at London, exchange at $54\frac{1}{4}d.$ per piece of eight?

	p.		*d.*		*p.*	*ri.*	*mar.*		£.	*s.*	*d.*	
If	1	:	$54\frac{1}{4}$	::	1426	4	26	:	322	9	$4\frac{3}{4}\frac{66}{272}$	Answer

Third, with ITALY.

In Italy they keep their accounts in livres, sols, and deniers, and exchange, by the piece of eight, or dollar, which is equal to 4*s.* 6*d.* at par.

12 Deniers	make one	Sol
20 Sols	make one	Livre
5 Livres	make one	Piece of $\frac{8}{8}$ at Genoa
6 Livre,	make one	Piece of $\frac{8}{8}$ at Leghorn

At Florence the exchange is in ducatoons, and at Venice by ducats, divided as follows, viz.

6 Solidi make one gross, and 24 gross one ducat,

E. 8. Suppose there be owing me, by a correspondent at Genoa, 640 dollars, how much sterling does it amount to, exchange at 52*d.* per dollar?

dol.		*d.*		*dol.*
1	:	52	::	640
		640		
		208		
		312		
		12)33280		
		2|0)277|3—4		

Answer £. 138 13 4

Again, by Practice.

s.	*d.*		*dol.*	*s.*	*d.*
4	0	$\frac{1}{5}$	640 at	4	4
0	4	$\frac{1}{12}$	128		
			10—13—4		
Ans. £.			138	13	4 as before

E. 9.

E. 9. A merchant remits 138*l.* 13*s*: 4*d.* ſterling to genoa; how many dollars muſt he receive there, exchange at 52*d.* per dollar?

	d.		*dollar.*		*£.*	*s.*	*d.*
As	52	:	1	::	138	13	4
					20		
					2773		
					12		

52)33280(640 Dollars, Anſwer

Note. In St. George's bank at Genoa, accounts are kept in piaſters or pezzoes, which are divided into ſolidi and denarii, as the pound ſterling.—Some merchants keep their accounts in lires or liras, ſolide, and denare, divided as before; this money is only one-fifth in value of the bank money.

To change current money into bank, and bank into current, they muſt be proportioned thus; As 100 with the agio (that is the difference) added to it, is to 100 bank, ſo is any given ſum current, to its value in bank: and as 100 is to 100 with the agio added to it, ſo is the bank money given to its value current.

E. 10. Change 110 guilders 12 ſtivers current, into bank florins, agio $\frac{1}{5}$ =4 per cent.

	guil.		*guil.*		*guil.*	*ſt.*		*guil.*	*ſt.*	*gr.*	*pen.*	
As	104	:	100	::	110	12	:	106	6	1	6	the Anſwer.

E. 11. London is indebted to Genoa in 1710*l.* 16s. 4*d.* for how many pezzoes may Genoa value on London, exch. at 74½*d.* per pezzoe?

	d.		*pez.*		*£.*	*s.*	*d.*		*pez.*	
As	47½	:	1	::	1710	16	4	:	8644	the Anſwer.

E. 12. Change 8644 *pez.* 2*s.* 6*d.* into ſterling money, exchange at 47½*d.* per pezzoe.

	pez.		*d.*		*pez.*		*£.*	*s.*	*d.*	
As	1	:	47½	::	8644	:	1710	16	4	Anſwer

Fourth, with PORTUGAL.

Accounts are kept in Portugal in milreas and reas, and they exchange by the milrea, which London gives from 5*s.* to 6*s.* 9*d.* for the ſame.

400 Reas make one cruſadoe, and 1000 reas one milrea.

E. 13. A merchant at Liſbon remits to his correſpondent in London 500 milreas, exch. at 5*s.* 6*d.* how much ſterling muſt he receive?

5*s*, = $\frac{1}{4}$)500 at 5*s.* 6*d.*

6*d.* = $\frac{1}{10}$ 125

12—10

Anſwer *£.* 137 10

		m.		*s.*	*d.*		*m.*		*£.*	*s.*
Or thus,	As	1	:	5	6	::	500	:	137	10 Anſwer as above.

E. 14.

E. 14. How many milreas will 1566*l.* 6*s.* 6*d.* amount to, exchange at 64*d.* per milrea?

	d.	*mil.*		£.	*s.*	*d.*
As	64 :	1	::	1566	6	6

20
———
31326
12
———
64 { 8)375918
———
8) 46989—6
———
5873—5 } = 46
1000
———
64 { 8)46000
———
8) 5750
———
718—6

Answer 5873 milr. 718 $\frac{6}{8}$ reas

Fifth, with HOLLAND, FLANDERS, and GERMANY.

In these countries their accounts are kept, sometimes in pounds, shillings, and pence, as in England, and sometimes in guilders, stivers, and pennings. In Holland and Flanders the money is distinguished by the name of Flemish; exchange being made with London from 30*s.* to 38*s.*. Flemish per pound sterling?

8 Pennings	make one	Groat
2 Groats		Stiver
6 Stivers		Shilling
20 Stivers		Flor. or guilder
2½ Florins		Rix dollar
6 Florins		Pound Flemish
5 Guilders		Ducat

To change Flemish money into sterling, and on the contrary, sterling into Flemish, is the same with that of France, only what was French there, will be Flemish here.

E. 15. A merchant in Rotterdam remits 282*l.* 5*s.* 3*d.* Flemish, to be paid in London, how much sterling money must he draw for, exchange at 34*s.* 4*d.* per pound sterling?

	s.	*d.*		£.		£.	*s.*	*d.*
As	34	4	:	1	::	282	5	3

12
———
412

20
———
5645
12 £. *s.* *d.*
———
412)67743(164 8 $5\frac{3}{4}\,\frac{316}{412}$ Answer

E. 16. Suppose a merchant delivered in London 164*l.* 8*s.* $5\frac{3}{4}$*d.* to receive the value at Amsterdam in Flemish money; how many pounds must he receive there, exchange at 34*s.* 4*d.* Flemish per pound sterling?

	£.		*s.*	*d.*		£.	*s.*	*d.*		£.	*s.*	*d.*	
As	1	:	34	4	::	164	8	$5\frac{3}{4}$	:	282	5	3	the answer

To reduce Flemish pounds, shillings and pence, into guilders,

RULE. Divide the whole sum when reduced into pence Flemish by 40 (the number of pence in on guilder) and the quotient will be guilders;

guilders; the remainder (if any) divide by 2 (the pence in one ſtiver) and the quotient will be ſtivers.

E. 17. In 423*l.* 8*s.* Flemiſh, how many guilders?

```
 £.   s.
 423  8
  20
 ----
 8468
   12
4|0)10161|6
-----------  guil. ſtiv.
2540 16/40 = 2540  8 Anſwer
```

E. 18. In 2540 guild. 8 ſtivers, how many Flemiſh pounds?

```
guil. ſtiv.  d.
2540   8 = 16
  40
 ------
12)101616
 --------
2|0) 846|8
 --------
£.423  8s. Anſwer
```

Sixth, with VENICE.

Money of exchange here is always underſtood to be ducats in bank; which is imaginary, 100 whereof make 120 ducats current money; ſo that the difference betwixt bank and current money is an agio of 20*l.* per cent. though the brokers have invented another agio to be added, which is more or leſs, according to bargain.

The courſe of exchange of a ducat of the bank of Venice is from 45 to 50*d.* ſterling.

E. 19. Venice draws on London for 2350 ducats banco, exchange at 47*d.* per ducat, how much ſterling money will pay the draught?

```
     du     d.      du      £.   s.  d.
As   1  :  47  ::  2350  :  460  4   2  Anſwer
```

Seventh, with POLAND and PRUSSIA.

Dantzic and Koningſberg, exchange, with London by way of Amſterdam and Hamburgh; 270 Poliſh groſch being equal to 1*l.* groſs banco in Holland 110, Poliſh groſch being equal to 1 rix-dollar banco of Hamburgh.

18 Penningen	make one	Groſch
3 Groſch		Ditkin
2 Ditkins		Sixer
3 Sixers		Tymph
7½ Groſch		Arch de Halber
4 Arch de Halbers		Florin or guilder
3 Florins or guilders		Current Dollar
4 Gilders		Specie Dollar

E. 20. Change 2342 florins into ſterling money, 270 groſchi Poli, per pound Flemiſh, and 34*s.* 4*d.* Flemiſh per pound ſterling?

```
      G.P.    £.       Flor.
As    270  :  1   ::   2342
                         30
                 ----------- £.   s.  d.
             27|0) 7026|0(260  4   5¾ Flemiſh
                   s.  d.   l.       l.   s.  d.
Again, as          34  4  :  1  ::  260   4   5¾
Or, as 1648   :   1   ::  249815  :  151l. 11s. 8¾d. Anſwer.
```

Eighth, with RUSSIA.

3 Copecs	make 1	Altine	2 Polpolitons	make 1	Poltin
10 Copecs		Grievener	2 Poltins		Rubble
25 Copecs		Polpoliton	2 Rubbles		Ducat

The Ruſſian rubbles are converted into florens current money of Amſterdam, and the current into bank money, according to agio of three or five per cent. and bank money into ſterling, according to agio of three or five per cent. and bank money into ſterling according to the courſe of exchange between England and Amſterdam.

E. 21. In 6420 rubbles, 42 copecs, exchange 122 copecs per rix-dollar current, agio 3 per cent. 34*s.* 6*d.* Flemiſh per pound ſterling, how much ſterling money?

```
      6420  42
      ×100
122) 642042(5262 78/122 Rix-dollars
     610
     ---
      320
      244
      ---
       764
       732
       ---
        322
        244
        ---
         78
```

$5262\frac{78}{122}$ Rix-dollars
$\times 2\frac{1}{2}$

$$\left.\begin{matrix} 10525\frac{34}{122} \\ 2631\frac{39}{122} \end{matrix}\right\}\text{Florins current}$$

	Fl. cur.		*Flor. ba.*				
	As 103	:	100	::	$13156\frac{73}{122}$		
Or, as	12566	:	100	::	1605105	:	$12773\frac{2491}{6283}$ Flor. ba.

Now $12773\frac{2491}{6283} \times 40 = 510935\frac{5395}{6283}$ Pence

And 34*s.* 6*d.* = 414*d.* Then,

d.		*l.*		*d.*		*l.*	*s.*	*d.*
As 414	:	1	::	$510935\frac{5395}{6288}$	:	1234	2	$10\frac{1}{2}$ Anſwer

Ninth, with IRELAND.

In Ireland they keep their accounts in pounds, ſhillings, and pence Iriſh, divided as in England: but having no coins of their own, they are ſupplied by the different countries with which they traffic.

The par of exchange between England and Ireland is 100*l.* ſterling for 108*l.* 6*s.* 8*d.* Iriſh; or 1*s.* Engliſh for 13*d.* Iriſh.

The courſe of exchange is from 5 to 12 per cent, according to the balance of trade.

E. 22. Dublin draws upon London for 370*l.* 7*s.* 3*d.* Iriſh exchange at 12 per cent. how much ſterling muſt London pay Dublin to diſcharge this bill?

	l.		*l.*		*l.*	*s.*	*d.*
As	112	:	100	::	370	7	3

	d.		*l.*		*d.*		*l.*	*s.*	*d.*
Or, as	26880	:	100	::	8888700	:	330	13	$7\frac{1}{4}\frac{1536}{2688}$ Anſ.

E. 23.

E. 23. London remits to Ireland 330*l.* 13*s.* $7\frac{1}{4}$*d.* sterling; how much Irish must London be credited with, exchange at 12 per cent?

	l.		*l.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*	
As	100	:	112	::	330	13	$7\frac{1}{4}$	:	370	7	3	the Answer

Tenth, with AMERICA and the WEST-INDIES.

Accounts are kept, and the money divided, as in England; their money is called currency.

The scarcity of cash obliges them to substitute a paper currency for carring on their trade; which being subject to casualties, suffer a very great discount for sterling in the purchase of bills of exchange.

E. 24. Philadelphia is indebted to London 4168*l.* 16*s.* $10\frac{1}{2}$*d.* currency, what sterling may London reckon to be remitted, when the exchange is 150 per cent.?

	l.		*l.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*	
As	150	:	100	::	4168	16	$10\frac{1}{2}$	:	2779	4	7	Answer

E. 25. A, at Paris, draws on B, of London, 1200 crowns, at 55*d.* sterling per crown; for the value whereof B draws again on A 56*d.* sterling per crown, besides commission $\frac{1}{2}$ per cent. Did A gain or lose by this transaction, and what?

	l.		*s.*		*l.*		*s.*	*d.*	
If	100	:	10	::	275	:	27	6	the commission

Then 275*l.* + 1*l.* 7*s.* 6*d.* = 276*l.* 7*s.* 6*d.* = 66330*d.*

Therefore, as 56*d.* : 1 crown :: 66330*d.* : $1184\frac{13}{28}$ crowns

Consequently, $1200 - 1184\frac{13}{28} = 15\frac{15}{28}$ crowns, A's gain, Answer

E. 26. V, of Amsterdam, draws on X, of Hamburgh, at 67*d.* Flemish per dollar, of 32 sols Lubeck; aud on Y of Nurembergh, at 70*d.* Flemish per florin, of 63 crutzers current. If V has orders to draw on X, in order to remit to Y at the said prices, how would run the exchange between Hamburgh and Nuremberg?

	d.		*sols.*		*d.*		*sols.*	
As	67	:	32	::	70	:	$33\frac{29}{67}$	Lubeck, per florin, answer

E. 27. M, of Amsterdam, orders N, of London, to remit O, of Paris, at 54*d.* sterliug per crown, and to draw on P, of Antwerp, for the value, $33\frac{1}{2}$*s.* Flemish per pound sterling; but as soon as N received the commission, the exchange was on Paris at $54\frac{1}{2}$*d.* per crown; pray at what rate of exchange ought N to draw on P to execute his orders, and be no loser?

	d.		*s.*	*d.*		*d.*		*d.*		*s.*	*d.*	
If	54	:	33	6	::	$54\frac{1}{2}$	:	$398\frac{34}{109}$	=	33	$2\frac{34}{109}$	Flem. ans.

```
                         d.            s.  d.           d.
At length thus, as      54     :       33  6    ::    54½
                         2             12               2
                       ———            ———             ———
                       108            402             109
                                      108
                                     ————
                                     3216
                                    402
                                   —————
                          109)43416(398d. 34/109 Answer
                                   327
                                   ————
                                   1071
                                    981
                                   ————
                                    906
                                    872
                                   ————
                                     34
```

XXXI. COMPARISON of WEIGHTS and MEASURES.

IS when the weights or meaſures of different countries are compared together; and is a very neceſſary rule (of great importance to the merchant) to be aquainted with.

RULE. Place the numbers alternately under each other, in two perpendicular columns, ſo that there may not be found in either column two terms of one kind; then the numbers in the leſſer column muſt be multiplied together for a diviſor; and the numbers in the greater column, where the odd term is, for a dividend; the quotient will be the anſwer. The work may often be abridged by throwing out numbers that are alike in both columns.

Example 1. If 6 pounds of ſugar be equal in value to 7 pounds of raiſins; 5 pounds of raiſins to 4 yards of ribbon; 10 yards of ribbon to 40 nutmegs, and 7 nutmegs to 18 pence; what is 3 pounds of ſugar worth?

```
 6 Sugar        7 Raiſins         Then, per rule,  7
 5 Raiſins      4 Ribbon                           4
10 Ribbon      40 Nutmegs                        ———
 7 Nutmegs     18 Pence                           28
 —              3 Sugar                           40
                                                ————
                                                1120
      And 6                                       18
          5                                    —————
         ——                                    20160
         30                                        3
         10                                    —————
        ———                              21|00)604|80(28
        300                                    42
          7                                    ———
       ————                                    184
       2100                                    168
                                               ———
                                                16
```

Anſwer 28 $\frac{16}{21}$ Pence

E. 2. If 100*lb.* at Copenhagen be equal to 80 *lb.* at Rome, and 100*lb.* at Rome be equal to 114*lb.* at Madrid; how many pounds at Madrid are equal to 180*lb.* at Copenhagen?

l.		*lb.*	
100	=	80	Then, 80 × 114 × 180 = 1641600
100	=	114	And 160 × 100 = 10000
		180	

Alſo 10000) 1641600 ($164\frac{4}{25}$ pounds, the anſwer

E. 3. Suppoſe 100*lb.* of Portugal be equal to 92*lb.* of Antwerp, and 100*lb.* of Antwerp be equal to 110*lb.* at Lyons; how many pounds at Lyons are equal to 60*lb.* of Portugal?

100	92	Then 92 × 110 × 60 = 607200
100	110	And 100 × 100 = 10000
	60	

Alſo 10000) 607200(60 $\frac{72}{100}$ pounds, the anſwer.

XXXII. POSITION;

OR,

THE RULE OF FALSE.

IS ſo called, becauſe we ſuppoſe, or make a poſition of ſome uncertain numbers, in order that by reaſoning from them we may gain the true number ſought; and becauſe thoſe poſitions are altogether at random, or adventure, the rule is alſo called falſe.

The uſe of this rule, before the common knowledge of algebra, was much more conſiderable than ſince; becauſe that art ſupplies theorems for reſolving all kinds of queſtions in this rule in a better and more curious manner than here.

Some authors have entirely diſcarded it, and others poſtpone it, as obſolete, and of little uſe ſince algebra; but, in my opinion, it is a very good approximation, and in exponental equations, as well as many other things, ſucceeds better than any other method, and is very uſeful in ſolving many intricate problems, not only in arithemetic and algebra, but in the more abſtruſe parts of the mathematics (as Mr. Emerſon remarks in his Cyclomatheſis, page 151) where he ſays, in many difficult problems, there is hardly any other way to come at a ſolution, but by this method of trial and error.

Queſtions in this are performed by one or two ſuppoſitions; if by one, the rule is ſaid to be of ſingle poſition; if two ſuppoſitions are neceſſary, it is called double poſition.

SINGLE POSITION.

RULE. Make choice of ſome fit number, and proceed with this, according to the nature of the queſtion, as if it were the true number, and if you find the reſult either too much, or too little, you may then find the anſwer by the rule of three, viz.

As

As the result of this position is to the position, so is the given number to the number required.

EXAMPLE 1. What sum is that, of which the half, third, and fourth, make 520*l*?

Suppose the sum to be 96
Then the $\frac{1}{2}$ is 48
The $\frac{1}{3}$ is 32
The $\frac{1}{4}$ is 24

Result 104

Then, if 104 : 96 :: 520

$$\begin{array}{r} 520 \\ \hline 192 \\ 480 \\ \hline 104)49920(480 \text{ Ans.} \\ 416 \\ \hline 832 \\ 832 \\ \hline 0 \end{array}$$

For the half of 480 = 240
The third .... = 160
And the fourth = 120
Sum = 520 Proof

E. 2. A, B, and C, buy a parcel of timber, which cost 48*l.* and it is agreed that B should pay a third part more than A, and C a fourth part more than B; what sum must each pay?

Suppose A pays 3
Then B's part is 4
And C's 5

Result 12

$$\text{Then, as } 12 : 48 :: \left\{\begin{array}{l} 3 : 12 \text{ A's share} \\ 4 : 16 \text{ B's share} \\ 5 : 20 \text{ C's share} \end{array}\right.$$

£. 48 Proof

E. 3. A schoolmaster being asked how many scholars he had, answered, if I had as many, half as many, and one-fourth as many, I should have 198; how many had he?

Suppose he had 16
Then, as many 16
Half as many 8
One fourth ditto 4

44

If 44 : 16 :: 198

$$\begin{array}{r} 16 \\ \hline 1188 \\ 198 \\ \hline 44)3168(72 \text{ Scholars Answer} \end{array}$$

E. 4. *An old woman of above threescore and ten,*
Has buried four husbands, and married again
To Jerry the mugman, a bagpiper rare!
And none can with him for his music compare;
The music he play'd pleas'd the old woman much,
Till she hopp'd, and she caper'd about without cruth.
Though wrinkled and wither'd—no tooth in her head,
Yet money she had, and she got married:
To his bagpipes she mov'd, with one foot in the grave,
For all her delight was a husband to have!
The sum of both ages one hundred years are,
Wanting five—and one-fourth of her age I declare,
Is the age of her husband;—now Tyro *you'll find*
The bagpiper's age, with his spouses so kind.

Suppose

Suppoſe the wife's age to be 60
Then the huſband's will be 15

Sum 75

Then, as 75 : 60 :: 95 The ſum of both their ages

60

75)5700(76 The wife's age

Then, per queſtion, $76 \div 4 = 19$ The huſband's age

Proof 95

E. 5. A man overtaking a maid driving a flock of geeſe, ſaid to her, How do you do, ſweetheart? where are you going with theſe 80 geeſe? No, Sir, ſaid ſhe, I have not 80; but if I had as many more, half as many more, and 20 geeſe beſides, I ſhould have 80; how many geeſe had ſhe?

Suppoſe ſhe had	20	Then 80	As 50 : 20 :: 60
Then as many	20	— 20	60
One-half as many	10	—	5|0)120|0
	—	60	
Sum	50		24 Her flock, Anſwer

XXXIII. DOUBLE POSITION.

IS when two ſuppoſitions are uſed, becauſe here the numbers cannot be parted to find the anſwer as before; therefore, when we make two ſuppoſitions, and miſs in both, obſerve the nature of the errors, whether they be greater or leſs than the number propoſed; and accordingly mark them with the ſigns + or —; and place them againſt their proper ſuppoſitions; but if with either of the ſuppoſitions we find the number that anſwers the queſtion, the work is done.

When the errors are equal, and have unlike ſigns, half the ſum of the ſuppoſitions is the number ſought.

RULE. As the difference of the errors, if alike, or their ſum, if unlike, is to the difference of the ſuppoſitions; ſo is either of the errors to a fourth number, which added to the ſuppoſition over-againſt it, if leſs, or ſubtracted from it, if more, gives the number ſought.

EXAMPLE 1. A man agreed to thraſh 60 buſhels of corn, part of it wheat, and part oats, at the rate of 2*d.* per buſhel for the wheat, and $1\frac{1}{2}$*d.* for the oats; at laſt he received 8*s.* for his labour; how much of each did he thraſh?

Suppoſe there were 30 buſhels of wheat, price 60 pence
Then there are 30 buſhels of oats, price 45 pence

Too much 105
There ſhould only be 96

Firſt error - + 9

Again,

Again, suppose 20 bushels of wheat, price = 40 pence
Then there will be 40 bushels of oats, price = 60 pence

Too much 100
96

Second error .... + 4

Sup. *er.*		
30 × 9 20 × 4 180 120	Then 180 — 120 60 Diff. of prod.	Also 9 — 4 5 Diff. of the errors

Therefore 5)60

12 *bu.* of wheat

Consequently there is 48 Ditto of oats

Total 60

PROOF.

bu.		*d.*		*d.*
	12 at	2	=	24
And	48 at	$1\frac{1}{2}$	=	72
	Sum		=	96 = 8*s.*

E. 2. A gentleman finding several beggars at his door, gave each of them 3*d.* a-piece, and had 5*d.* remaining; he would have given them 4*d.* a-piece, but wanted 7*d.* to do it; how many beggars were there?

Suppose the No. of beggars	14	And	14
	3		4
	42		56
	+ 5		− 7
His money	47		49 His money also
			47
		The first error	+ 2

Again, suppose the number to be	10	And	10
	3		4
	30		40
	+ 5		− 7
	35		33
	33		
Second error	— 2		

Sup.		*er.*			
14	×	2	2		28
10		2	2		20
20		28	4	)	48
					12 Beggars, the Answer

E. 3. Double my money for me, said A to B, and I will give thee 6*d.* out of the stock, with the remainder; he applied in like manner to C, with equal success, and gave him also 6*d.* he repeated this proposal to D, and then 6*d.* was all he had to give. Pray what sum had he to begin with?

Suppose

Suppose he had 10*d*.
Then 10 + 10 = 20
Also 20 — 6 = 14
14 + 14 = 28
28 — 6 = 22
22 + 22 = 44
44 — 6 = 38
Too much

Again, suppose he had 7*d*.
Then 7 + 7 = 14
14 — 6 = 8
8 + 8 = 16
16 — 6 = 10
10 + 10 = 20
20 — 6 = 14
Too much also.

Sup. er.
10 × 38
7 × 14
266 140

Then 38 266
14 140
24) 126 ($5\frac{1}{4}$*d.* Answer
120
6
4
24) 24 ($\frac{1}{4}$

E. 4. *When first the marriage knot was tied*
Betwixt my wife and me,
My age did her's as far exceed,
As three times three doth three;
But when ten years, and half ten years,
We man and wife had been,
Her age came up as near to mine
As eight is to sixteen.
What both our ages was, I pray,
Now tell me, on the wedding-day?

Suppose her age to be - 13
Then (per quest.) he will be 13 × 3 = 39
13 + 10 + 5 = 28
39 + 10 + 5 = 54 } Subt.
28 × 2 = 56 }
First error — 2

Again, suppose her age to be...... 17
Then (per quest.) he will be 17 × 3 = 51
17 + 10 + 5 = ... 32
51 + 10 + 5 = ... 66 } Subt.
32 × 2 = ... 64 }
Second error + 2

Sup. er.
13 × 2
17 × 2
34 26

2 34
2 26
4) 60
15 Years, her age when married

Then 15 × 3 = 45 years, his age when married
15 + 10 + 5 = 30 her age } At the time proposed.
45 + 10 + 5 = 60 his age }

For, as ; 16 :: 30
30
8)480
60 Proof

Note. You may observe, that when the errors happen to be alike in quantity, but unlike in quality (as in the above solution) the answer may be more easily obtained than by proceeding as above (as I hinted at

Y

at the beginning of this section) for in such case, half the sum of the suppositions will be the number sought, as in this solution the sum of the suppositions $13+17=30$, half whereof is 15, the same as above.

EXAMPLE 5.

A farmer with a plowman doth agree,
That thirty days his servant he should be;
Each day he wrought the farmer is to pay
Him sixteen pence; but when he was away,
Five groats he is for each day to abate.
The time expired, they their accompts do state,
Whereby the master nothing is to give,
Nor has the servant any to recieve.
How many days he wrought I do demand,
And how many he play'd I'd understand.

	d.	
Suppose he worked 15 days, his wages at 16*d*. per day =	240	subt.
Then (per q.) he was idle 15 da. which at 20*d*. per day =	300	
First error, too little	60	

Again, sup. he wr. 20 days, his wages at 16*d*. per day =	320
Then (per q.) he was idle 10 days, which at 20*d*. per day =	200
Second error, too much +	120

Sup.		*er.*			
15	×	60	Then 60	1800	
20		120	120	1200	
				days.	*hours.*
1200		1800	18|0)	300|0(16	8 the Answer

E. 6. A thief breaking into an orchard, stole from thence a certain number of apples, and at his coming forth he met with three men, one after another, who threatened to accuse him of theft; and, in order to appease them, he gave unto the first man half the apples he had stolen, who returned him back 12 of them; then he gave unto the second half of those he had remaining, who returned him back 7 of them; and unto the third person he gave half the residue, who returned him back 4; at last getting safe away, he finds he has 20 left. How many had he at first?

Suppose he had - - -	60	Again, suppose he had - -	92
Gave the first 30 — 12 =	18	Then, 46 — 12 =	34
Remains -	42	Remains - - -	58
Gave the second 21 — 7 =	14	29 — 7 =	22
Remains -	28	Remains - - -	36
Gave the third 14 — 4 =	10	18 — 4 —	14
Too little	18	Too much - -	22
Should be	20	Should be - - -	20
First error	— 2	Second error +	2

Sup.

$$\begin{array}{ccc}
\textit{Sup.} & & \textit{er.} \\
60 & \times & 2 \\
92 & & 2 \\
\hline
184 & & 120
\end{array}
\qquad
\begin{array}{rcl}
\text{Then } 2 & & 184 \\
+ \; 2 & + & 120 \\
\hline
4 &) & 304 \\
\hline
\end{array}$$

Anſwer 76 what he had at firſt

Or, thus $60 + 92 = 152$, which $\div 2 = 76$, the Anſwer as before

E. 7. *A man that was idle, and minded to ſpend*
Both money and time, went to drink with a friend;
He ſaid to his hoſt, if you'll now to me lend
As much coin as I have, then my ſix-pence I'll ſpend.
His hoſt lent the money, his ſix-pence he ſpent,
And having ſo done, to another houſe went,
Where the ſame he requeſted, and the ſame ſum he ſpent:
He went to a third houſe, where, Landlord, cries he,
*Lend me as much money as here you ſee,**
Which having received, his ſix-pence he ſpent,
So all being gone, home the fuddle-cap went
To caſt up his reck'nings; but his head aching ſore,
He beg's you to do it, and he'll do ſo no more;
What had he at firſt, and how much on ſcore?

	d.		d.	Sup.		er.
Suppoſe he had -	8	Again, ſuppoſe he had -	7	8	×	22
Then $8 + 8 =$	16	Then $7 + 7 =$	14	7		14
Alſo $16 - 6 =$	10	$14 - 6 =$	8			
$10 + 10 =$	20	$8 + 8 =$	16	154		112
$20 - 6 =$	14	$16 - 6 =$	10			
$14 + 14 =$	28	$10 + 10 =$	20			
$28 - 6 =$	22	$20 - 6 =$	14			
Too much		Too much				

Then, $22 - 14 = 8$, and $154 - 112 = 42$ ∵ $42 \div 8 = 5\frac{1}{4}$ Anſ.

E. 8. *Gentlemen, of you I muſt enquire,*
How the poll ſtood for the knights of our ſhire?
The number of votes, as I have ſeen,
Was five thouſand two hundred and nineteen;
Which amongſt four was juſt ſo divided,
As one the ſecond and third exceeded
By twenty-two and four-ſcore, bating ſeven,
The fourth by no more than ſix-ſcore and ten:
Then how many votes had each candidate?
You cannot in finding much trouble your pate.

* Shewing what he had left.

Suppoſe

	Votes.		Votes.
Suppose the first had - -	2000	Again, suppose the first had	1600
The 2d. 2000—22 =	1978	The 2d. 1600 — 22 =	1578
The 3d. 2000—73 =	1927	The 3d. 1600 — 73 =	1527
The 4th. 2000—130 =	1870	The 4th. 1600 — 130 =	1470
Too much - -	7775	Too much - -	6175
Should be - -	5219	Should be - -	5219
First error -	+ 2556	Second error -	+ 569

$$\begin{array}{ccc} 2000 & \times & 2556 \\ 1600 & & 956 \\ \hline 4089600 & & 1912000 \end{array}$$

Then $2556—956=1600$, and $4089600—1912000=2177600$ ∴ $2177600\div1600=1361$, the number of votes the first candidate had

The second had $1361—22=1339$; the third had $1361—73=1288$; and the fourth had $1361—130=1231$ votes, the answer. For $1361+1339+1288+1231=5219$ proof.

E. 9. There is a fish, whose head is 9 inches long; the tail as long as his head and half his body, and his body is as long as both his head and tail; I demand the whole length of the fish?

Suppose the body to be 12 inches

Then $\overline{12\div2}+9=15$ tail, also $15+9=24$, too much by 12

Again, suppose the body to be 14 inches

Then $\overline{14\div2}+9=16$ tail; also $16+9=25$, too much by 11

$$\begin{array}{ccc} \textit{Sup.} & & \textit{er.} \\ 12 & \times & 12 \\ 14 & & 11 \\ \hline 168 & & 132 \end{array} \qquad \begin{array}{rrr} \text{Then} & 12 & 168 \\ & 12 & 132 \\ \hline & 0\ ) & 36\ (\\ & & 36 \end{array}$$

36 Length of the body

And $\overline{36\div2}+9=27$ length of the tail; therefore $36+27+9=72$, length of the whole fish, answer.

E. 10. *A painter of skill and much fame in the town,*
Had procur'd himself work for more hands than his own;
He employ'd an assistant, to help him in part,
A proficient in every branch of his art.
O'er a glass of good wine upon terms they debate,
And the bottle was drained while they state and unstate,
For as plenty of Baccus' enlivening juice,
Does most commonly projects and whimsies produce;
So when that their spirits grew warm with the liquor,
Fresh maggots were started, and fancies grew quicker;
They were long in contriving what both sides could please,
And at length the proposals agreed on were these:
For a single year's service the man should be t'yd:
And for every day that he was full employ'd
Seven shillings per day should his wages be paid;
But for all such as those when he rested or play'd,

He

He should forfeit three shillings; the year was compleat,
Neither master nor man was in each other's debt.
Now, what time he neglected, ye artists, is sought,
And how much for his master in painting he wrought?

First, suppose he wrought 100 days; then $365 - 100 = 265$ days he was idle

$$\left.\begin{array}{ll}\text{Therefore,} & 100 \times 7 = 700 \\ \text{And, -} & 265 \times 3 = 771\end{array}\right\} \text{Subtract} = 95, \text{ too little}$$

Again, suppose he wrought 108 days, then $365 - 108 = 257$ days he played

$$\left.\begin{array}{ll}\text{Therefore,} & 108 \times 7 = 756 \\ \text{And, -} & 265 \times 3 = 771\end{array}\right\} \text{Subtract} = 15, \text{ too little}$$

$$\begin{array}{rcr} \textit{Sup.} & & \textit{er.} \\ 100 & \times & 95 \\ 108 & & 15 \\ \hline 10260 & & 1500 \end{array}$$

Then $10260 - 1500 = 8760$
And $95 - 15 = 80 \therefore 8760 \div 80 = 109\frac{1}{2}$ days he wrought
And $365 - 109\frac{1}{2} = 255\frac{1}{2}$ days idle, the answer
For $109\frac{1}{2} \times 7 = 255\frac{1}{2} \times 3 = 766\frac{1}{2}$ Proof

E. 11. If $\frac{2}{3}$ of my age be added to $\frac{4}{6}$ thereof, and that sum multiplied by 4, that product divided by 8, and that quotient made less by 8, the remainder will be 14, what was my age at the time of making this question?

Suppose my age to be 21

$$\begin{array}{rr} \text{Then} & 21 \\ & 2 \\ \hline & 3)42 \\ \hline & 14 = \frac{2}{3} \\ + & 14 = \frac{4}{6} \\ \hline & 28 \\ \times & 4 \\ \hline & 8)112 \\ \hline & 14 \\ & 8 \\ \hline \text{Too little} & 6 \\ \text{Should be} & 14 \\ \hline \text{First error} & -\ 8 \end{array} \text{ Subtract}$$

Again, suppose my age to be 27

$$\begin{array}{rr} \text{Then} & 27 \\ & 2 \\ \hline & 3)54 \\ \hline & 18 = \frac{2}{3} \\ + & 18 = \frac{4}{6} \\ \hline & 36 \\ \times & 4 \\ \hline & 8)144 \\ \hline & 18 \\ & 8 \\ \hline \text{Too little} & 10 \\ \text{Should be} & 14 \\ \hline \text{Sec. er.} & -\ 4 \end{array}$$

$$\begin{array}{rcr} \textit{Sup.} & & \textit{er.} \\ 21 & \times & 8 \\ 27 & & 4 \\ \hline 216 & & 84 \end{array} \qquad \begin{array}{rr} 8 & 216 \\ 4 & 84 \\ \hline 4\ ) & 132 \\ \hline \text{Ans.} & 33 \text{ Years} \end{array}$$

Note. It will sometimes shorten the work by making a cypher and unit the two suppositions.

E. 12. What number is that, which being multiplied by 6, the product increased by 18, and the sum divided by 9, the quotient will be 20?

First, suppose 30 to be the number sought; then $\frac{30 \times 6 + 18}{9} = 10 \times 2 + 2 = 22$; but ought to have been 20; therefore the error is 2 in excess.

Again,

Again, ſuppoſe 18 the number ſought; then $\frac{18\times 6+18}{9}=2\times 6+2=14$; but ought to be 20; therefore the error is 6 in defect, and the errors are of different kinds or affections.

$$\begin{array}{ccc} \textit{Sup.} & & \textit{er.} \\ 30 & \times & 2 \\ 18 & & 6 \\ \hline 36 & & 180 \end{array}$$

Then $180+36=216$, and $6+2=8 \therefore 216\div 8=27$, the number ſought,

But to work this by the note in the preceeding page, ſuppoſe firſt 0; then $\frac{0\times 6+18}{9}=\frac{18}{9}=2$; but ought to be 20; therefore the error is 18 in defect.

Again, ſuppoſe 1; then $\frac{1\times 6+18}{9}=\frac{2+6}{3}=\frac{8}{3}=2\frac{2}{3}$; but ſhould have been 20; therefore the error is $17\frac{1}{3}$ in defect alſo, and the errors are of the ſame kind.

Whence per rule, $\frac{0\times 17\frac{1}{3} \backsim 18\times 1}{18-17\frac{1}{3}}=\frac{18}{\frac{2}{3}}=9\times 3=27$, the number ſought.

XXXIV. ARITHMETICAL PROGRESSION.

IS when a rank or ſeries of numbers increaſe or decreaſe by a common difference, or by a continual adding or ſubtracting ſome equal numbers,

As, $\left\{\begin{array}{l} 1,\ 2,\ 3,\ 4,\ 5,\ 6, \\ 6,\ 5,\ 4,\ 3,\ 2,\ 1, \end{array}\right\}$ Here the common difference is 1.

Or 1, 3, 5, 7, 9, 11; here the common difference is 2.

Alſo 30, 25, 20, 15, 10, 5; here the common difference is 5.

The numbers that compoſe a rank or ſeries of progreſſionals, are called its terms, whereof the firſt and laſt are called extremes, and any two equally diſtant from them, means. Now when the number of terms are even, as 1, 3, 5, 7, 9, 11, the ſum of the two extremes will be equal to the ſum of any two means that are equally diſtant from the extremes, viz. 1, 3, 5, 7, 9, 11,

$$1+11=5+7=3+9=12.$$

When the number of terms are odd, as 4, 10, 16, 22, 28, the double of the middle figure or term will be equal to the ſum of the extremes, or to any two means equally diſtant from the middle term.

Viz. 4, 10, 16, 22, 28,

$$16+16=32, \qquad 22+10=32.$$

In this rule there are five things to be conſidered, viz.

1. The firſt term, commonly the leaſt.

2. The

2. The laſt term, commonly the greateſt.
3. The number of terms.
4. The common exceſs, or difference.
5. The aggregate, or ſum of all the terms.

Any three of which being given, the other two may be eaſily found.

PROPOSITION 1. When the two extremes, and the number of terms are given, to find the ſum of all the ſeries or terms,

RULE. Multiply the ſum of the two extremes by half the number of terms, or multiply the ſum of the two extremes into the number of terms, and divide the product by 2; the quotient will be the ſum of all the ſeries.

E. 1. How many ſtrokes does the hammer of a clock ſtrike in 12 hours;

$1+12=13$ the ſum of the eztremes
6 half the No. of terms

Anſ. 78 Strokes

Or thus:

$1+12=13$ ſum of the extremes
12 the numb. of terms

2)156

Anſ. 78 Strokes, as before

E. 2. Suppoſe 100 ſtones were placed in a right line, a yard diſtant from one another, and the firſt ſtone was a yard from the baſket; I demand how many miles he muſt travel that gathers them ſingly into the baſket?

$2+200=202$ The ſum of the extremes
$\times$ 50 Half the number of terms

Yds. —— *Miles*

A mile $=$ 1760)10100($5\frac{3}{4}$ wanting 20 yards, the diſtance ran,
8800 Anſwer

1300 Yards

E. 3. A butcher buys 100 ſheep, and gave for the firſt ſheep 1*s*. and for the laſt 9*l*. 19*s*. I demand what he gave for the 100 ſheep?

Firſt 9*l*. 19*s*.	$=$	199 Shillings
Then $1+199$	$=$	200 The ſum of the extremes
And $100\div2$	$=$	50 Half the number of terms

2|0)1000|0

Anſ. £. 500

PROPOSITION 2. When the two extremes, and number of terms are given, to find the common difference,

RULE. The difference of the two extremes divided by the number of terms leſs 1, the quotient will be the common difference.

E. 4. One who had 12 children, that differed alike in their ages; the youngeſt was 5 years old, the eldeſt 27; what was the difference of their ages, and the age of each?

$27-5=22$ The difference of the extremes
And $12-1=11$ The number of terms leſs 1
$\therefore$ $22\div11=2$ The common difference

Which added to the age of the youngeſt, and ſo on to the reſt, will give their ſeveral ages, viz. $5+2=7$ the age of the ſecond, and ſo for the reſt.

E. 5.

E. 5. A debt is to be diſcharged at 10 different payments in arithmetical progreſſion; the firſt payment is to be 5*l.* and the laſt 50*l.* what is the whole debt, and what muſt each payment be?

Firſt $50-5=45$ Difference of the extremes
Then $10-1=9$ Number of terms leſs 1
$\because$ $45\div 9=5$ The common difference
Conſequently $5+10+15+20+25+30+35+40+45+50=275$*l.* the whole debt.

Prop. 3. When the two extremes and common difference are given, to find the number of terms,

RULE. Divide the difference of the two extremes by the common exceſs; add unity or 1, to the quotient, and the ſum will be the number of terms.

E. 6. A man being aſked how many children he had, anſwered, my youngeſt child is five years old, and the eldeſt 27, and that he had increaſed one in his family every two years; how many children had he?

Firſt $27-5=22$ Difference of the extremes
Then $22\div 2=11$ Number of terms leſs 1
$\because$ $11+1=12$ Children, the anſwer

E. 7. A perſon was to go a journey, and his firſt day's travel is to be 6 miles, and the laſt 60, every day increaſing his journey three miles, how many days would he be in compleating the ſame?

Firſt $60-6=54$ Difference of the extremes
And $54\div 3=18$ $\because$ $18+1=19$ Days, the anſwer

Prop. 4. When the laſt term, the common difference, and the number of terms are given, to find the firſt term,

RULE. Multiply the number of terms leſs 1, by the common difference; the product ſubtracted from the laſt term leaves the firſt.

E. 8. A man in 19 days went from Birmingham to a certain place; every days journey was greater than the preceding one by three miles, his laſt day's journey was 60 miles, what was the firſt?

Firſt $19-1=18$ Number of terms leſs 1
Then 18×3 (the common difference) $=54$
And $60-54=6$ The firſt days journey, anſwer

Prop. 5. When the number of terms, common difference, and ſum of all the terms are given, to find the firſt term,

RULE. Divide the ſum of all the ſeries by the number of terms, and from that quotient ſubtract half the product of the common difference, multiplied by the number of terms leſs 1, gives the firſt term.

E. 9. An amiable lady being in company with a very agreeable young gentleman, told him that in two years and a half ſhe ſhould receive the whole of her fortune, which was 1000*l.* that next quarter-day ſhe ſhould receive the firſt payment, and each payment after would

would exceed the former by 20*l*. " Now Sir, (ſays ſhe) I am free to give you the firſt payment, provided you will tell me what it is from the given data. But the young gentleman being unſkilled in numbers, could not comply with her propoſal, but leaves it to the ſtudy of the ingenious arithmetician to reſolve this queſtion, and tell him the firſt payment.

Firſt $1000 \div 10$ (the quarters in $2\frac{1}{2}$ years) $= 100$

And $10 - 1 = 9$, which multiplied by 20 (the common difference) the product is 180: then $180 \div 2 = 90$, which deducted from 100, leaves 10*l*. the anſwer.

Or thus: From $1000 \times 2 = 2000$
Take $10 \times 20 \times 9 = 1800$
Diviſor $10 \times 2 = 2|0)20|0$
£.10 The anſwer as above

PROP. 6. When the firſt term, number of terms, and the common difference, are given, to find the laſt term,

RULE. Subtract the common difference from the product of the number of terms, multiplied by the common difference; the remainder added to the firſt term will give the laſt.

E. 10. What is the laſt term of an arithmetical progreſſion, beginning at 6, and continuing by the increaſe of 3 to 21 places?

Firſt $21 \times 3 = 63$, and $63 - 3 = 60$

Then $60 + 6 = 66$, the laſt term required.

E. 11. What is the laſt term of an arithmetical progreſſion, beginning at 1, and continuing by the increaſe of two to 50 places?

Firſt $50 \times 2 = 100$; then $100 - 2 = 98$

And $98 + 1 = 99$, the laſt term required.

PROP. 7. The firſt term, common difference, and number of terms given, to find the ſum of all the terms.

RULE. From the product of the number of terms in the common difference, ſubtract the common difference, and to the remainder add the double of the firſt term; half the product of that ſum multiplied by the number of terms, gives the ſum of all the terms or ſeries.

E. 12. Suppoſe I agree with a pump-maker to ſink a well 30 yards deep, upon theſe terms, viz. to pay him three ſhillings for the firſt yard, five for the ſecond, ſeven for third, &c. raiſing two ſhillings for every yard; what will the whole amount to?

Firſt $30 \times 2 = 60$; alſo $60 - 2 = 58$

Again $58 + 9 = 64$; and $64 \times 30 = 1920$

$\therefore 1920 \div 2 = 960s. = 48l.$ the anſwer.

PROP. 8. The firſt term, the number of terms, and ſum of all the terms given, to find the common difference,

RULE. Divide the double ſum of all the ſeries by the number of terms, and from the quotient ſubtract double the firſt term; divide the

remainder by the number of terms leſſened by unity, the quotient will be the common difference.

E. 13. A perſon travelled from London to York, being 180 miles, in 6 days, and every day travelled equally further than the preceeding day; it is known that the firſt day he travelled 6 miles, how many miles did he travel each of the other days?

Firſt $360 \div 6 = 60$, and $60 - 12 = 48$; alſo $6 - 1 = 5$

Then $48 \div 5 = 9\frac{3}{5}$ miles, the common difference required

∵ $9\frac{3}{5}$ Added to 6, and every other term reſpectively gives as follows,

viz.	6	for the firſt	Days journey. Q. E. F.
	$15\frac{3}{5}$	ſecond	
	$25\frac{1}{5}$	third	
	$34\frac{4}{5}$	fourth	
	$44\frac{2}{5}$	fifth	
	54	ſixth	
Proof	180	Miles	

Prop. 9. When one perſon or thing moves with an equal, and another the ſame way by a progreſſive motion, to find in what time the firſt will be overtaken.

RULE. To double the ſpace gone each day by the purſued, add the common difference of the purſuer's day's journey; from that ſum ſubtract double the ſpace he travelled the firſt day, and divide the remainder by the common difference, the quotient will give the number of days, in which the purſued will be overtaken by the purſurer.

E. 14. A noted highwayman having commited a robbery, and ſuſpecting a purſuit, rode off at the rate of 40 miles a day; a thieftaker (one of Sir John Fielding's men) upon the ſcent, follows him in a progreſſive motion only 30 miles the firſt day, 34 the ſecond, 38 the third, and ſo on, increaſing every day's journey 4 miles; in how many days will the highwayman be overtaken?

Firſt $80 + 4 = 84$, and $84 - 60 = 24$

Then $24 \div 4 = 6$ days, the anſwer

For $6 \times 40 = 240$ miles, the ſpace travelled by the robber

Then by Prop. 7. $6 \times 4 = 24$, alſo $24 - 4 = 20$, and $20 + 60 = 80$

∵ $80 \div 2 \times 6 = 240$ miles when the thief-taker comes up with the highwayman.

XXXV. GEOMETRICAL PROGRESSION.

IS when any rank or ſeries of numbers increaſe by one common multiplier, or decreaſe by one common diviſor.

As 4, 8, 16, 32, 64, 128; here the common multiplier or ratio is 2.

Alſo, 729, 243, 81, 27, 9, 3; here the com. diviſor or ratio is 3.

In any ſeries of numbers in geometrical progreſſion, the product of the two extremes are equal to the product of any two means, that are equally diſtant from the extremes.

As

As 3, 9, 27, 81, 243, 729.
Here $3\times729=9\times243=27\times81=2187$.

When the number of terms are odd, the middle term multiplied into itſelf will be equal to the product of the two extremes, or any two means equally diſtant from the ſaid mean or middle term.

As 3, 6, 12, 24, 48. $12\times12=6\times24=48\times3=144$.

In geometrical progreſſion, five things are to be obſerved, as in arithmetical progreſſion, viz.

1. The firſt term. 2. The laſt term. 3. The number of terms. 4. The ratio. 5. The ſum of the terms.

Any three of theſe being known, the reſt may be eaſily found.

If over any rank of geometrical numbers, you place a ſeries of arithmetical ones beginning with 0, the addition and ſubtraction of the indices, anſwer to the multiplication and diviſion of the numbers they ſtand over.

Thus $\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7 \text{ Indices} \\ 1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128 \text{ Numbers in geomet. progreſſ.} \end{cases}$

That is $\begin{cases} \text{As } 2+3=5 \text{ which is the indice of } 32 \\ \text{So } 4\times8=32 \text{ the 5th term in geometrical progreſſion.} \end{cases}$

Again $\begin{cases} \text{As } 2+4=6 \\ \text{So } 4\times16=64 \text{ the 6th term.} \end{cases}$

Now by theſe indices and a few of the firſt terms, the laſt term, or any diſtant one, may be ſpeedily found, without producing the whole ſeries.

Proposition 1. When the firſt term is unity, the ratio and number of terms being known, to find the laſt of any remote term,

RULE. Find a few of the leading terms, over which place their indices, as before directed; then find what figures of the indices, when added together, will give the index of the term wanted; multiply the numbers ſtanding under ſuch indices into each other, the laſt product will be the term required.

Note. When the indices begin with a cypher, the ſum of the indices made choice of muſt be always one leſs than the number of terms given in the queſtion, becauſe 1 in the indices ſtands over the ſecond term.

Example 1. A boy agrees for 14 oranges, to pay only the price of the laſt, reckoning a farthing for the firſt, a half-penny for ſecond, &c. doubling the price to the laſt; how much did he give for them?

Firſt $\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6 \text{ Indices} \\ 1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64 \text{ Terms} \end{cases}$

Then $\begin{cases} 2+5=7 \\ 4\times32=128 \end{cases}$ Alſo $\begin{cases} 7+6=13 \\ 128\times64=8192 \text{ } qrs. \end{cases}$

Which is the 14th term, becauſe the indices are leſs than the terms by one. And 8192 *qrs.* = 8*l.* 10*s.* 8*d.* Anſwer.

E. 2. A man bought a horſe, and by agreement was to give what the laſt nail would come to, at a farthing for the firſt nail, two for the

 ſecond,

ſecond, four for the third, &c. There were four ſhoes, and 8 nails in each ſhoe? I demand the price of the horſe?

$$\text{First}\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8 \text{ Indices} \\ 1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128,\ 256 \text{ Terms} \end{cases}$$

$$\text{Then}\begin{cases} 7+7=14 \\ 128\times128=16384=14\text{th term} \end{cases}$$

$$14+14=28$$
$$16384\times16384=268435456=28\text{th term}$$
$$\times 8=3\text{d term}$$
$$=2147483648 \text{ Farthings}$$

Or 32d term, which reduced to pounds, will give 2236962*l.* 2*s.* 8*d.* the price of the horſe, anſwer.

PROP. 2. In any ſeries, not proceeding from unity, the ratio and firſt term being given, to find any remote term, without producing all the intermediate terms.

RULE. Proceed as in the laſt propoſition, only obſerve to divide every product by the firſt term, and the quot. will be the term required.

E. 3. A perſon dying left 11 children, to whom, and to his executor, he bequeathed in manner following, viz. to his executor, for ſeeing his will performed, 10*l.* the youngeſt child to have 30*l.* and ſo on, every child to exceed the next younger in triple proportion; what will be the ſhare of the eldeſt?

$$\text{First}\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6 \text{ Indices} \\ 10,\ 30,\ 90,\ 270,\ 810,\ 2430,\ 7290 \text{ Terms} \end{cases}$$

$4+6=10$ Number of terms leſs 1

Then $810\times7290=5904900$, which $\div$ 10, the firſt term, gives 590490*l.* the eldeſt child's fortune.

PROP. 3. When the firſt term, ratio, and number of terms are given, to find the ſum of all the terms.

RULE. Find the laſt term as before, from which take the firſt, divide the remainder by the ratio, leſs one, and to that quotient add the laſt term, and you have the ſum required.

E. 4. A gentleman married, and received of his father-in-law one guinea, on condition that he was to have a preſent every month for the firſt year, which ſhould be double ſtill to what he had the month before; what was the young lady's portion;

$$\text{First}\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6 \text{ Indices} \\ 1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64 \text{ Terms} \end{cases}$$

$$\text{Then}\begin{cases} 6+5=11 \text{ The number of terms leſs one} \\ 64\times32=2048 \text{ The laſt term} \end{cases}$$

And $2048-1\div2-1=2047$

Alſo $2047+2048=4095$ Guineas

$1s.=\frac{1}{20})4095\quad 0$

$\qquad\qquad 204\quad 15$

Anſ. £. 4299 15 The young lady's portion

Note

Note. If the ratio of any rank or ſeries of proportionals be double, the difference of the greateſt and leaſt terms are equal to the ſum of all except the greateſt; if the ratio be triple, the exceſs or difference is double the ſum of all except as aforeſaid; if quadruple, triple; if quintuple, quadruple, and ſo on.

E. 5. A laceman well verſed in numbers, agreed with a gentleman to ſell him 20 yards of rich gold-brocaded lace, for 2 pins the firſt yard, 6 for the ſecond, 18 for the third, and ſo on in triple proportion; I demand how much the lace produced? The pins afterwards ſold at a farthing per 100; alſo whether the laceman gained or loſt by the ſale thereof, ſuppoſing the ſaid lace to have been bought at 10*l.* 1*s.* per yard?

$$\text{Firſt}\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5 \text{ Indices} \\ 2,\ 6,\ 18,\ 54,\ 162,\ 486, \text{ Terms}\end{cases}$$

Then $5+5=10$

$486\times486=236196$ the 10th term

And $5+4=9$

$486\times162=78732$ the 9th term

∴ 236196	
× 78732	
472392	
708588	
1653372	
1889568	
1653372	
18596183472	laſt term
— 2	Firſt ditto
Ratio $3-1=2$)18596183470	
This added to the laſt term - 9298091735	gives
The ſum of all - - - 1|00)278942752|07	the terms

	£	s.	d.
Value of the pins 278942752 $\frac{7}{100}$ qrs. =	290565	7	4
Lace comes to 10*l.* 1*s.* × 5 × 4 (=20) =	201	0	0
Anſwer, The laceman gained - - £.	290364	7	4

Prop. 4. When the firſt term and ratio of any infinite decreaſing geometrical ſeries, or infinite ſeries of decreaſing proportionals are given, to find the ſum of the ſeries:

RULE. Divide the ſquare of the firſt term by the difference between the ſaid firſt term and the ſecond term in the ſeries; the quotient will be the ſum of the ſeries.

Note. A geometrical ſeries that decreaſeth ad infinitum, or in other words, an infinite ſeries of decreaſing proportionals, are ſuch whoſe laſt or leaſt term is a cypher, or leſs than any thing aſſignable, and its number of terms inexpreſſible.

E. 6. A great ſhip purſues a ſmall one, ſteering the ſame way, at the diſtance of ſix leagues from it, and ſails twice as faſt as the ſmall ſhip; how far muſt the great ſhip ſail before it overtakes the leſſer?

Firſt 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, &c. ad infinitum

Then $6\times6=36$, ſquare of the firſt term

And $36\div3$ the ſecond term $=12$ leagues, the anſwer.

E. 7.

E. 7. *Suppose a round ball for to move in the air,*
In a certain proportion which I shall declare;
Let the first hour be 12 miles, the next to move 10,
And so in proportion from whence it began,
As 12 is to 10. Now try if you can
Tell the miles it will move, suppose it to be
Continued in motion to ETERNITY?

First $12 \times 12 = 144$ square of the first term
And $12 - 10 = 2$ difference of the first and second term
Then $144 \div 2 = 72$ miles, the answer.

Otherwise thus; As 2 : 10 :: 12 : 60 miles, the sum of all the terms except the first or greatest; to which add 12 the first term, aud the sum will be 72 miles, the answer as above.

XXXVI. PERMUTATION,

OR,

VARIATION.

IS the changing or varying the order of things, in respect of their places.

RULE. Multiply all the given terms into one another continually, whose first term, or common difference, is unity or 1, and the last product will be the number of changes or variations required.

E. 1. A young scholar coming into town for the conveniency of a good library, demands of a gentleman with whom he lodged, what his diet would cost for a year? who told him 10*l.* but the scholar not being certain what time he should stay, asked him what he must give him for so long as he could place his family (consisting of seven persons besides himself) in different positions every day at dinner; the gentleman thinking it would not be long, tells him 5*l.* to which the scholar agrees; what time did he stay with the gentleman?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ days, answer.

Days in a year = 365)40320(110 years 170 days, the time the scholar (was to stay
365
382
365
170

E. 2. *At Birmingham we've a church that's nearly new,*
Which beauteous pile can be outvied by few:
Here sacred grandeur captivates the eye,
Trav'lers admire the same as they pass by;
To grace this structure, there's a lofty tower,
With ten fine bells, which harmonize each bower.

How

How many changes may be rung declare,
On these ten bells, and likewise tell me fair
How long they would be ringing them once o'er,
Allowing six seconds per change not more?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800$, number of changes

Then $3628800 \times 6 = 21772800$ seconds

Seconds.

In a day there are 864|00)217728|00(252 days
1728
4492
4320
1728
1728
0

Answer 252 days

E. 3. An accomptant told a gentleman, who had constantly 8 persons at his table, that he would gladly make a ninth, and was willing to give 20 guineas for his board, so long as he could place the said company at dinner, differntly from any one day before; this being accepted, what did his entertainment cost him per year?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ days

Then, as 362880 : 20 :: 365
21
20
40
420
12
5040
365
25200
30240
15120
362880)1839600($5\frac{25200}{362880}d.$
1814400 Answer
25200

Answer $5\frac{5}{72}d.$ in its lowest terms

E. 4. *A famous gen'ral having serv'd his king,*
Who always from the wars did victory bring,
For his good service (with a pleasant smile)
Ask'd of his king one farthing for each file
Of ten men in a file, which he could then
Make with a body of one hundred men.
The king considering his brave actions past,
And seeming modesty of his request,
Gave his consent; to what will it amount
In sterling money? Take your pen and count.

Note. To solve questions of this nature, you must place the given quantity by itself decreasing it gradually by unity, so often as there are quantities in the combinations; placing them one after another with a sign of multiplication between them, which numbers must be multiplied into one another for a dividend; then placing an unit with the

the like number of places, increasing by unity till you arrive at the number to be combined; which multiply continually for a divisor, and the quotient will be the number of combinations sought, thus:

$$\frac{100}{1}\times\frac{99}{2}\times\frac{98}{3}\times\frac{97}{4}\times\frac{96}{5}\times\frac{95}{6}\times\frac{94}{7}\times\frac{93}{8}\times\frac{92}{9}\times\frac{91}{10}=$$

$$\frac{62815650955529472000}{3628800}=17310309456440 \textit{ qrs.}$$ which are equal to 18031572350*l.* 9*s.* 2*d.*

E. 5. *Two gamesters one day, at dice they did play,*
And being full merry with wine;
Says B *unto* A, *what odds will you lay,*
I cast not six aces this time?
Says A *then to B, ten to one I'll lay thee,*
With six dice, the six aces you cast not:
Pray youths shew, and here let me know,
For the odds on the cast, Sirs, they know not.

First $6\times6\times6\times6\times6\times6=46656=6$ different combinations
And $1\times2\times3\times4\times5\times6=720$ variations
Then $46656-720=45936$ chances against A
But a A laid 10 to 1, therefore 7200 chances to B
Therefore A's chance to that of B's is as 45936 : 7200; or as 6.38 : 1, the Answer.

Practical Arithmetic.

PART II.

VULGAR FRACTIONS.

A FRACTION is some part or parts of an integer, or whole thing, represented by 1; as $\frac{3}{4}$ is a fraction, denoting three-fourth parts of an integer or 1. Every fraction consists of two numbers, placed one above the other, with a line between them, as in this fraction $\frac{2}{3}$; the the lower number 3 is called the denominator, and shews how many parts the integer is divided into; the upper number 2, is called the numerator, and expresses how many of these parts the fraction consists of. And both numerator and denominator are called terms of the fraction.

A Vulgar Fraction is either proper, improper, single, compound, or mixed.

A proper

A Proper Fraction is that wherein the numerator is leſs than the denominator, as $\frac{3}{4}$; and is called proper with reſpect to the relative integer, becauſe it expreſſes a quantity leſs than it.

An Improper Fraction is ſuch whoſe numerator is equal to, or greater than its denominator, as $\frac{4}{4}$, $\frac{4}{3}$, and is called improper with reſpect to the relative integer, becauſe it expreſſes a quantity greater than it.

A Single Fraction is that which conſiſts of but one numerator, and one denominator, and is referred immediately to ſome integer, as $\frac{3}{4}$, or $\frac{1}{2}$ of any thing.

A Compound Fraction is the fraction of a fraction, conſiſting of two or more ſimple fractions referred to one another in order, and the laſt referred to ſome integer, as $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{6}$, &c.

A Mixed Number is compoſed of a whole number and fraction, as $6\frac{1}{2}$, $34\frac{2}{3}$, $152\frac{6}{8}$, &c.

XXXVII. REDUCTION OF VULGAR FRACTIONS.

Case 1.

To reduce a fraction to another of equal value.

RULE. Multiply or divide both terms of the fraction by the ſame number, and you will have a new fraction equivalent to that given.

Example. Let the fraction be $\frac{4}{6}$; now (per rule) multiply both terms of the given fraction by 5, thus:

$$\begin{array}{r} 4 \\ \underline{5} \\ 20 \end{array} \qquad \begin{array}{r} 6 \\ \underline{5} \\ 30 \end{array}$$

Whence the new fraction $\frac{20}{30} = \frac{4}{6}$. Again Divide both terms of the fraction by 5, thus:

$5)\frac{20}{30}(\frac{4}{6}$ The fraction given.

Case 2. To reduce a whole number to the form of a fraction,

RULE. Set 1 under it for a denominator.

Example. Suppoſe 8, 6, 4, 36, were numbers to be reduced to fractions. Then (per rule) they become $\frac{8}{1}$, $\frac{6}{1}$, $\frac{4}{1}$, $\frac{36}{1}$, the fractional quantity required.

Case 3. To reduce a whole number to a fraction of a given denomination,

RULE. Multiply the whole number by the given denominator, and under the product write the ſame denominator.

Example. Reduce 9 into a fraction whoſe denominator ſhall be 6.

$$\begin{array}{r} 9 \\ \underline{6} \\ 54 \end{array}$$

Numerator; Then $\frac{54}{6}$ is the fraction required.

CASE 4. To reduce a compound fraction to a single one of the same value,

RULE. Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

EXAMPLE. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction.

$$\begin{array}{rr} & 2 \qquad 3 \\ & \underline{3} \qquad \underline{4} \\ & 6 \qquad 12 \\ & \underline{4} \qquad \underline{5} \\ \text{N.} & 24 \qquad 60 \ \text{D.} \end{array}$$

Then $\frac{24}{60}$ is the single fraction.

Note. N. stands for numerator, and D. for denominator.

CASE 5. To reduce any mixed number to an improper fraction,

RULE. Multiply the whole number by the denominator of the fraction, and to the product add the numerator for a new numerator, which place over the denominator.

EXAMPLE. Reduce $26\frac{3}{8}$ to an improper fraction.

$$\begin{array}{r} 26\frac{3}{8} \\ \underline{8} \\ 211 \end{array}$$

211 N. $\therefore$ $\frac{211}{8}$ is the fraction required.

CASE 6. To reduce an improper fraction to its equivalent whole or mixed number,

RULE. Divide the numerator by the denominator, and the quotient is the whole number. Then what remainder there is, place it over the denominator and annex this fraction to the quotient before found.

EXAMPLE. Reduce $\frac{211}{8}$ to its equivalent whole or mixed number:

$$\begin{array}{r} 8)211 \\ \hline 26\frac{3}{8} \end{array}$$

Ans. $26\frac{3}{8}$

Reduce $\frac{631}{16}$ to its equivalent whole or mixed number.

$$\begin{array}{r} 16)631(39\frac{7}{16} \ \text{Ans.} \\ \underline{48} \\ 151 \\ \underline{144} \\ 7 \end{array}$$

Note. The preceeding six cases being so exceeding easy, I thought more than one example in each case would be quite unnecessary.

CASE 7. To find the greatest common measure or divisor for the numerator and denominator of any given fraction, or for any two numbers,

RULE. Divide the greater term by the lesser, and the last divisor by the remainder, and so on continually till nothing remain; the last divisor is the common measure required.

EXAMPLE

EXAMPLE 1. What is the greateſt common meaſure of $\frac{252}{364}$?

$$\begin{array}{l} 252)364(1 \\ \underline{252} \\ 112)252(2 \\ \quad \underline{224} \\ \quad 28)112(4 \\ \qquad \underline{112} \\ \qquad 0 \end{array}$$

Anſwer 28 is the laſt diviſor, which is the greateſt number that will divide both numerator and denominator without a remainder.

When there are mixed numbers given, they muſt be reduced to a common denominator; then proceed with the two new numerators to find their greateſt common meaſure, make that the numerator, and under put the common denominator, which fraction will be the greateſt common meaſure required.

E. 2. What is the greateſt common meaſure of $14\frac{4}{5}$ and 32;

$$\begin{array}{lll} 14\frac{4}{5} & 32 & 74)160(2 \\ \underline{5} & \underline{5} & \underline{148} \\ 74\text{ N.} & 160\text{ N.} & 12)74(6 \\ & & \underline{72} \\ & & 2)12(6 \\ & & \underline{12} \end{array}$$

Anſwer $\frac{2}{5}$ is the greateſt common meaſure of $14\frac{4}{5}$ and 32.

CASE 8. To reduce a fraction to its leaſt, or loweſt terms,

RULE. Find the greateſt common meaſure, by which divide both terms of the fraction; the quotients will give the fraction required.

EXAMPLE 1. Reduce $\frac{2832}{12848}$ to its loweſt terms.

$$\begin{array}{l} 2832)12848(4 \\ \underline{11328} \\ 1520)2832(1 \\ \underline{1520} \\ 1312)1520(1 \\ \underline{1312} \\ 208)1312(6 \\ \underline{1248} \\ 64)208(3 \\ \underline{192} \\ 16)64(4 \\ \underline{64} \end{array}$$

Then $16)\frac{2832}{12848}(=\frac{177}{803}$ Anſ.

Note. If the laſt remainder is 1, the fraction is already in its loweſt terms.

E. 2. Reduce $\frac{204}{228}$ to its lowest terms.

$$\begin{array}{l} 204)228(1 \\ \quad\underline{204} \\ \quad\;\; 24)204(8 \\ \qquad\quad \underline{192} \\ \qquad\quad 12)24(2 \end{array}$$

Then $12)\frac{204}{228}(=\frac{17}{19}$ Answer

When the terms of the fraction are even numbers they may be divided by 2 continually, thus: $\frac{24}{136}$, being continually halved, is $\frac{24}{136}=\frac{12}{68}=\frac{6}{34}=\frac{3}{17}=$ in value to the given fraction $\frac{24}{136}$.

When both terms end with 5; or one with 5, and the other with a cypher, divide both by 5, thus: $5)\frac{65}{80}(=\frac{13}{16}$ Answer.

When both terms end with cyphers, cut off an equal number in both. Thus, $\frac{1|00}{24|00}$ becomes $\frac{1}{24}$ the terms required.

If you can discern any number that will divide both terms, divide by that number.

Reduce $\frac{24}{112}$ to its lowest terms. $8)\frac{24}{112}(=\frac{3}{14}$ Answer

Likewise $7)\frac{119}{168}(=\frac{17}{24}$ Answer

CASE 9. To reduce fractions of different denominations to fractions of equal value, that shall have one common denominator,

RULE. Multiply each numerator by all the denominators except its own, for a new numerator; then multiply all the denominators together for a new denominator.

EXAMPLE 1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, to a common denominator.

2	3	4	3
4	3	4	4
8	9	16	12
5	5	3	5
40 N.	45 N.	48 N.	60 D.

Therefore $\frac{2}{3}=\frac{40}{60}$, $\frac{3}{4}=\frac{45}{60}$, and $\frac{4}{5}=\frac{48}{60}$ Answer.

E. 2. Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, to a common denominator.

1	1	1	1	3
4	3	3	3	4
4	3	3	3	12
5	5	4	4	5
20	15	12	12	60
6	6	6	5	6
120 N.	90 N.	72 N.	60 N.	360 Common denominator.

Answer $\frac{120}{360}$, $\frac{90}{360}$, $\frac{72}{360}$, $\frac{60}{360}$, are all of the same value with the respective original ones, and have one common denominator.

E. 3.

E. 3. Reduce $14\frac{2}{3}$, 7, and $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{9}$, and $\frac{4}{7}$ to 1 common denominator.
First $14\frac{2}{3}=\frac{44}{3}$, and $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{9}=\frac{30}{216}=\frac{5}{36}$ in its lowest terms.
Then we have these fractions $\frac{44}{3}$, $\frac{7}{1}$, $\frac{5}{36}$ and $\frac{4}{7}$ to be reduced.

44	7	5	4	3
7	3	1	36	1
308	21	5	144	3
1	36	3	1	36
308	126	15	144	108
36	63	7	3	7
1848	756	105 N.	432 N.	756 Com. denom.
924	7			
11088 N.	5292 N.			

Answer $14\frac{2}{3}=\frac{11088}{756}=\frac{44}{3}$; $\frac{7}{1}=\frac{5292}{756}$; $\frac{5}{36}=\frac{105}{756}$; and $\frac{4}{7}=\frac{432}{756}$.

Note. If one fraction be equivalent to another, it will hold as the numerator of the one is to its denominator, so is the numerator of the other to its denominator; or as one numerator to the other, so is one denominator to the other. For in the above example it is said $\frac{4}{7}=\frac{432}{756}$.

To prove which say, if 432 : 756 :: 4 : 7 Proof

$$\begin{array}{r} 4 \\ \hline 432)3024(7 \\ 3024 \end{array}$$

CASE 10. Several fractions being given; to find as many whole numbers, in the same proportion,

RULE. Reduce the fractions to a common denominator, then the several numerators will be to one another as the fractions given.

EXAMPLE. Suppose $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, were given, to find whole numbers, in the same proportion.

1	1	1	2
4	2	4	4
4	2	4	8
6	6	2	6
24 N.	12 N.	8 N.	48 Denominator

Then the fractions given are reduced to $\frac{24}{48}$, $\frac{12}{48}$, $\frac{8}{48}$
∴ $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, are in the same proportion to one another as 24, 12 and 8

CASE 11. To reduce coins, weights, measures, &c. into fractions,

RULE, Reduce the given quantity to the lowest denominations mentioned, and make it the numerator; then reduce the whole of the integer, which the given numbers are parts of, and make it the denominator, and you have the fraction required.

EXAMPLE

EXAMPLE 1. Let it be required to reduce 4*s.* 2*d.* to the fraction of a pound sterling.

s.	*d.*		*s.*	
4	2		20	
12			12	
50 N.			240 D.	Answer $\frac{50}{240}$.

E. 2. Reduce $8\frac{1}{2}$*d.* to the fraction of a shilling.

d.	*d.*	
$8\frac{1}{2}$	12	
2	2	
17 N.	24 D:	Anf. $\frac{17}{24}$.

E. 3. Reduce 3 roods, 6 poles to the fraction of an acre.

R.	*P.*	*R.*	
3	6	4	
40		40	
126 N.		160 D.	Anf. $\frac{126}{160}$.

E. 4. Reduce 2 *Cwt.* 1 *qr.* 4 *lb.* to the fraction of 1 *Cwt.*

C.	*qr.*	*lb.*	*C.*	
2	1	4	1	
4			4	
9			4	
28			28	
256 N.			112 D.	Anf. $\frac{256}{112}$.

Note. After the above manner may any other weights, measures, &c. be reduced to fractions.

CASE 12: To reduce fractions of one denomination to another, retaining the same value.

RULE. 1. If the fraction given is to be brought from a less to a greater denomination, multiply the denominator by all the denominations, from that given to that sought.

2. If the fraction given is to be brought from a greater to a less denomination, multiply the numerator by all the denominations, from that given to that sought.

EXAMPLE 1. Reduce $\frac{2}{8}$*l.* to the fraction of a penny.

2
20
40
12
480 N.

Anf. $\frac{480}{8} = \frac{60}{1}$ in its lowest terms.

E. 2. Reduce $\frac{3}{8}$ of a penny to the fraction of a pound.

8
12
96
20
1920

Anf. $\frac{3}{1920} = \frac{1}{640}$ in its lowest term.

E. 3.

E. 3. Reduce $\frac{3}{7}$*lb.* troy, to the fraction of a penny-weight.

$$\begin{array}{r} 3 \\ 12 \\ \hline 36 \\ 20 \\ \hline 720 \text{ N.} \end{array}$$

Anſ. $\frac{720}{7}$.

E. 4. Reduce $\frac{9}{12}$ of a ſhilling to the fraction of a farthing.

$$\begin{array}{r} 9 \\ 12 \\ \hline 108 \\ 4 \\ \hline 432 \text{ N.} \end{array}$$

Anſ. $\frac{432}{12}=\frac{36}{1}$, in its loweſt terms.

E. 5. Reduce $\frac{4}{12}$ of a pound to the fraction of 1 hundred weight.

$$\begin{array}{r} 12 \\ 28 \\ \hline 336 \\ 4 \\ \hline 1344 \text{ D.} \end{array}$$

Anſ. $\frac{4}{1344}=\frac{1}{336}$, in its loweſt ter.

E. 6. Reduce $\frac{2}{3}$ of a pound to the fraction of a guinea.

$$\begin{array}{rr} 2 & 3 \\ 20 & 21 \\ \hline 40 \text{ N.} & 63 \text{ D.} \end{array}$$

Anſ. $\frac{40}{63}$ the fraction required.

E. 7. Reduce $\frac{3}{4}$ of a dram to the fraction of a hundred weight.

$$\begin{array}{r} 4 \\ 16 \\ \hline 64 \\ 16 \\ \hline 384 \\ 64 \\ \hline 1024 \\ 28 \\ \hline 8192 \\ 2048 \\ \hline 28672 \\ 4 \\ \hline 114688 \text{ D.} \end{array}$$

Anſ. $\frac{3}{114688}$.

CASE 13. To find the proper quantity of a fraction in the known parts of an integer,

RULE. Multiply the numerator by the number of parts contained in the integer, and divide the product by the denominator, the quotient ſhews the known parts. If there be any remainder, multiply it by the next inferior denomination, and divide by the denominator as before; continue this work till you come at the loweſt denomination.

EXAMPLE 1. What is the value of $\frac{7}{9}$ of a guinea?

$$\begin{array}{rr} 21 & \\ 7 & \\ \hline 9)147 & \\ \hline 16 & 3 \\ & 12 \\ \hline & 9)36 \\ \hline & 4 \end{array}$$

Anſ. 16*s.* 4*d.*

E. 2. What is the value of $\frac{5}{8}$ of 4*s.* 5*d*?

$$\begin{array}{rrr} & s. & d. \\ & 4 & 5 \\ & & 5 \\ \hline 8)1 & 2 & 1 \\ \hline £.0 & 2 & 9\frac{1}{8} \end{array}$$ Anſwer

E. 3.

E. 3. Required the value of $\frac{135}{480}$ of a pound sterling.

```
       135
        20
      ----
480)2700(5s.
    2400
    ----
     300
      12
    ----
480)3600(7d.
    3360
    ----
     240
       4
    ----
480)960(2qrs.
    960
    ----
Ans. 5s. 7½d.  0
```

E. 4. What is the value of $\frac{484}{1394}$ of a moidore?

```
        484
          9×3=27
      -----
       4356
          3
      -----
1394)13068(9s.
     12546
     -----
       522
        12
     -----
1394)6264(4d.
     5576
     ----
      688
        4
     ----
1394)2752(1qr.
     1394
     ----
     1358
Answer 9s. 4¼d. 1358/1394
```

E. 5. What is the value of $\frac{4}{5}$ of a three-pound-twelve?

```
      £.  s.
      3  12
          4
     ------
   5)14   8
     -----------
Ans. £. 2  17  7⅕
```

E. 6. What is the quantity of $\frac{6}{8}$ of an acre?

```
       6
       4
    8)24
Ans.   3 Rds.
```

E. 7. What is the value of $\frac{5}{8}$ of a day?

```
          5
         24
     8)120
Answer   15  Hours
```

Note. After the same manner the value of any fraction may be found.

XXXVIII. ADDITION OF VULGAR FRACTIONS.

RULE.

REDUCE all the given fractions to simple fractions of the same integer and denominator, if not so already; then the sum of the numerators being made a numerator to the common denominator, makes the fractional sum sought, which may be further reduced, as seems most expedient, or the case will admit.

E. 1. What is the sum of $\frac{2}{8}$ and $\frac{3}{8}$?

```
 2
 3
--
 5      Ans. 5/8
```

E. 2. What is the sum of $\frac{5}{10}$ and $\frac{8}{10}$?

```
 5
 8
--
13      Ans. 13/10
```

E. 3. What is the sum of $\frac{3}{4}$ of $\frac{5}{9}$ and $2\frac{1}{2}$?

First $\frac{3}{4}$ of $\frac{5}{9} = \frac{15}{36}$, and $2\frac{1}{2} = \frac{5}{2}$; these reduced to a common denominator are equal to $\frac{30}{72}$, $\frac{180}{72}$, whose sum is $\frac{210}{72} = \frac{35}{12}$ in its lowest terms.

E. 4. What is the sum of $\frac{1}{3}$ of $\frac{1}{4}$, and $\frac{3}{8}$, and $1\frac{1}{4}$?

First $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, also $1\frac{1}{4} = \frac{5}{4}$, then $\frac{1}{12}$, $\frac{3}{8}$ and $\frac{5}{4}$ reduced to a common denominator, are $\frac{2}{24}$, $\frac{9}{24}$, and $\frac{30}{24}$, the sum whereof is $\frac{41}{24}$, Answer.

E. 5. What is the sum of $\frac{3}{5}$ of a pound, $\frac{5}{10}$ of a shilling, and $\frac{7}{8}$ of a penny?

First $\frac{5}{10}$ of a shilling $= \frac{5}{200}$ of a pound, and $\frac{7}{8}$ of a penny $= \frac{7}{1920}$ of a pound. Then $\frac{3}{5}$, $\frac{5}{200}$, and $\frac{7}{1920}$ are reduced to $\frac{5760}{9600}$, $\frac{240}{9600}$, $\frac{35}{9600}$, whose sum is $\frac{6035}{9600}$ of a pound $= \frac{1207}{1920}$ in its lowest terms $=$ 12*s.* $6\frac{3}{4}$*d.* $\frac{960}{1920}$ Answer.

E. 6. *Admit I sail where billows roar, and plough the raging sea,*
And steering to a foreign shore, a prize falls in my way;
When having chang'd a full broadside, as Rodney us'd to do,
Or Hood, and many more beside, who boldly dar'd the foe:
Suppose this prize ten thousand pound, three-fiftieths is my share,
Another sailor's share is found, his part two-eightieths are,
I purchase this, then what's to me, the total worth define?
And you shall with Minerva be, and in her temple shine.

First find the value of $\frac{3}{50}$ and $\frac{1}{40}$ of 10000*l.* and add them together, thus:

$$
\begin{array}{rl}
4|0) & 1000|0 \\
 & \underline{\quad 3} \\
5|0) & 3000|0 \\
\text{Add} & \left\{\begin{array}{l} 600 = \frac{3}{50} = \text{The sailor's own share} \\ 250 = \frac{1}{40} = \text{The purchased share} \end{array}\right. \\
\text{Answer } £. & \overline{850}
\end{array}
$$

Note. Reduction of fractions being well understood, addition will be very easy; the reason of which will be obvious, if we consider that the givien fractions being such, or reduced to such a state, that all the numerators represent things of the same denomination, both absolute and relative; their sum must therefore be a number of such parts as the common denominator expresses of the same common integer.

XXXIX. SUBTRACTION OF VULGAR FRACTIONS.

RULE.

PREPARE the fractions as directed in addition; then subtract one numerator from the other, and their differcnce will be a new numerator, under which subscribe the common denominator.

EXAMPLE 1. From $\frac{4}{5}$ take $\frac{2}{5}$?

From 4
Take 2
2 The remainder is $\frac{2}{5}$ Ans.

E. 2. From $\frac{18}{44}$ take $\frac{12}{44}$?

18
-12
6 The remainder is $\frac{6}{44}$ Ans.

E. 3.

	E. 3. From	$389\frac{6}{8}$		E. 4. From	$8968\frac{8}{12}$
Take	- - - -	$142\frac{2}{8}$	Take	- - -	$1442\frac{9}{12}$
Remains	- -	$247\frac{4}{8}$	Remains	- -	$7525\frac{11}{12}$
Proof	- - -	$389\frac{6}{8}$	Proof	- - -	$8968\frac{8}{12}$

To work the 4th example, say 9 from 8 I cannot, but 12, the parts the integer is divided into, I borrow to 8, is 20; then 9 from 20, there remains 11, which set down as a numerator to the denominator 12, and carry 1 to the 2, and proceed as in common subtraction, the answer will be $7525\frac{11}{12}$.

E. 5. From $12\frac{3}{4}$ take $9\frac{1}{3}$.

First $12\frac{3}{4} = \frac{51}{4}$, $9\frac{1}{3} = \frac{28}{3}$, then subtract $\frac{28}{3}$ from $\frac{51}{4}$.

51	28	4	∴ From	153	
3	4	3	Take	112	
153 N.	112 N.	12 D.	Remains	41	Ans. $\frac{41}{12} = 3\frac{5}{12}$

		£.	s.	d.		£.	s.	d.
E. 6. From		82	$9\frac{3}{7}$	0	=	82	$9\frac{9}{21}$	0
Take	- - -	60	$14\frac{2}{3}$	$8\frac{5}{6}$	=	60	$14\frac{14}{21}$	$8\frac{5}{6}$
Difference	- - - -					21	$14\frac{15}{21}$	$3\frac{1}{6}$

Note. Such examples as the last, perhaps, may never happen in business; but as such are useful exercises for learners, I thought an example of this sort might be agreeable to my ingenious readers.

XL. MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

PREPARE the given numbers (if they require it) by the rules of reduction; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

Note. Multiplication of fractions decrease the value, in the same proportion as whole numbers increase it, which seems to contradict the definition of multiplication; but this difficulty will vanish, if we consider that the more any integral number is increased, the farther is the figure in the highest place removed from unity; and the more any part of an integer is decreased, the farther will its value also be removed from its relative unit; consequently, as it is the nature of integers to increase, and of fractions to decrease, the purpose of multiplication is equally answered in both cases; the reason of which will more plainly appear by the following examples.

E. 1.

E. 1. Multiply 5*s*. by 5*s*. as the fraction of a pound sterling? First 5*s*.$=\frac{1}{4}$ of a pound; therefore $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$ of a pound, which, by Case 13, sect. XXXVII. will be found equal to 1*s*. 3*d*.

E. 2. Multiply $\frac{4}{5}$ by $\frac{6}{9}$.

$$\begin{array}{cc} 4 & 9 \\ 6 & 5 \\ \hline 24\ \text{N.} & 45\ \text{D.} \end{array}$$

Answer $\frac{24}{45}$.

E. 3. Multiply $\frac{8}{32}$ by $\frac{24}{84}$.
First $\frac{8}{32}=\frac{1}{4}$, and $\frac{24}{84}=\frac{2}{7}$; then the fractions to be multiplied are $\frac{1}{4}$ and $\frac{2}{7}$.

$$\begin{array}{cc} 1 & 4 \\ 2 & 7 \\ \hline 2\ \text{N.} & 28\ \text{D.} \end{array}$$

Answer $\frac{2}{28}=\frac{1}{14}$.

If you have a mixed number or fraction to multiply by a whole number; multiply the whole number by the whole number, and then multiply the numerator by the said whole number, and divide by the denominator, and add this quotient to the former product.

E. 4. Multiply $\frac{3}{4}$ by 8.

First $\frac{3\times8}{4}=\frac{24}{4}$ the product, then

$$\begin{array}{r} 3 \\ 8 \\ \hline 4)24 \\ \hline 6 \end{array}$$

Answer 6 the product

E. 5. Multiply $3\frac{4}{7}$ by 13.

$$\begin{array}{r} 3 \\ 13 \\ \hline 39 \end{array} \qquad \begin{array}{r} 13 \\ 4 \\ \hline 7)52 \\ \hline 7-\frac{3}{7} \end{array}$$

$$\begin{array}{r} 39 \\ +7\frac{3}{7} \\ \hline 46\frac{3}{7} \end{array}$$

$46\frac{3}{7}$ the product

E. 6. Multiply 14*l*, 10*s*. $10\frac{2}{5}$*d*. by 4.

£.	*s*.	*d*.	
14	10	$10\frac{2}{5}$	
		4	
58	3	$5\frac{3}{5}$	Product

To work this example, say, 4 times 2 is $8=\frac{8}{5}=1\frac{3}{5}$, which 1 I carry to the product of pence; and proceed as in common multiplication. The operations in this this rule are so easy, that more examples would be unnecessary.

XLI. DIVISION OF VULGAR FRACTIONS.

RULE.

PREPARE the fractions (as before directed) by the rules of reduction, then multiply the denominator of the divisor by the numerator of the dividend, for a new numerator, and the numerator of the divisor into the denominator of the dividend for a new denominator.

EXAMPLE I. Divide $\frac{5}{8}$ by $\frac{3}{5}$. $\frac{3}{5})\frac{5}{8}(\frac{25}{24}=1\frac{1}{24}$ Answer

To work this example, I multiply 5, the numerator of the dividend, into 5, the denominator of the divisor, the product is 25, the numerator for the quotient; then I multiply 8, the denominator of the dividend, into 3, the numerator of the divisor, the product is 24, the denominator of the quotient. ∴ $\frac{25}{24}$ is the quotient required.

E. 2. Divide $\frac{4}{6}$ by $\frac{2}{3}$.
$\frac{2}{3})\frac{4}{6}(\frac{12}{12} = 1$ Anſwer.

E. 3. Divide $\frac{8}{12}$ by $\frac{4}{9}$.
$\frac{4}{9})\frac{8}{12}(\frac{72}{48}$ Anſwer.

E. 4. Divide $\frac{8}{9}$ of 1*s*. by $\frac{3}{5}$ of 1*l*.
Firſt $\frac{8}{9}$ of a ſhilling $= \frac{8}{180}$ of 1*l*.
Then $\frac{8}{180})\frac{8}{9}(\frac{72}{1440} = 20$ Anſwer.

E. 5. Divide $\frac{3}{4}$ of $\frac{2}{5}$ by $\frac{4}{8}$.
Firſt $\frac{3}{4}$ of $\frac{2}{5} = \frac{6}{20} = \frac{3}{10}$
Then $\frac{4}{8})\frac{3}{10}(\frac{24}{40} = \frac{3}{5}$ Anſwer.

E. 6. Divide $6\frac{2}{3}$ by $2\frac{3}{8}$.
Firſt $6\frac{2}{3} = \frac{20}{3}$, and $2\frac{3}{8} = \frac{19}{8}$
Then $\frac{19}{8})\frac{20}{3}(\frac{160}{57} = 2\frac{46}{57}$ Anſwer

E. 7, Divide 8 by $\frac{3}{5}$.
$\frac{3}{5})\frac{8}{1}(\frac{40}{3}$ Anſwer

E. 8. Divide $\frac{3}{4}$ of 1*l*. by $\frac{2}{3}$ of 1*s*.
Firſt $\frac{2}{3}$ of 1*s*. $= \frac{1}{30}$ of a pound
Then $\frac{1}{30})\frac{3}{4}(\frac{90}{4} = \frac{45}{2} = 22$*l*. 10*s*. Anſ.

E. 9. Divide $\frac{2}{3}$ of 1*s*, by $\frac{3}{4}$ of 1*l*.
Firſt $\frac{2}{3}$ of 1*s*. $= \frac{1}{30}$ of 1*l*.
Then $\frac{3}{4})\frac{1}{30}(\frac{4}{90} = \frac{2}{45} = 10\frac{1}{2}$*d*. $\frac{10}{15}$ A.

1. If it can be done, divide the numerator of the dividend by the numerator of the diviſor, and the denominator by the denominator, for the quotient.

E. 10. Divide $\frac{8}{15}$ by $\frac{2}{3}$. $\frac{2}{3})\frac{8}{15}(\frac{4}{5}$ the quotient.

2. If the two numerators, or the two denominators, can be divided by any number, take the quotients inſtead thereof.

E. 11. Divide $\frac{3}{39}$ by $\frac{6}{13}$. $\frac{3}{39})\frac{6}{13}(=6$
For $6 \div 3 = 2$, and $39 \div 13 = 3$ $\therefore$ $3 \times 2 = 6$ the Anſwer.

3. A fraction is divided by a whole number by multiplying the denominator of the fraction by the whole number.

E. 12. Divide $\frac{12}{16}$ by 8.

$$\begin{array}{r} 16 \\ 8 \\ \hline 128\text{ D.} \end{array}$$

Anſwer $\frac{12}{128}$

4. If the denominators are equal, place the numerator of the dividend over the numerator of the diviſor for the quotient.

E. 13. Divide $\frac{6}{18}$ by $\frac{3}{18}$. The quotient is $\frac{6}{3} = \frac{2}{1} = 2$ Anſwer

Note. If you divide by a proper fraction, the quotient is always a greater number than the dividend; contrary to whole numbers and likewiſe contrary to the ſtrict ſenſe of the word diviſion, which imports the leſſening of a thing. Conſequently, if the diviſor is a proper fraction, multiplication prevails; but diviſion prevails if the diviſor is an improper fraction, as may be ſeen in Ex. 8 and 9.

XLII. RULE OF THREE DIRECT,

IN VULGAR FRACTIONS.

RULE 1.

PREPARE the fractions as before directed, and then ſtate and work your ſum as in whole numbers, only multiplying and dividing fraction wiſe, viz. according to the dirctions in multiplication and diviſion of fractions.

RULE 2. Multiply the denominator of your firſt number into the numerators of the ſecond and third for a new numerator; then multiply the

the numerator of the first number into the denominator of the second and third, for a new denominator, and place it under the new numerator, for the answer, which reduce to its proper quantity.

EXAMPLE 1. If $\frac{2}{3}$ of a pound of hops cost $8\frac{3}{4}d.$ how many pounds may be bought for $25l.$?

First $8\frac{3}{4}d. = \frac{35}{4}$ of $\frac{1}{12} = \frac{35}{48}s.$ and $25l. = \frac{25}{1}$ of $\frac{20}{1} = \frac{500}{1}s.$

s. lb. s.

Then, per rule 1, as $\frac{35}{48} : \frac{2}{3} :: \frac{500}{1}$

$$\begin{array}{rl} 500 & \quad 3 \\ 2 & \quad 1 \\ \hline 1000 \text{ N.} & \quad 3 \text{ D.} \end{array}$$

$\frac{25}{48})\frac{1000}{3}(\frac{48000}{105} = \frac{3200}{7} = 457\frac{1}{7}$ Answer

Or thus, $8\frac{3}{4}d. = \frac{35}{4}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{35}{960} = \frac{7}{192}l.$ and $25l. = \frac{25}{1}$

£. lb. £.

Then, by rule 1, as $\frac{7}{192} : \frac{2}{3} :: \frac{25}{1}$

$$\begin{array}{rl} 25 & \quad 3 \\ 2 & \quad 1 \\ \hline 50 \text{ N.} & \quad 3 \text{ D.} \end{array}$$

$\frac{7}{192})\frac{50}{3}(\frac{9600}{21} = \frac{3200}{7} = 457\frac{1}{7}l.$ = as before.

Again for variety, by rule 2, thus:

$$\begin{array}{rl} 192 & \quad 7 \\ 2 & \quad 3 \\ \hline 384 & \quad 21 \\ 25 & \quad 1 \\ \hline 1920 & \quad 21 \text{ D.} \\ 768 & \\ \hline 9600 \text{ N.} & \end{array}$$

Ans. $\frac{9600}{21} = \frac{3200}{7} = 457\frac{1}{7}lb.$ the same as before.

Note. This is the most expeditious, and easiest method, as appears by the work.

E. 2. If $2\frac{2}{5}$ yards of cloth cost $3\frac{3}{4}l.$ what will $4\frac{4}{5}$ yards cost, at the same rate?

First, $2\frac{2}{5} = \frac{12}{5}$; $3\frac{3}{4} = \frac{15}{4}$; and $4\frac{4}{5} = \frac{24}{5}$.

Then, as $\frac{12}{5} : \frac{15}{4} :: \frac{24}{5}$

$$\begin{array}{rl} 5 & \quad 12 \\ 15 & \quad 4 \\ \hline 75 & \quad 48 \\ 24 & \quad 5 \\ \hline 300 & \quad 240 \text{ D.} \\ 150 & \\ \hline 1800 \text{ N.} & \end{array}$$

Answer $\frac{1800}{240} = \frac{15}{2} = 7l.\ 10s.$

E. 3. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a pound, what will $\frac{3}{5}$ of an English ell cost, at that rate?

First $\frac{1}{4}$ of a yard $= \frac{1}{4}$ of $\frac{4}{5}$ of an ell $= \frac{4}{20}$

Ell. £. *Ell*.

Then, if $\frac{4}{20} : \frac{2}{3} :: \frac{3}{5}$

$$\begin{array}{rl} 20 & \quad 4 \\ 2 & \quad 3 \\ \hline 40 & \quad 12 \\ 3 & \quad 5 \\ \hline 120 \text{ N.} & \quad 60 \text{ D.} \end{array}$$

Ans. $\frac{120}{60} = \frac{2}{1} = 2l.$

E. 4. If $1\frac{1}{2}$ herring cost $1\frac{1}{2}d.$ how many may be bought for $11d$?

First $1\frac{1}{2} = \frac{3}{2}$; $1\frac{1}{2}d. = \frac{3}{2}$, and $11 = \frac{11}{1}$; then

H. *d*. *H*.

If $\frac{2}{3} : \frac{3}{2} :: \frac{11}{1}$

$$\begin{array}{rl} 2 & \quad 3 \\ 3 & \quad 2 \\ \hline 6 & \quad 6 \\ 11 & \quad 1 \\ \hline 66 \text{ N.} & \quad 6 \text{ D.} \end{array}$$

Ans. $\frac{66}{6} = \frac{11}{1} = 11$ herrings

E. 5.

E. 5. Suppose a merchant makes an assurence upon a ship and cargo, bound to the West Indies, value 2700*l.* 10*s.* and agrees to pay 10 guineas per cent. what comes the charges of the assurance to?

First $10\frac{1}{2} = \frac{21}{2}l.$ and $2700\frac{1}{2} = \frac{5401}{2}$; Likewise $100 = \frac{100}{1}$; then

	£.		£.		£.
If	$\frac{100}{1}$	:	$\frac{21}{2}$	::	$\frac{5401}{2}$

$$\begin{array}{r} 1 \\ 21 \\ \hline 21 \\ 5401 \\ \hline 5401 \\ 10802 \\ \hline 113421 \text{ N.} \end{array} \qquad \begin{array}{r} 100 \\ 2 \\ \hline 200 \\ 2 \\ \hline 400 \text{ D.} \end{array}$$

Or thus, by rule 1.

$$\begin{array}{r} 5401 \\ 21 \\ \hline 5401 \\ 10802 \\ \hline 113421 \text{ N.} \end{array} \qquad \begin{array}{r} 2 \\ 2 \\ \hline 4 \end{array}$$

$\frac{100}{1}) \frac{113421}{4} (\frac{113421}{400} = 283l.\ 11s.\ 0\frac{1}{2}d.$ the answer as before.

Ans. $\frac{113421}{400}l. = 283l.\ 11s.\ 0\frac{1}{2}d.$

E. 6. Three workmen can do a piece of work in certain times viz. A can do it in 3 weeks, B can do thrice the work in 8 weeks, and C five times in 12 weeks; in what time can they finish it jointly?

First it may be easily found that A can do $\frac{1}{3}$, B $\frac{3}{8}$, and C $\frac{5}{12}$ of the work in one week, which fractions being reduced to a common denominator, make $\frac{8}{24}$, $\frac{9}{24}$, and $\frac{10}{24}$, whose sum $= \frac{27}{24} = \frac{8}{9}$, being the work they all can do when working together in one week; then:

	Work.		*Days.*		*Work.*
If	$\frac{8}{9}$	:	$\frac{6}{1}$	::	$\frac{1}{1}$

$$\begin{array}{r} 8 \\ 6 \\ \hline 48 \\ 1 \\ \hline 48 \text{ N.} \end{array} \qquad \begin{array}{r} 9 \\ 1 \\ \hline 9 \\ 1 \\ \hline 9 \text{ D.} \end{array}$$

Ans. $\frac{48}{9} = \frac{16}{3} = 5\frac{1}{3}$ Days

XLIII. THE RULE OF THREE INVERSE, IN VULGAR FRACTIONS.

RULE. 1.

PREPARE the fractions as before directed, and then proceed as in Section XIII.

RULE 2. Multiply the denominator of the third number into the numerator of the first and second for a new numerator; then multiply the numerator of the third number into the denominator of the first and second, for a denominator, which place under the numerator for the answer, and find the proper quantity as before directed.

EXAMPLE

EXAMPLE 1. What quantity of shalloon, that is $\frac{3}{4}$ yard wide will line $7\frac{1}{2}$ yards of cloth, that is $1\frac{1}{2}$ yard wide?

First by reduction $1\frac{1}{2}=\frac{3}{2}$, and $7\frac{1}{2}=\frac{15}{2}$; then, if $\frac{3}{2} : \frac{15}{2} :: \frac{3}{4}$

15	2
3	2
45 N.	4 D.

$\frac{3}{4})\frac{45}{4}(\frac{180}{12}=\frac{15}{1}=15$ yards, Ans.

E. 2. If 8 men can do a piece of work in $16\frac{3}{4}$ days, in how many days will 24 men do the same?

As $\frac{8}{1} : 16\frac{3}{4}=\frac{67}{4} :: \frac{24}{1}$

67	4
8	1
536 N.	4 D.

$\frac{24}{1})\frac{536}{4}(\frac{536}{96}=5\frac{53}{96}$ Days, Ans.

E. 3. How many yards of cloth, at 8*s.* 6*d.* per yard, must be given for $26\frac{3}{8}$ yards, at 5*s.* 7*d.* per yard?

First 8*s.* 6*d.* $=\frac{17}{2}$, and $26\frac{5}{8}=\frac{213}{8}$ yards; also 5*s.* 7*d.* $=\frac{67}{12}$; then, As $\frac{67}{12} : \frac{213}{8} :: \frac{17}{2}$

213	12
67	8
1491	96 D.
1278	
14271 N.	

$\frac{17}{2})\frac{14271}{96}(\frac{28542}{1632}=\frac{14271}{816}=17\frac{133}{272}$ yards Ans.

E. 4. Suppose B lends C $100\frac{2}{3}$*l.* for $6\frac{2}{3}$ months, what sum must C lend B, for $3\frac{5}{6}$ years, to requite him?

First $100\frac{2}{3}=\frac{302}{3}$*l.* and $6\frac{2}{3}=\frac{20}{3}$ months; or $\frac{20}{3}$ of $\frac{1}{12}=\frac{20}{36}=\frac{5}{9}$ years; also $3\frac{5}{6}=\frac{23}{6}$ years.

Then as $\frac{5}{9} : \frac{302}{3} :: \frac{23}{6}$

302	23
6	3
1812	69
5	9
9060 N.	621 D.

$\frac{9060}{621}=\frac{3020}{207}=14$*l.* 11*s.* $9\frac{1}{4}$*d.* $\frac{165}{207}$ Answer

E. 5. Suppose 12 men mow down a field of grass in $5\frac{3}{4}$ days, how many men at the same rate of working, will mow down the same in 3 days?

First $12=\frac{12}{1}$, and $5\frac{3}{4}=\frac{23}{4}$; also $3=\frac{3}{1}$; then as $\frac{23}{4} : \frac{12}{1} :: \frac{3}{1}$

23	3
12	4
276	12
1	1
276 N.	12 D.

$\frac{276}{12}=\frac{23}{1}=23$ men, the Ans.

E. 6. How many yards of matting, of $\frac{1}{2}$ yard wide, will be sufficient to cover a floor that is 16 feet wide, and 28 feet long?

First $\frac{1}{2}$ yard $=\frac{9}{2}$ feet, and $16=\frac{16}{1}$; also $28=\frac{28}{1}$; then

As $\frac{16}{1} : \frac{28}{1} :: \frac{9}{2}$

28	9
16	1
448	9
2	1
896 N.	9 D.

Answer $\frac{896}{9}=99\frac{5}{9}$ yards

XLIV. THE DOUBLE RULE OF THREE, IN VULGAR FRACTIONS.

RULE.

PREPARE the fractions by reduction, as before directed, and then proceed as in whole numbers. See section XIV.

EXAMPLE

EXAMPLE 1. If 12 men are hired to do a piece of work in 8 days, at 2*s*. 2*d*. per day, what will be the wages of 9 men for $20\frac{1}{2}$ days?

First 12 men at 2*s*. 2*d*. per day, $= 1l.\ 6s. = 1\frac{6}{20}l. = 1\frac{3}{10}l. = \frac{13}{10}$; and $20\frac{1}{2}$ days $= \frac{41}{2}$; then

$$\begin{array}{cccccc} \frac{12}{1} & : & \frac{13}{10} & :: & \frac{9}{1} \\ \frac{8}{1} & : & 0 & :: & \frac{41}{2} \end{array}$$

Now $\frac{41}{2} \times \frac{13}{10} \times \frac{9}{1} = \frac{4797}{20}$ dividend. And $\frac{12}{1} \times \frac{8}{1} = \frac{96}{1}$ the divisor.

Then $\frac{96}{1})\frac{4797}{20}(\frac{4797}{1920} = 2l.\ 9s.\ 11\frac{1}{2}d.\ \frac{96}{192}$ the Anſ.

Or by neglecting the denominators, to find the numerator, and multiplying them into the other numerators, thus:

$$\begin{array}{rl} 41 & \\ 9 & \\ \hline 369 & \\ 13 & \\ \hline 4797 & \text{N.} \end{array} \qquad \begin{array}{rl} 2 & \\ 10 & \\ \hline 20 & \\ 12 & \\ \hline 240 & \\ 8 & \\ \hline 1920 & \text{D.} \end{array}$$

Anſ. $\frac{4797}{1920} = 2l.\ 9s.\ 11\frac{1}{2}d.\ \frac{96}{192}$ as before.

E. 2. What principal, put to intereſt, will gain 40*l*. in 8 months, at 5 per cent. per annum? Firſt 8 months $= \frac{2}{3}$ of a year.

$$\text{Then as } \begin{array}{cccccc} \frac{5}{1} & : & \frac{100}{1} & :: & \frac{2}{3} \\ \frac{1}{1} & : & 0 & :: & \frac{40}{1} \end{array}$$

Now $\frac{100}{1} \times \frac{40}{1} \times \frac{2}{3} = \frac{1200}{2}$ the dividend. And $\frac{5}{1} \times \frac{1}{1} = \frac{5}{1}$ the diviſor.

$\frac{5}{1}(\frac{12000}{2}(\frac{12000}{10} = 1200l.$ the Anſ.

Again by two ſtatings,

First as $\frac{5}{1} : \frac{100}{1} :: \frac{40}{1}$

Then $\frac{100}{1} \times \frac{40}{1} = \frac{4000}{1}$

And $\frac{5}{1})\frac{4000}{1}(\frac{4000}{5} = 800l.$

Again as $\frac{1}{1} : \frac{800}{1} :: \frac{2}{3}$

Then $\frac{800}{1} \times \frac{1}{1} = \frac{800}{1}$

And $\frac{2}{3})\frac{800}{1}(\frac{2400}{2} = 1200l.$ the Anſwer, as before.

Note. This laſt ſtating is in inverſe proportion.

E. 3. Six men with their wives, upon calculation, found that their expences for three months paſt amounted to 26*l*. 19*s*. 4*d*. I demand what time 14*l*. 15*s*. may be ſpent by 36 men in the like proportion?

Firſt $26l.\ 9s.\ 4d. = 26\frac{29}{30} = \frac{809}{30}l.$ and $14l.\ 15s. = 14\frac{3}{4} = \frac{59}{4}l.$

$$\begin{array}{cccccc} \frac{12}{1} & : & \frac{3}{1} & :: & \frac{809}{30} \\ \frac{30}{1} & : & 0 & :: & \frac{59}{4} \end{array}$$

Now $\frac{809}{30} \times \frac{36}{1} = \frac{4854}{5}$ the diviſor

And $\frac{59}{4} \times \frac{12}{1} \times \frac{3}{1} = \frac{531}{1}$ the dividend

Then $\frac{4854}{5})\frac{531}{1}(\frac{2655}{4854} = 16\frac{331}{809}$ Days, the anſwer.

Note. The numerator is multiplied by 30, the days in a month, and then valued as taught in reduction.

QUESTIONS

Questions for Exercise in Vulgar Fractions.

Quest. 1. Four figures of 6 may be so placed and disposed of, as to denote and read for 67, neither more nor less; pray how is that to be done?

First $\frac{6}{6}=1$, then 66

$$\begin{array}{r} 1 \\ \hline 67 \end{array}$$

Answer 67

Quest. 2. A lad having got 4000 nuts, in his return home was met by Mad Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole stock. Raving Ned lights on him afterwards, and forced $\frac{2}{5}$ of $\frac{5}{8}$ of the remainder from him; unluckly, Positive Jack found him, and required $\frac{7}{10}$ of $\frac{17}{20}$ of what he had left. Smiling Dolly was by promise to have $\frac{3}{4}$ of a quarter of what nuts he brought home; how many then had the boy left?

First his whole stock = 4000
Then $\frac{5}{8}$ of $\frac{2}{3}$ of 4000 = $1666\frac{2}{3}$ Mad Tom took

$2333\frac{1}{3} = \frac{7000}{3}$ left
$\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{7000}{3}$ = $583\frac{1}{3}$ Raving Ned took

1750 Left
$\frac{7}{10}$ of $\frac{17}{20}$ of 1750 = $1041\frac{1}{4}$ Positive Jack took

$708\frac{3}{4} = 708\frac{48}{64}$ Left
$\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{2835}{4}$ = $132\frac{57}{64}$ Dolly had

Answer - $575\frac{55}{64}$ Left.

Quest. 3. If the scavenger's rate at $1\frac{1}{2}d.$ in the pound, comes to $6s.$ $7\frac{1}{2}d.$ where they ordinarily assess $\frac{4}{5}$ of the rent; what will the king's tax for that house be, at $4s.$ in the pound, rated at the full rent?

First $1\frac{1}{2}d. = \frac{1}{160}$, $6s.\ 7\frac{1}{2}d. = \frac{159}{480} = \frac{53}{160}l.$ and $4s. = \frac{1}{5}l.$

Then as $\frac{1}{160} : \frac{1}{1} :: \frac{53}{160}$

$\frac{1}{160})\frac{53}{160}(\frac{8480}{160} = 53l. = \frac{4}{5}$ of the rent

$53 \div 4 =$ $13\frac{1}{4} = \frac{1}{5}$ ditto

£. $66\frac{1}{4}$ = the whole rent

Again, as $\frac{1}{1} : \frac{5}{1} :: 66\frac{1}{4} = \frac{265}{4}$

$$\begin{array}{r} 265 \\ 1 \\ \hline 265\ \text{N.} \end{array} \qquad \begin{array}{r} 5 \\ 4 \\ \hline 20\ \text{D.} \end{array}$$

Answer $\frac{265}{20} = 13l.\ 5s.$

Quest. 4. X, Y, and Z, can, working together, complete a stair-case in 12 days; Z is man enough to do it alone in 24 days, and X in 34; in what time then could Y get it done himself?

First $\frac{1}{34} = \frac{12}{408}$ X, $\frac{1}{24} = \frac{17}{408}$ Z;

Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$, the work performed in one day by X and Z;

 and

and $\frac{1}{12}=\frac{34}{408}$ performed in one day by all three working together;

Therefore $\frac{34}{408}-\frac{29}{408}=\frac{5}{408}$ performed in one day by Y.

Work. Day. Work.

Then as $\frac{6}{408}$: 1 :: 1 $\frac{5}{408})\frac{1}{1}(\frac{408}{5}=81\frac{3}{5}$ the Answer

Quest. 5. Miss Kitty told her brother George, that though her fortune on her marriage took 19312*l.* out of the family, it was but $\frac{3}{5}$ of 2 years rent; heaven be praised for this yearly income! pray what was it?

As $\frac{3}{5}$: $\frac{19312}{1}$ :: $\frac{5}{5}$

$\frac{3}{5})\frac{19312}{1}(\frac{96560}{3}=32168$*l.* 13*s.* 4*d.* Two years rent, which $\div 2 =$ 16093*l.* 6*s.* 8*d.* the yearly income required.

Quest. 6. A politician having about him a certain number of crowns, said, if $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}$ of what he had, were added together, they would make just Wilkes's number (45); how many crowns had he about him?

First $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}=\frac{18}{72}+\frac{24}{72}+\frac{12}{72}=\frac{54}{72}=\frac{24}{36}$

Then as $\frac{27}{36}$: $\frac{45}{1}$ :: $\frac{36}{36}$

```
 36        36
 45         1
---        --
180        36 D.
144
----
1620 N.
```

$\frac{27}{36})\frac{1620}{36}(\frac{58320}{972}=\frac{6}{1}=60$ Crowns, the answer.

Practical Arithmetic.

PART III.

XLV. DECIMAL FRACTIONS.

DECIMALS are different from whole numbers; for whole numbers increase from the right-hand towards the left in a ten-fold proportion from unity or one; and decimals decrease from unity in the same proportion from the left-hand towards the right; the following table makes this evident.

Units	1,	Unit or integer
Primes	,1	One tenth part of the integer
Seconds	,01	One hundredth part
Thirds	,001	One thousandth part
Fourths	,0001	One ten thousandth part
Fifths	,00001	One hundred thousandth part
Sixths	,000001	One millionth part
Sevenths	,0000001	One ten millionth part
Eighths	,00000001	One hundred millionth part
Ninths	,000000001	One thousand millionth part

So

So that decimal fractions are of ſeveral denominations or names, as primes, ſeconds, thirds, &c. and becauſe the denominator is always 1, with as many cyphers annexed as there are dicimal places; for this reaſon the numerator or decimal is always wrote alone, without the denominator, ſo if I would expreſs the twenty-five hundredth parts of any thing, which vulgarly ſtands thus $\frac{25}{100}$, becauſe the denominator is 1, with as many cyphers prefixed as there are decimals or places in the numerator, it is always expreſſed thus ,25; and $\frac{123}{1000}$ thus ,123; and $\frac{6848}{10000}$ thus ,6848, &c. And becauſe vulgar fractions are the foundation of decimals, I ſhall ſhew (in its proper place) the manner of reducing them to decimals, by which means all thoſe computations hitherto deemed ſo intricate, may be performed with the utmoſt eaſe and pleaſure.

A finite decimal is that which ends at a certain number of places; but an infinite, is that which no where ends.

A circulating or recurring decimal is that wherein one or more figures are continually repeated.

Thus 84,56666, &c. is called a ſingle circulate or recurring decimal.

And 147,642642, &c. is called a compound recurring decimal.

In all operations, if the reſult conſiſts of ſeveral nines, reject them, and make the next ſuperior place a unit more. Thus for 12,2999 write 12,3; and for 32,99 write 33, &c.

XLVI. ADDITION OF DECIMALS.

RULE.

PLACE primes under primes, ſeconds under ſeconds, &c. whether they be cyphers or ſignificant figures; when the work is done, make a point or dot with your pen between the whole numbers (if there be any) and decimals; this is known by cutting off ſo many places to the right hand as your greateſt decimal fraction contains.

EXAMPLES.

21,42	3,121	,31214	61,2182
1,0	2,14	,0214	4,1041
23,4	34,11	,36212	,342
561,21	410,2	,514	6,13
3,424	34,13	,231	78,41
1,212	4,521	,41642	3,4265
611,666	488,222	1,85708	153,6308

£.	s.	d.		£.
59	7	$7\frac{3}{4}$	=	59,3822916
57	17	5	=	57,8708333
57	13	4	=	57,6666666
25	6	8	=	25,3333333
45	13	4	=	45,6666666
245	18	$4\frac{3}{4}$	=	245,9197916
				20
	Shillings			18,3958320
				12
	Pence			4,7499840
				4
	Farthings			2,9999360

Note. When all or any of the decimals repeat a single digit, make the repetends conterminous, and add 1 to the sum of the first, or right-hand column, for every nine that is contained in it.

Agreeing with the above nearly.

XLVII. SUBTRACTION OF DECIMALS.

RULE.

PLACE the greater number uppermost, the points under points, tenths under tenths, &c. then subtract as in whole numbers, placing the points of separation under the other points.

EXAMPLES.

From	,864213	36,1214
Take	,128191	,81642
Remains	,736022	35,30498
Proof	,864213	36,1214

In subtracting integers and decimals, observe the following order:

	£.		£.	s.	d.	q.
Lent	1730,027	=	1730	0	6	1,92
Received	1681,8352	=	1681	16	8	1,792
Remains	48,1918	=	48	3	10	0,128
	20					
Shillings	3,8360					
	12					
Pence	10,0320					
	4					
Farthings	128					

Agreeing exactly with that on the right hand; for the decimal ,1918 of a pound is equal to 3*s.* 10*d.* 0*q.* ,128.

If a single digit is repeated, borrow 9 in the first repeating place when necessary.

From

	£.	s.	d.		£.
From	7849	6	8	=	7849,333
Take	6979	13	4	=	6979,666
Remains	869	13	4	=	869,666

XLVIII. MULTIPLICATION OF DECIMALS.

RULE.

MULTIPLY the decimals, as if they were whole numbers, and from the product cut off as many decimal places, as there are in both numbers. If there be not so many places, make them out with cyphers on the left-hand of the product.

EXAMPLES.

,3042	,3042	,3042
,2015	20,15	2015,
15210	15210	15210
3042	3042	3042
6084	6084	6084
,06129630	6,129630	612,9630

Note. I have made use of the same figures throughout each of these examples; yet the reader will find the values of the products are very different.

CONTRACTIONS.

It frequently happens in business, that one ot both the factors consist of many decimal places; so that to work them all would be very troublesome, and when done, but little to the purpose, because a less number of places may do the business as well; therefore use the following

RULE. 1. Transpose all the figures of the multiplier in a contrary order to the common way, viz. let the units place stand to the left-hand.

2. The units place of the multiplier must stand under that place of the multiplicand whose decimal place you intend to retain the product.

3. Begin as in common multiplication, always having regard to the increase of that figure on the right-hand, the figure that stands over your multiplier; making use of no more places of your multiplier than those which stand even with your multiplicand to the left-hand.

E. 1. Let it be required to multiply 3,14159 by 24,8253, and to retain 4 decimal places in the product.

3,14159

3,14159 Multiplicand	The operation at length: 3,14159
3528,42 Multiplier inverted	24,8253
628318	9 | 42477
125663	157 | 0795
25132	628 | 368
628	25132 | 72
157	125663 | 6
9	628318 |
77,9907 Product	77,9909 | 19227

Note. As the allowance for what may be carried from the columns neglected is altogether a guess, we may very often make the product less than it ought to be by 1 or 2, as appears by the above example; to avoid which, make one or two columns more than the number of decimal places you would have in the product, and cut them off at pleasure

E. 2. Multiply 75,4678 by 6,05408, so as to retain only three places of decimals in the product?

75,4678	
80450,6	
452806	
3773	
301	
456,880	

If the multiplier is a decimal fraction, put a cypher in the units place, and set the other figures, in order from that on the left-hand.

E. 3. Multiply ,68479 by ,0785 to have 5 decimal places in the product.

,68479
5870,0
4793
547
34
,05374

From these examples it is manifest how advantageous these contractions are to shorten the work of long calculations and computations, which the experienced practitioner finds too often occur, in arithmetic, algebra, and geometey.

To multiply by 10, 100, 1000, &c. remove the decimal point so many steps further to the right-hand, as there are cyphers in the multiplier. As 86,564 × 100 = 8656,4; and 45 × 1000 = 450, &c.

XLIX. DIVISION of DECIMALS.

RULE.

DIVIDE as if they were whole numbers; then cut off as many decimal places in the quotient, as the number of decimal places in the dividend exceeds the number in the divisor; if there are not so many in the divisor, prefix so many cyphers.

In dividing a whole number by a whole number, if any thing remains, annex cyphers to the remainder, and continue the division as far

ſar as you pleaſe; ſo you will have a decimal in the quotient of as many places as you annexed cyphers, and the whole quotient thus found will be a mixed number.

There are nine caſes, which take in the following order, by which the learner will eaſily acquire a true notion of the ground and nature of decimals.

CASE 1. A whole number given to be divided by a whole number.

579268,)314159265,00000(542,33837,
2896340
2452526
2317072
1354545
1158536
1960090
1737804
2222860
1737804
4850560
4634144
2164160
1737804
4263560
4054876
Remains 208684

In this example here are five cyphers added to the dividend, which produce five decimal places in the quotient.

CASE 2. A whole number given to be divided by a mixed number.

579,268)314159265,0000(542338,3
2896340
2452526
2317072
1354545
1158536
1960090
1737804
2222860
1737804
4850560
4634144
2164160
1737804
Remains 426356

In the laſt example three cyphers are added to the right-hand of the whole number in the dividend, which makes the quotient a whole number; and becauſe there is a remainder, you may go on again, by adding cyphers at pleaſure; ſo the quotient will be a mixed number.

CASE 3. A mixed number given, to be divided by a whole number.

579268,) 3,14159265 (,00000542
2896340
2452526
2317072
1354545
1158536
Remains 196009

In this example here are five cyphers prefixed to the quotient, that they might be equal to the decimal places of the dividend.

CASE 4. A mixed number given to be divided by a mixed number.

57,9268)31,4159265(,542
2896340
2452526
2317072
1354545
1158536
Remains 196009

In this example the decimal places in the dividend exceed thoſe in the diviſor by three, therefore the quotient is a decimal.

CASE

CASE 5. A whole number given to be divided by a decimal fraction.

,579268)314159265,000000(542338373
2896340
2452526
2317072
1354545
1158536
1960090
1737804
2222860
1737804
4850560
4634144
2164160
1737804
4263560
4054876
2086840
1737804
Remains 349036

In this example here are six cyphers annexed to the dividend to answer the decimal places of the divisor, that the quotient might be a whole number.

CASE 6. A mixed number given, to be divided by a decimal fraction.

,579268)3,14159265(5,42
2896340
2452526
2317072
1354545
1158536
Remainder 196009

CASE 7. A decimal fraction given, to be divided by a whole number.

579268),314159265(,000000542
2896340
2452526
2317072
1354545
1158536
Remainder 196009

CASE 8. A decimal fraction given, to be divided by a mixed number.

5,79268),314159265(,0542
2896340
2452526
2317072
1354545
1158536
Remainder 196009

CASE 9. A decimal fraction given, to be divided by a decimal fraction.

,579268),314159265(,542
2896340
2452526
2317072
1354545
1158536
Remainder 196009

If any whole, mixed, or decimal number, is given to be divided by 10, 100, 1000, &c. you only remove the separating ppint towards the left-hand so many places as there are cyphers in the divisor, contrary to what was taught in multiplication.

Thus, 1523 ÷ 10 = 152,3; and 1523 ÷ 1000 = 1,523, &c.

To work any case of division by multiplication, and on the contrary, any case of multiplication by division; and this in many instances will be found very useful.

RULE

RULE. Divide a unit with cyphers annexed by the given multiplier, and the quotient is the divisor sought.

EXAMPLE. Suppose I have 7315 to multiply by any other number, as 125; but have a desire to divide the said number, and to have a quotient equal to the product of those two numbers; query, the divisor.

Given 125)1,000(,008 the divisor sought.
1000
0

Then 7315
× 125
36575
14630
7315
Product 914375

And ,008)7315,000(914375 Quotient, equal to the product.
72
11
8
35
32
30
24
60
56
40
40

Suppose I have 7315 given, to be divided by any other number ,008; but would multiply the said number, and have a product equal to the quotient of the same number divided by ,008; query the multiplier.

RULE. Divide an unit with cyphers annexed by the given divisor, and the quotient will be the multiplier sought.

Thus ,008)1,000(125

The remainder of the work is only the reverse of the former, and therefore need not be repeated.

From the foregoing examples relating to division it may be observed, that the first figure of every quotient must possess the same place (with respect to its value) as that figure of the dividend doth, which stands over the units place of the first figure's product, which is an excellent rule to value quotients, obtained by the following,

CONTRACTION. When the divisor consists of many places of decimal parts, the work may be much abbreviated by the following,

RULE. Consider in what place the first figure of the quotient ought to stand, and find its value or denomination; taking as many of the left-hand figures as you intend to have figures in the quotient for the first divisor; then take as many figures of the dividend as will answer them. In dividing, omit, or point off one figure at each operation; at the same time, have a due regard to the increase, which would arise from the figure or figures so ommited.

EXAMPLE 1

76,84375)630,92878(8,210541
······ 61475000

1617878
1536875

81003
76843

4160
3842

318
307

11
7

4

In this example, 8 is multiplied into 76,84375; then 2 is multiplied into 76,8437, carrying 1 from the last figure pointed off, and so you must proceed with the remainder of the figures in the divisor until they are all pointed off.

If the dividend contains many places of decimals, there is no occasion for using but a few of the first.

E. 2. 57,92|68)3,1415|9265(,0542
28963

2452
2317

135
115

20

The common method.

57,9268)3,1415|9265(,0542
28963|40

2452|526
2317|072

135|4545
115|8536

19|6009

Note. Though much labour may be saved by this method yet it is only useful when the decimals in the dividend contain many places, and then take all the divisor.

E. 3. 24,324)842,31415216342(,00034629

Note. As these contractions, and those taught in multiplication, answer the same end in almost all operations as the method of circulating or recurring decimals; therefore, to have treated on them, would be swelling this treatise for no purpose but curiosity only.

L. REDUCTION of DECIMALS.

CASE 1.

TO reduce a vulgar fraction to a decimal,

RULE. Add cyphers to the numerator, representing so many places of decimals, and divide by the denominator; the quotient will be the decimal fraction required.

EXAMPLE 1. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, to decimals.

4)1,00 — ,25   2)1,0 — ,5   4)3,00 — ,75

Answer ,25$=\frac{1}{4}$; ,5$=\frac{1}{2}$, and ,75$=\frac{3}{4}$

E. 2.

E. 2. Reduce $\frac{1}{3}$ to a decimal.

$$\begin{array}{r} 3)\underline{1,0000} \\ ,3333 \end{array}$$ &c. ad infinitum

E. 3. Reduce $\frac{5}{16}$ to a decimal.

$$16\left\{\begin{array}{l} 2)5,0 \\ \hline 8)2,5 \end{array}\right.$$

Anfwer ,3125

Or thus, 16)5,0000(,3125

$$\begin{array}{r} 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline 0 \end{array}$$

E. 4. Reduce $13\frac{4}{7}$ to a decimal, or mixed number.

First, $13\frac{4}{7}=\frac{95}{7}$; then

$$\begin{array}{r} 7)\underline{95} \\ \underline{13,571428} \end{array}$$

Anfwer $13\frac{4}{7}=13,571428$

E. 5. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5}$ to a decimal. Firft, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5}=\frac{6}{40}$ then 40)6,00(,15 Anfwer

$$\begin{array}{r} 40 \\ \hline 200 \\ 200 \\ \hline 0 \end{array}$$

E. 6. Reduce $\frac{164}{395}$ to a decimal.

395)164,000(,415 Anf.

$$\begin{array}{r} 1580 \\ \hline 600 \\ 395 \\ \hline 2050 \\ 1975 \\ \hline 75 \end{array}$$

Note. If the decimals will not terminate, but there will ftill be a remainder, it will be exact enough in moft cafes, and the remainder may be rejected after the decimal has been carried on to 4 or 5 places.

CASE 2. To reduce coins, weights, meafures, &c. into decimals,

RULE 1. Reduce the given money, weights, &c. into the loweft denomination or name mentioned, for a dividend; then reduce the integer into the fame denomination for a divifor, the quotient will be the decimal required.

RULE 2. Place the numbers of the feveral denominations under each other, begining with the leaft, and divide each by fuch a number that will raife it to the next fuperior name, placing each quotient as a decimal part of the next dividend before it be divided, and the final quotient will be the anfwer.

EXAMPLE

EXAMPLE 1. Reduce 18*s.* 6¾*d.* to the decimal of a pound sterling,

```
                 s.   d.
By rule 1, thus 18   6¾
                12
               ---
               222
                 4
£.   qrs.     ---
1 = 960)891,000000(,928125 The decimal required,
        864
        ---
         270
         192
         ---
          780
          768
          ---
           120
            96
           ---
            240
            192
            ---
             480
             480
             ---
               0
```

By rule 2 thus:

```
 4 | 3,00
12 | 6,75
2|0| 18,5625
   ---------
     ,928125 Decimal as above
```

Note. By rule 2, the three farthings are reduced to the decimal of a penny (which = ,75) and set on the right of 6*d.* then 6,75 pence to the decimal of a shilling (=,5625) then 18,5625 shillings to the decimal of a pound,

E. 2. Reduce 15*s.* 9*d.* to the decimal of a pound.

```
           s.  d.
           15  9
           12
£.   d.   ---
1 = 24|0)189,0000(,7875 the
                (decimal required
```

By rule 2, thus:

```
 12 | 9,00
2|0 | 15,75
    ------
Answer  ,7875 same as before
```

E. 3. Reduce ¾ of a penny to the decimal of a pound.

First $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{3}{960} = \frac{1}{320}$*l.* then,

```
32|0)1,000000(,003125 the deci-
                    (mal required
```

By rule 2 thus:

```
 4 | 3,00
12 | ,75
2|0| ,0625
Answer ,003125 as before
```

E. 4. Reduce 11*dwts.* to the decimal of a pound troy.

First 1*lb.* = 240*dwts.* then,

```
     dwts.
24|0)11,0000(,4583 the decimal
                 (required
```

E. 5. Reduce 10 drams to the decimal of a pound avoirdupoise.

First 1*lb.* = 256*drs.* then,

```
256)10,00000(,03906 the decimal
                  (required
```

E. 6.

E. 6. Reduce 9 inches to the decimal of a yard.
First 1 yard = 36 inches; then, 36)9,00(,25 the decimal required.

E. 7. Reduce $3\frac{1}{4}$ inches to the decimal of a foot.

$3\frac{1}{4}$
4

Foot. *qrs*
1 = 48) 13,0000(,2708$\frac{16}{48}$ the decimal required.

E. 8. Reduce 6 furlongs to the decimal of a league.
First 1 league is 3 miles = 24 furl. then 24)6,00(,25 the decimal req.

E. 9. Reduce 12 gallons 2 quarts of wine, to the decimal of a hogshead.

12 2
4

Hhd. *qts.*
1 = 252)50,00000(,1984 the decimal required.

E. 10. Reduce 3 quarts 1 pint of ale, to the decimal of a barrel.

3 1
2

pts.
1 Barrel = 256)7,00000(,02734$\frac{96}{256}$ Answer.

E. 11. Reduce 4 inches to the decimal of a foot.

12)4,000

,333 &c.

E. 12. Reduce 36 poles to the decimal of an acre.
First 1 acre = 160 poles; then 16|0)36,000(,225 the answer.

E. 13. Reduce 4 bushels 2 pecks to the decimal of a chaldron.
First 4 bushels 2 pecks = 18 pecks, and a chaldron. = 144 pecks;
Then 144)18,000(,125 the decimal required.

E. 14. Reduce 12 minutes to the decimal of an hour.

Min.
1 Hour = 60)12,0(,2 the decimal required.

E. 15. Reduce 2 qrs. 25 pounds, to the decimal of a hundred.

2 25
28

Cwt. *lb.*
1 = 112)81,0000(,7232$\frac{16}{112}$ Answer.

E. 16. Reduce 12 days to the decimal of a Julian year.
First 12 days = 288 hours, and 365 days 6 hours = 8766 hours;
Then 8766)288,00(,0328$\frac{4752}{8766}$ the decimal required.

E. 17. Reduce 440 yards to the decimal of a mile.
First 1 mile = 1760 yards; then 1760)440,00(,25 Answer.

In

In order to the application of decimals, we ought to have ready calculated the decimal of any integer of money, weight, meaſure, &c. anſwering to every number, ſimple or mixed, of inferior denomination, and of leſs value than that integer; which decimals being orderly collected and diſpoſed, make what we call decimal tables, by which any decimal required may be readily found, or alſo the value of any given decimal in known inferior ſpecies.

To find every decimal by a ſeparate application of the foregoing rule, would be a very tedious work (though it is a general and complete one)

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

TABLE I.
COIN
1 £. Ster. the Integer.

Sh.	De.	Sh.	Dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence	Decimals
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farth.	Decimals
3	,003125
2	,002083
1	,001042

TABLE II.
Troy Weight.
1*lb.* the Integer.

This Table will ſerve for Inches, Months, or Dozens.

Ounces	Decimals
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Pen. wt.	Decimals
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains	Decimals
20	,003472
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173
½	,000086

TABLE III.
Avoirdupoiſe.
112*lb.* the Integer.

Qrs.	Decimals
1	,25
2	,5
3	,75

Pds.	Decimals
20	,178571
14	,125
13	,116071
12	,107143
11	,098214
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ozs.	Decimals
10	,00558
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001674
2	,001116
1	,000558

Drs.	Decimals
10	,000348
9	,000313
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034
½	,000027

TABLE IV.
Avoirdupoiſe Weight
1*lb.* the Integer.

Ounces	Decimals
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams	Decimals
8	,03125
7	,027343
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

TABLE V.
Liquid Meaſure.
1 Tun the Integer.

Gallons	Decimals
10	,039682
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

Pints	Decimals
4	,001984
3	,001488
2	,000992
1	,000496

A Hogshead the Int.

Gallons	Decimals
30	,47619
20	,31746
10	,15873
9	,142857
8	,126984
7	,111111
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints	Decimals
3	,005952
2	,003968
1	,001984

TABLE VI.

Measure

LIQUID. DRY.

1 Gal. 1 Quar. Int

Pints	Decimals	Bu.
4	,5	4
3	,375	3
2	,25	2
1	,125	1

Q. Pt.	Decimal	Pec.
3	,09375	3
2	,0625	2
1	,03125	1

Decimals	Qr. Pk.
,023437	3
,015615	2
,007812	1

Decimals	Pints
,005859	3
,003906	2
,001953	1

TABLE VII.

Long Measure.

1 Mile the Integer.

Yards	Decimals
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170545
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,054091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001139
1	,000568

Feet	Decimals
2	,0003787
1	,0001894

Inches	Decimals
6	,0000947
3	,0000474
2	,0000315
1	,0000158

TABLE VIII.

TIME.

1 Year the Integer.

Days	Decimals
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013699
4	,010959
3	,008219
2	,005479
1	,002739

1 Day the Integer

Hours	Decimals
20	,833333
10	,416666
9	,375
8	,333333
7	,291666
6	,25
5	,208333
4	,166666
3	,125
2	,083333
1	,041660

Min	Decimals
50	,034722
40	,027777
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

TABLE IX.

Cloth Measure.

1 Yard the Integer.

Qrs.	Decimals
3	,75
2	,5
1	,25

Nails	Decimals
3	,1875
2	,125
1	,0625

TABLE X.

Lead Weight.

1 Fother the Integer

Hund.	Decimals
10	,51282
9	,461538
8	,410256
7	,358974
6	,307692
5	,25041
4	,205128
3	,153846
2	,102564
1	,051282

Qrs.	Decimals
2	,025641
1	,01282

Pounds	Decimals
14	,0064102
13	,0059523
12	,0054945
11	,0050366
10	,0045789
9	,0041208
8	,003663
7	,0032051
6	,0027472
5	,0022893
4	,0018315
3	,0013736
2	,0009157
1	,0004578

TABLE XI.

Of Motion.

A Sign of the Zodiac the Integer.

Do	Decimals
1	,033333
2	,066666
3	,1
4	,133333
5	,166666
6	,2
7	,233333
8	,266666
9	,3
10	,333333
11	,366666
12	,4
13	,433333
14	,466666
15	,5
16	,533333
17	,566666
18	,6
19	,633333
20	,666666
21	,7
22	,733333
23	,766666

Note. The uſe of the preceding tables is ſo obvious and natural, even by a bare inſpection, that I preſume it is needleſs to ſay any thing about that; the following examples being ſufficient to teſtify the great uſe and excellency of ſuch tables, and will at the ſame time give the learner a clear knowledge of the uſe of them.

EXAMPLE 1. What is the decimal part of a pound for 15*s.* 9*d*?

In table I. you find againſt	15 Shillings - -	,75
	9 Pence : - -	,0375
	The anſwer is -	,7875

E. 2. What decimal part of a pound is 18*s.* 6¾*d*?

In table I you find againſt	18 Shillings -	,9
	6 Pence - - .	,025
	3 Farthings -	,003125
	The anſwer is - - -	,928125

E. 3. What decimal part of a pound troy is 7 ounces?
In table II. you find againſt 7 ounces - ,583333 Anſwer

E. 4. What decimal part of an hundred weight is 12 pounds 4 oz?

In table III. you find againſt	12 Pounds - - -	,107143
	4 Ounces - - -	,002232
	Anſwer - - -	,109375

E. 5. What decimal part of a mile is 300 yards 2 feet?

In table VII. you find againſt	300 Yards - -	,170454
	2 Feet - -	,0003787
	The anſwer - -	,1708327

By the preceding tables all the ſpecies of money, weight, meaſure, &c. contained therein, by the above method are immediately turned into decimals, and are then worked with the ſame pleaſure and facility as whole numbers.

CASE 3. To find the value of any decimal fraction, in money, weight, meaſure, &c.

RULE. Multiply the given decimal by the parts of the next inferior denomination, and cut off towards the right-hand of the product ſo many figures as there are places in the given decimal, and thoſe on the left will be integers; then multiply the remaining decimals by the next inferior denomination, and cut off for decimals as before; thus proceed till you have brought it into the loweſt parts of the integer. A few examples will make this plain to the young practitioner.

EXAMPLE 1. What is the value of ,725 of a pound ſterling?

	,725
	20
Shillings	14,500
	12
Pence -	6,0 Anſwer 14*s.* 6*d.*

Note

Note. As often as cyphers fall on the right-hand of your work, always drop them, for they are of no value.

E. 2. What is the value of ,72083 of a crown?

	5	
Shillings	3,60416	
	12	
Pence -	7,24999	Answer 3s $7\frac{1}{4}d$.
	4	
Farthings	,99999	

Note. If the multiplicand be a compound repetend, and the multiplier only a single digit, to the product of the first figure on the right-hand, add as many units as there are tens in the product of the left-hand place of the repetend.

Thus in the above example, ,72083 being a repetend as above described, I multiply by 5, the shillings in a crown, saying 5 times 3 is 15, there being only one ten in that product, I set down 6, which is one more, and then proceed as in common multiplication with the remainder of the multiplicand. Again ,60416 I multiply by 12, the pence in a shilling, saying 12 times 6 is 72, there being seven tens in that product, I add 7 to the 2 remaining, which makes 9, which I set down, and proceed as before with the remainder of the multiplicand, continuing thus till the work is finished.

E. 3. What is the value of ,36 of a shilling?

,36
12
4,32
4
1,28 Answer $4\frac{1}{4}d$.

E. 5. What is the value of ,775 of an ounce troy?

,775
20
dwts. 15,5
24
grs. 12,0 Ans. 15*dwts* 12*grs.*

E. 7. What is the value of ,175 of a hundred weight?

,175
4
,700
28
lb. 19,6 Answer 19*lb.* 9*oz.*

E. 4. What is the value of ,9 of a guinea?

,9
21
18,9
12
10,8
4
3,2 Answer 18*s.* $10\frac{3}{4}d$.

E. 6. What is the value of ,3375 of a ton?

,3375
20
Cwt. 6,7500
4
qrs. 3,00 Ans. 6 *Cwt.* 3*qrs.*

E. 8. What is the value of ,8375 of an acre?

,8375
4
Roods 3,3500
40
Perches 14,0 Ans. 3*r.* 14*per.*

E. 9.

E. 9. What is the value of ,933593 of a barrel of ale London measure?

,933593
32

1867186
2800779

Gall. 29,874976
4

Quarts 3,499904
2

Pints ,999808

Anf. 29*galls.* 3*qts.* 1*pt.* nearly

E. 10. What is the value of ,342 of a day?

,342
24

1368
684

Hours 8,208
60

Minutes 12,48
60

Seconds 28,8
60

Thirds 48,0

	Hours.	*min.*	*sec.*	*thirds.*
Anfwer	8	12	28	48

E. 11. What is the value of ,241 of a chaldron of coals?

,241
36

1446
723

Bufhels 8,676
4

Pecks 2,704

Anfwer 8 *bufhels*, 2 *pecks*.

E. 12. What is the value of ,53373 of a year?

,53373
13

160119
53373

Months 6,93849
4

Weeks 3,75396
7

Days 5,27772
24

111088
55544

Hours 6,66528
60

Minutes 39,9168
60

Seconds 55,008

	Mo.	*w.*	*d.*	*h.*	*m.*	*sec*
Anfwer	6	3	5	6	39	55+

Thefe examples I think fufficient to fhew the method of reducing decimals into the known parts of any fpecies of quantity.

LI. EXTRACTION OF THE SQUARE ROOT.

EXTRACTION of the fquare root, is finding fuch a number that being multiplied by itfelf fhall give the refpective power, out of which the root is to be extracted; as if 36 be propofed to be extracted, you will find its root to be 6, for $6\times6=36$, the given number.

TABLE.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

To

To extract the square root of any number, observe the following

RULE. 1. Begin at the units place, and point the given number into periods of two figures each.

2. Find the greatest square that is contained in the first period, towards the left-hand; set the root in the quotient, and subtract the square from the figures of that period.

3. To the remainder bring down the two figures under the next point for a dividend.

4. Double the quotient or root, and place it for a divisor; seek how often the divisor is contained in the dividend (reserving the units place) and put the answer in the quotient, and also on the right-hand of the divisor; then multiply the divisor by the last figure put in the quotient (as in common division) the product subtract from the dividend, and to the remainder bring down the next period, and proceed thus till all the figures or periods are brought down.

Note. If at last there be no remainder, the quotient will be the true root; but if any thing remain, annex two cyphers, and work as has been taught above, and for every two cyphers thus annexed, there will be one decimal place in the root.

Instead of doubling the quotient every time for a divisor, you may always add the last quotient figure to the last divisor, for a new divisor, and proceed as before.

EXAMPLE 1. Let it be required to extract the square root of 393129?

```
      393129(627 Root
      36
  122)331
  + 2 244
 1247)8729
      8729
```

EXPLANATION. The number being separated or pointed into periods of two figures each, then the nearest square to 39 the first period, is 36, whose root 6, I place in the quotient, and subtract the square 36 from 39, the remainder is 3.

Then I bring down 31, the next point, and annex it to 3, and the new dividend is 331, then I double the quotient 6 for a divisor, which is 12, and seek how oft 12 in 33? the answer is 2, which I place in the quotient, and also after 12; then the divisor becomes 122, which multiplied by 2, the product is 244, which subtracted from 331, the remainder is 87.

Lastly, I bring down 29, the next point, and the dividend is 8729; then I double the quotient 62, which is 124, for a new divisor, and seek how oft 124 in 872? the answer is 7 times. Then I multiply 1247 by 7, and subtract the product 8729 from the last dividend, and there remains nothing; therefore 393129 is found to be a square number, and 627 its root.

To prove the work; if you square the root, and to that product add the remainder (if any) that sum shall be equal to the number first given, thus: $627 \times 627 = 393129$, the given divisor in the last example.

E. 2. What is the ſquare root of 321489 ?

```
     . . .
     321489(567 Root
     25
 106)714
  +6 636
1127)7889
     7889
     ....
```

E. 3. What is the ſquare root of 814602573 ?

```
      . . . . . . .  . .
      814602573,0000(28541,24
      4                  [Root
   48)414
      384
  565)3060
      2825
 5704)23525
      22816
57081)70973
      57081
570822)1389200
       1141644
5708244)24755600
        22832976
Remains 1922624
```

E. 6. What is the ſquare root of 2 ?

```
   .  . . . . . .
   2,000000000000(1,414213 Root
   1
 24)100
    96
 281)400
     281
 2824)11900
      11296
 28282)60400
       56564
 282841)383600
        282841
2828423)10075900
        8485269
Remains 1590631
```

Note. If the root of a mixed number is propoſed to be extracted, make the number of decimal places even, by annexing cyphers to the right-hand of the given ſquare, that a point may fall on the units place of the whole number.

E. 4. What is the ſquare root of 436,5 ?

```
  .  .  . . . .
  436,50000000(20,8925 Root
  4
408)3650
    3264
4169)38600
     37521
41782)107900
      83564
417845)2433600
       2089225
Remains 344375
```

E. 5. What is the ſquare root of ,000729 ?

```
      . . .
   ,000729(,027 Root
       4
    47)329
       329
       ...
```

Note. When the root is to be extracted to a great number of places the work may be much abbreviated by proceeding by the common method, till you have one figure more than half the number there is to be in the root, and then dividing the remainder according to the contraction in diviſion of decimals. See the above example worked by this method.

E. 7.

E. 7.

```
2,000000000000(1,414213   Root, the
1                         same as
24)100                    before.
   96
281)400
    281
2824)11900
     11296
2828)604
     565
      39
      28
      11
       8
Remainder 3
```

E. 8.

```
2,000000000000(1,414213   Root as
1                         before.
24)100
   96
281)400
    281
2824)11900
     11296
2828)6040
     5656
     3840
     2828
     1020
     8484
Remainder 1636
```

Note. If common division be ufed, you muft bring down as many figures, as there where periods to come down when you began with divifion. See the laft Example.

Numbers like thofe in Example 8, are called furds, whofe fquare root cannot be exactly found; but by annexing cyphers as above, you may come extremely near the truth, and the further you proceed, the more exact will the root be; but for common purpofes four or five places of decimals are fufficient.

To Extract the Square Root of Vulgar Fractions.

RULE. Reduce the fraction or fractional parts to their loweft terms, and if it be a mixed number, to an improper fraction; then extract the fquare root of the numerator for a new numerator, and the fquare root of the denominator for a new denominator. But if the fraction be not a compleat power, then reduce it to a decimal, and proceed as taught before.

Example 1. What is the fquare root of $\frac{36}{81}$?
Firft $\frac{36}{81}$ in its loweft terms is $= \frac{4}{9}$; then $\sqrt{\frac{4}{9}} = \frac{2}{3}$ the root required.

E 2. What is the fquare root of $\frac{2704}{4225}$?
Firft $\frac{2704}{4225} = \frac{16}{25}$ in its loweft terms; then $\sqrt{\frac{16}{25}} = \frac{4}{5}$ the root required.

E. 3. What is the fquare root of $\frac{9216}{12544}$?
Firft $\frac{9216}{12544} = \frac{36}{49}$ in its loweft terms; then $\sqrt{\frac{36}{49}} = \frac{6}{7}$ the root required.

LII. THE USE OF THE SQUARE ROOT.

CASE 1.

TO find a mean proportion between any two given numbers,

RULE. Multiply the two given numbers together, and extract the fquare root of the product, which root will be the mean proportional fought.

Example

EXAMPLE 1. What is the mean proportional between 7 and 9?

First 9×7=63;

```
Then 63(7,93 Anſwer
     49
149)1400
    1341
1583)5900
     4749
     1151
```

E. 2. What is the mean proportional between 36 and 64?

First 36×64=2304;

```
Then 2304(48 Anſwer
     16
  88)704
     704
       0
```

Therefore, as 36 : 48 :: 48 : 64

CASE 2. To find the ſide of a ſquare, equal in area to any given ſuperficies,

RULE. Extract the ſquare root of the givien ſuperficies, which root will be the ſide of the ſquare ſought.

E. 3. If the area of a circle be 33124, I demand the ſide of a ſquare, whoſe ſuperficial content ſhall be equal thereto?

```
    33124(182 Anſwer.
    1
28)231
   224
362)724
    724
    ...
```

E. 4. A gentleman has a piece of ground in the form of a parallelogram, whoſe longeſt ſide is 134 chains, and ſhorteſt 80 chains, which he intends to change for a ſquare piece of ground of the ſame area, which is to be incloſed out of a large field; you are required to find the length of the ſide? First 134×80=10720; then:

```
     10720,00(103,5 Anſwer.
     1
203)0720
     609
2065)11100
     10325
       775
```

CASE 3. To find the diameter of a circle, equal in area to an ellipſis, whoſe tranſverſe and conjugate axes are given,

RULE. Multiply the two axes of the ellipſis together; and the ſquare root of the product is the diameter of a circle equal to the ellipſis.

E. 5.

E. 5. Suppose the transverse axes of an ellipsis be 36, and the conjugate 23,5, what is the diameter of a circle equal thereto?

First 23,5 × 36=8460; then:

```
  .  .  .  .
  8460,0000(91,97 Answer
  81
 181)360
     181
1829)17900
     16461
18387)143900
      128709
       15191
```

Case 4. Having the area of a circle, to find the diameter,

RULE. As 355 : 452 :: or, as : 1,273239 :: the area to the square of the diameter; or, multiply the square root of the area by 1,12837, and the product will be the answer.

E. 6. Required the diameter of a circle, that will comprehend within its circumference the quantity of an acre of land?

First, an acre of land contains 4840 square yards, then 355 : 452 :: 4840 : 6162,4788 square of the diameter.

```
 .  .  .  .
6162,4788(78,5  Yards, the
 49             (diameter.
148)1262
    1184
1565)7847
     7825
     2288
```

Note. ,7854, and 3,1416, are areas of circles, whose diameters are 1 and 2, and ,079577 is the area of a circle, whose circumference is 1; likewise 452, and 1,273239, are squares of the diameters of circles, whose areas are 355; and 1, and 1,12831, is the diameter of a circle, whose area is equal to a square whose side is 1.

E 7. *In the midst of a medow well stored with grass,*
I took just three acres to tether my horse;
How long must the cord be, that feeding all round,
He mayn't graze less or more than three acres of ground?

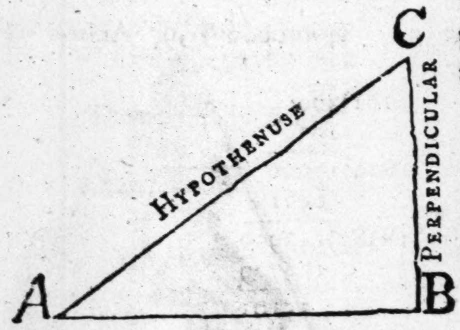
First 4840 × 3=14520 yards, the content of three acres; then, as 355 : 452 :: 14520 : 18487,4 yards square of the diameter.

```
 .  .  .  .  .
18487,4000(135,96 Diameter.
 1
23)84
   69
265)1587
    1325
2709)26240
     24381
27186)185900
      163116
       22784
```

Therefore 2)135,96 the Diameter.
67,98 Yards, length (of the cord required

Case 5. Any two sides of a right-angled triangle, A, B, C, being given, to find the remaining side.

1. The



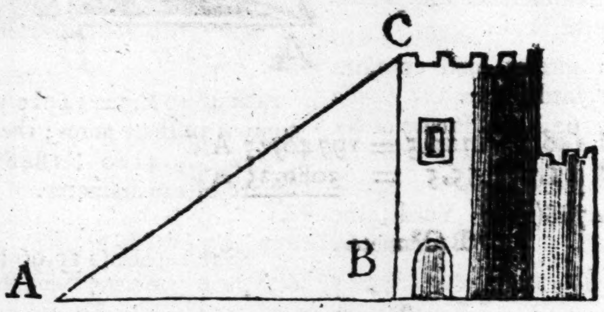
1. The baſe and perpendicular being given, to find the hypothenuſe,

RULE. Square each ſide, add the ſquares together, and the ſquare root of this ſum gives the hypothenuſe required.

2. If the hypothenuſe and one ſide be given, to find the other ſide.

RULE. From the ſquare of the hypothenuſe, ſubtract the ſquare of the given ſide, the ſquare root of the remainder gives the ſide required.

E. 8. The top of a caſtle from the ground is 45 yards high, and ſurrounded with a ditch 60 yards broad; what length muſt a ladder be, to reach from the outſide of the ditch to the top of the caſtle;



In the above figure, A B = the breadth of the ditch = 60 yards; B C = 45 yards, the height of the caſtle; and A C the length of the ladder required. First $60 \times 60 = 3600$

And $45 \times 45 = 2025$

5625(75 Yards = A C, the lenght (of the ladder.
49
145)725
725
. . .

E. 9. At Matlock, near the Peak, in Derbyſhire, where are many ſurpriſing curioſities in nature, is a rock by the ſide of the river Derwent, riſing perpendicular to a wonderful height, which being inacceſſible, I endeavoured to meaſure, and found by a mathematical method, that the diſtance between the place of obſervation and the foot of the rock, to be $55\frac{1}{2}$ yards, and from the top of the rock to the ſaid place, to be $140\frac{1}{2}$ yards, (nearly); required the height of this ſtupendous rock?

In

In the annexed figure, A is the place of obſervation; A B the diſtance to the foot of the rock $= 55\frac{1}{2}$ yards; A C the diſtance from the top of the rock to the ſaid place $= 140\frac{1}{2}$ yards and B C the perpendicular height of the rock, which is required.

First $140,5 \times 140,5 = 19740,25$ A C²
And $55,5 \times 55,5 = \underline{3080,25}$ A B²

B C² = 16660(129,07 yards = B C, the height (required

```
          1
   22)   66
         44
  249) 2260
       2241
24907) 190000
       174349
        15651
```

E. 10. *A caſtle wall there was, whoſe height was found,*
To be one hundred feet from th' top to th' ground;
Againſt the wall a ladder ſtood upright,
Of the ſame length the caſtle was in height.
A waggiſh youth did the ladder ſlide,
(The bottom of it) ten feet from the ſide;
Now I would know how far the top did fall,
By pulling out the ladder from the wall?

In the annexed fig. BC = 100 feet, the height of the castle; A B = 10 feet, the distance of the ladder from the wall; Aa the ladder = BC, and aC the distance the ladder fell from the top of the castle; which is required.

First $100 \times 100 = 10000 = Aa^2$ the ladder

And $10 \times 10 = 100 = AB^2$ the ladder's distance from the wall

Difference $9900 = Ba^2$

Then $\sqrt{9900} = 99{,}49874 = Ba$.

$\therefore$ $100 - 99{,}49874 = {,}50126 = a$C, which is 6 Inches nearly. Ans.

To know what light is proper for any room.

RULE

Multiply the length, breadth, and height together, the square root of that sum is the quantity of light required.

E. 11. Suppose a room was 24 feet long, 16 broad, and 14 high, how much light would be proper for such a room?

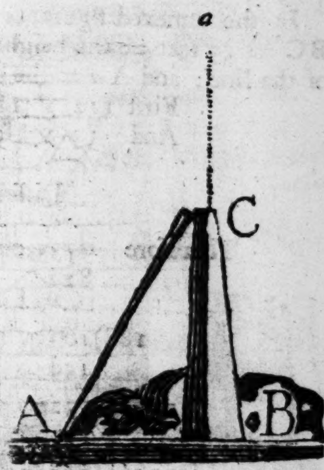
First $24 \times 16 \times 14 = 5376$

Then $\sqrt{5376} = 73{,}3$ feet, the quantity of light required.

E. 12. *As I was walking out one day,*
Which happened on the first of May,
As luck would have it, I did spy
A may-pole raised up on high,
The which at first me much surpriz'd,
Not being before-hand advertis'd
Of such a strange, uncommon sight;
I said, I would not stir that night,

Nor

Nor rest content until I'd found
Its height exact from off the ground;
But when these words I just had spoke,
A blast of wind the may-pole broke,
Whose broken piece I found to be
Exact in length yards sixty-three,
Which by its fall broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I can do,
The may-pole now being broke in two,
Unequal parts, to aid a friend,
Ye youths, pray then an answer send?



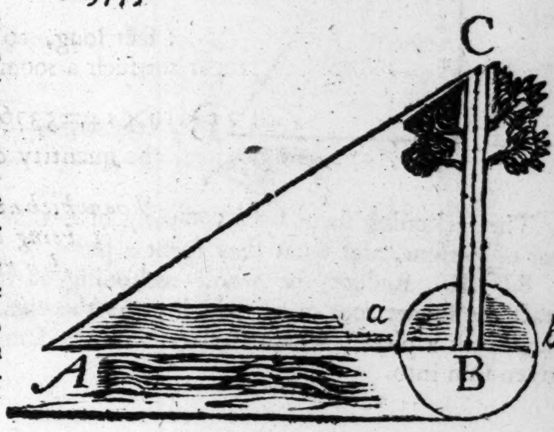
In the annexed Figure, A C = the length of the piece broken off, = 63 yards; A B = the distance the top of the piece fell from the bottom = 30 yards, and B C = the length of the pole that was left standing.

First $63 \times 63 = 3969 = AC^2$ or Ca^2
And $30 \times 30 = 900 = AB^2$

Difference 3069,00000000(55,3985 yards = B C
25
105) 569
525
1103) 4400
3309
11069) 109100
99621
110788) 947900
886304
1107965) 6159600
5539825
619775

Therefore 63
+55,3985
Answer 118,3985 Yards

E. 13. The height of an elm, growing in the middle of a circular island 30 feet in diameter, plumbs 53 feet, and a line stretched from the top of the tree straight to the hither edge of the water, 112 feet; what then is the breadth of the moat, supposing the land on the other side the water to be level?



In the annexed figure, $a\,b = 30$ feet $=$ the diameter of the island; BC $= 53$ feet $=$ the height of the elm; AC $= 112$ feet, the length of the line; and A$a =$ the breadth of the moat, which is required.

First $112 \times 112 = 12544 = AC^2$
And $53 \times 53 = 2809 = BC^2$

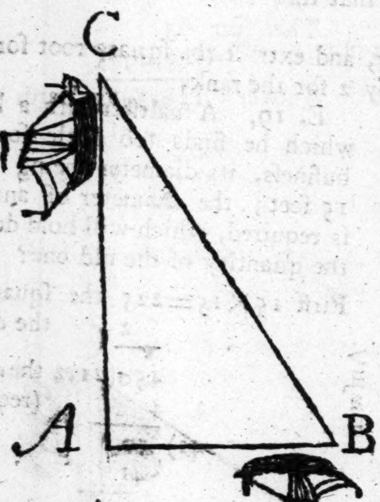
Difference $9735 = AB^2$

```
Therefore  9735,00 (98,66 = A B
           81       —15,00 Radius of the island
           ——       ——————
     188)1635       83,66 = A a the breadth of the moat
         1504                    (required, answer
    1966) 13100
          11796
   19726) 130400
          118356
          ——————
           12044
```

E. 14. Two ships set sail from the same port, one of them goes due east, 50 leagues; the other due north, 84; how far are they asunder?

In the following figure, A is the port where the two ships sailed from; one north to C $= 84$ leagues, the other east to B $= 50$ leagues; consequently BC is the distance they are from one another, which is required.

First $50 \times 50 = 2500 = AB^2$
And $84 \times 84 = 7056 = AC^2$




```
Sum 9556(97,75 leagues
    81        = BC, the
    ——          distance
 187) 1456
      1309
 1947) 14700
       13629
 19545) 107100
         97725
         —————
          9375
```

The reckoning spent by a company of persons, to find out the number of persons, and what they spent a-piece,

RULE. Reduce the whole reckoning to its lowest name, and extract the square root of it, which gives the number of persons, and what they spent a-piece; which is always of the same name you reduced the given sum into.

E. 15.

E. 15. A certain company being at a public-house, their reckoning came to 6*s*, $0\frac{1}{4}d$, the number of persons in company were equal to the farthings each spent; query, the number in company, and what each spent?

First 6*s*. $0\frac{1}{4}d.=289$ *qrs*

```
  . .
Then 289(17 men, answer
     1
    ---
 27)189
    189
    ...
```

```
          m.    qrs.   m.
Again, if 17 : 289 :: 1
17)289(17qrs.=4¼d. a-piece,
   17               (answer
   --
   119
   119
   ---
     0
```

E. 16. A company of men drinking till the reckoning came to 30*s*. 1*d*. I demand how many there were in company, and what they paid a-piece? Answer 19 men, paid 19*d*. a-piece.

E. 17. Suppose 75625 soldiers were ordered into a square battalia, how many men must there be in rank and in file?

```
      . . .
      75625(275 Men in rank and file, answer
      4
      ---
  47) 356
      329
      ----
 545) 2725
      2725
      ----
```

To place any number of men, so that the number of men in rank may be double to them in file,

RULE. Take half the number, and extract the square root for the file, which file you must multiply by 2 for the rank.

E. 18. Suppose 35912 men to be martialled in battle array, and the number of men in rank to be double to them in file; *query*, the number in rank and file?

```
First 2)35912
        . . .
        17956(134 men in file   }
        1      ×2               } Answer
        --     ---              }
   23)  79     268 men in rank  }
        69     ---
        --
  264)1056
      1056
      ....
```

E. 19. A maltster hath a kiln, which he finds too little for his business, its diameter being only 15 feet; the diameter of another is required, which will hold double the quantity of the old one?

```
First 15×15=225 the square of
              2      the diam.
           ------
           450(21,2 the diam.
           4          (required
           ---
      41)  50
           41
           ---
     422)  900
           844
           ---
            56
```

E. 20.

E. 20. A maltster hath a kiln, which he finds too large for his business, its diameter being 21,2 feet; the diameter of another, which will hold half the quantity, is required?

First 21,2×21,2=450 nearly; then 2)450

225(15 Feet, the answer.
1

25)125
125

By having the bung and head diameters of a cask given, to find the diagonal line,

RULE. Add the square of half the sum of the head and bung diameters, to the square of half the length; the square root of that sum is the diagonal of the cask.

E. 21. Let 25 be the bung, 22 the head diameter, and 30 inches the length of the cask; what is the diagonal line?

Head 22
Bung 25
Sum 47
Half 23,5
23,5
1175
705 Length 30
470 Half 15
15
Square 552,25
Add 225 Square 225

777,25(27,87 the diagonal (line
4
47)377
329
548)4825
4384
5567) 44100
38969
5131

E. 22. The semi-diameter of the earth being 3984,58 miles, and the perpendicular height of a mountain 3 miles; how far may it be seen at sea, the eye of the spectator being supposed to be on the surface of the water?

First 3984,58= Semi-diameter of the earth + 3 the height of the mountain = 3987,58 × 3987,58 =15900794,2564

Then 3987,58 × 3984,58 = 15888831,5164

And 15900794,2564—15888831,5164=11962,74

∴ √11962,74=109,36 miles the answer.

LIII.

LIII. EXTRACTION OF THE CUBE ROOT.

TO extract the cube root, is to find out a number, which being multiplied into itself, and then again into the product, produceth the given number.

As the cube root of 512 is 8, consequently $8 \times 8 \times 8 = 512$, the given number; and so of others, as in the following:

TABLE.

Roots	1	2	3	4	5	6	7	8	9
Cube	1	8	27	64	125	216	343	512	729

RULE. 1. Make a Point over every third figure given, beginning at the units place; seek the greatest cube to the first point on the left-hand (by the table) whose root place in the quotient; then subtract its cube from the period, and to the remainder (if any) bring down the three next figures, or your next period, and call it your dividend.

2. Find a divisor, by calling your quotient figure, with a cypher joined to it, r; then three times the square of r will be your divisor, seek how often it is contained in the dividend, and put the answer in the quotient, as in division, only with this difference, call the said quotient figure last put up e, and multiply your divisor by it, and place the produce underneath the dividend; then multiply the square of e, by three times r, and place it also under the dividend. Lastly, cube the figure you called e, and place it under the dividend; then add the three products together, which gives the subtrahend, which subtract from your last dividend, and to the remainder bring down the next period, and proceed as before.

EXAMPLE 1. What is the cube root of 32768?

$$
\begin{array}{rl}
\dot{3}2\dot{7}68(32 & \text{The root} \\
\underline{27} & \\
3r^2 = 2700)5768 & \text{Dividend} \\
5400 = 3rre & \\
360 = 3ree & \left.\vphantom{\begin{array}{l}a\\b\\c\end{array}}\right\} \begin{array}{l}\text{here } r = 30 \\ \text{and } e, 2\end{array} \\
8 = 3^e & \\
\overline{5768} & \text{Subtrahend, equal to the last dividend.} \\
\overline{\cdots\cdot} &
\end{array}
$$

EXPLANATION. The nearest cube to 32, the first period, is 27, which is set under, and subtracted therefrom, and 3, the root of the said cube, is placed in the quotient, and to the remainder 5, the period 768 is annexed,

annexed, which makes 5768 for a dividend; then a cypher is joined to the quotient figure 3, making 30, which is called r, and being squared, and that square multiplied by 3, produces 2700 for a divisor, which being contained twice in the dividend, 2 is placed in the quotient, and called e, by which the divisor is multiplied, and the product 5400 set under the dividend. Then 3 times $r = 90$, is multiplied by 4, the square of e, and the product 360 is placed under 5400; and lastly, 8 the cube of e, is placed under, and added to the other two numbers under the dividend; and the sum 5768 being the same as the dividend, and no more periods to be brought down, the work is finished, and 32768 is found to be a cube number, and 32 its cube root.

E. 2. What is the cube root of 21024576?

```
             .  .  .
            21024576(276 The root.
            8
3rr=1200)13024 Dividend
          8400 = 3rre  } here r = 20
          2940 = 3ree  } and e = 7
           343 = eee   }
         11683 Subtrahend.
3rr=218700)1341576
         1312200 = 3rre  } here r = 270
           29160 = 3ree  } and e = 6
             216 = eee   }
         1341576 Remainder.
         .......
```

E. 3. What is the cube root of 924?

```
              .
            924(9,7 The root
            729
3rr=24300)195000 Dividend
          170100 = 3rre  } here r = 90
           13230 = 3ree  } and e = 7
             343 = eee   }
          183673 Subtrahend.
Remains    11327
```

E. 4.

E. 4. What is the cube root of 92398647?

$$\begin{array}{rl}
 & 92398647(452{,}08 + \text{The root} \\
 & 64 \\
4800) & 28398 \text{ Dividend} \\
 & \left.\begin{array}{r}24000\\3000\\125\end{array}\right\} \begin{array}{l}\text{here } r = 40\\ \text{and } e = 5\end{array} \\
 & 27125 \text{ Subtrahend} \\
607500) & 1273647 \text{ Dividend} \\
 & \left.\begin{array}{r}1215000\\5400\\8\end{array}\right\} \begin{array}{l}\text{here } r = 450\\ \text{and } e = 2\end{array} \\
 & 1220408 \text{ Subtrahend} \\
6129120000) & 53239000000 \text{ Dividend} \\
 & \left.\begin{array}{r}49032960000\\8678400\\512\end{array}\right\} \begin{array}{l}\text{here } r = 45200\\ \text{and } e = 8\end{array} \\
 & 49041638912 \text{ Subtrahend} \\
 & 4197361088 \text{ Remains}
\end{array}$$

Now $452{,}08 \times 452{,}08 \times 452{,}08 + 4197361088 = 92398647$ Proof.

ANOTHER CONCISE METHOD OF EXTRACTING THE CUBE ROOT.

RULE 1. Point every third figure of the given number, beginning at the units place; then find the nearest cube to the first point, subtract, and bring down the three next figures in the next period to the remainder for a resolvend.

2. Square the quotient, and multiply it by 3, for a divisor; find how often it is contained in the resolvend, rejecting units and tens, and put the answer in the quotient.

3. Square this new figure, and put it on the right hand of the divisor; but if the new figure should be 1, 2, or 3, then put 01, 04, or 09, to the right hand.

4. Multiply the last figure in the quotient by 30, and multiply it by the former figures; add this product to the divisor, and multiply the sum by the last figure in the quotient; subtract that product from the resolvend, bring down the next three figures, and proceed as before.

E. 5. What is the cube root of 32768?

$$\begin{array}{rl}
 & 32768(32 \text{ The root} \\
 & 27 \\
2884) & 5768 \\
 & 5768 \\
 & \cdots
\end{array}$$

EXPLANATION. The square of $3 \times 3 = 27$, the divisor; and the square of 2 is 4, which (per rule) is 04, this put on the right-hand of the divisor

divisor

divisor 27, makes 2704; then $2\times30\times3=180$, which added to 2704, makes 2884, for a new divisor, which multiplied by 2, the last figure in the quotient, the product is 5768, to be set under the dividend and subtracted therefrom, and nothing remains; therefore 32768 is found to be a cube number, and 32 its cube root; the same as Example 1 in this section.

E. 6. What is the cube root of 618470208?

```
       .  .  .
     618470208(852 Root
     512
20425)106470
      102125
2172604)4345208
        4345208
        .......
```

E. 7. What is the cube root of 27407028375?

```
        .   .   .   .
       27407028375(3015 Root
       27
270901)407028
       270901
27225475)136127375
         136127375
         .........
```

M. de la Hire has given us a very odd property common to all powers, which *M. Carre* had observed with regard to the number 6, which is this: that all the natural cubic numbers, 8, 27, 64, 125, whose root is less than 6, being divided by 6, the remainder of the division is the root itself; and if we go further, 216, the cube of 6, being divided by 6, leaves no remainder, but the divisor 6 is the root itself. Again, 343, the cube of 7, being divided by 6, leaves 1, which added to the divisor 6, makes 7 the root, &c.

The above gentleman, on considering this property of 6, has found that all numbers, raised to any power whatever, have divisors, which have the same effect with regard thereto, that 6 hath with regard to cubic numbers.

For finding of these divisors, observe the following:

RULES. 1. If the exponent of the power of a number be even, i. e. if the number be raised to the second, fourth, sixth power, &c. it must be divided by 2; the remainder of the division, in case there be any, added to 2, or to a multiple of 2, gives the root of this number, corresponding to its power, i. e. the second, sixth, &c. root.

2. If the exponent of the power be an uneven number, i. e. if the number be raised to the third, fifth, seventh power, &c. the double of that exponent will be the divisor, which has the property mentioned.

Thus it is found in 6, double of 3, the exponent of the power of all the cubes; thus also 10 is the divisor, of all the numbers raised to the fifth power, &c.

To Extract the Cube of a Vulgar Fraction.

RULE. Extract the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator; and this new fraction will be the cube root of the given fraction.

The fractions must be reduced to their lowest terms; if it be a mixed number, to an improper fraction; and if a surd to a decimal.

EXAMPLE 1. What is the cube root of $\frac{27}{343}$?

First $\sqrt[3]{27}=3$; and the $\sqrt[3]{343}=7$; then $\frac{3}{7}$ is the root required.

E. 2

E. 2. What is the cube root of $\frac{352}{1188}$?
First $\frac{352}{1188} = \frac{8}{27}$; then $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ the root.

E. 3. What is is the cube root of $13\frac{4}{7}$?
First $13\frac{4}{7} = \frac{95}{7}$, or $\frac{95000}{7000}$; then $\sqrt[3]{\frac{95000}{7000}} = \frac{45}{19} = 2\frac{7}{19}$ the root.

The extraction of roots of higher powers, are of little or no use in practical arithmetic; I shall therefore leave this rule with the following observations.

1. The biquadrate of any number is found by extracting the square root of the given number first, and then the square root of that root.

2. The root of the square cubed, or sixth power of any number, is found by extracting the square root of the given number, then extract the cube root of that square root, which will give the sixth power required.

3. The root of the biquadrate squared, or eighth power, is found by extracting the square root of the given number, which will reduce it to a biquadrate, which proceed with as before.

4. The root of the cube cubed, or ninth power of any number, is found by extracting the cube root of the given number, and the result will be a cubic resolved; or extract the cube root also, which will be the root of the ninth power.

LIV. The USE of the CUBE ROOT.

CASE I.

TO find the side of a cube that will be equal in solidity to any given solid, as a globe, cylinder, cone, &c.

RULE. Extract the cube root of the solid content, of the given body, which root will be the side of the cube required?

Example 1. There is a stone of a cubic form, which contains 432 solid feet; what is the superficial content of one of its sides?

```
          432(7,55 + Side of the cube
          343
  15775)  89000
          78875
1698775)10125000
         8493875
Remains  1631125
```

Then 7,55 × 7,55 = 57,0025 the content req.

E. 2. The content of a globe is 1728 solid inches, what is the side of a cube equal thereto?

```
      1728(12 Inches, the side of the cube
      1
364)  728
      728
```

CASE 2. Having the dimenſions of any ſolid body, to find the dimenſions of another ſimilar ſolid, that ſhall be any number of times greater or leſs than the ſolid given,

RULE. Multiply the cube of each ſide by the difference between the ſolid given and that required, if greater, or divide by the difference if leſs than the ſolid given; then extract the cube root of each product or quotient, which will give the dimenſions of the ſolid required.

E. 3. Suppoſe the length of a ſhip's keel to be 125 feet, the breadth of the midſhip beam 25 feet, and the debth of the hold 15 feet; I demand the dimenſions of another ſhip of the ſame form, that ſhall carry three times the burthen?

First - 125
125
625
250
125
15625
125
78125
31250
15625
1953125
3
5859375(180,28 Keel
1
604)4859
4832
9730804)27375000
19461608
974593744)7913392000
7796749952
Remains 116642048

Secondly 25
25
125
50
625
25
3125
1250
15625
3
46875(36,05 Midſhip [beam
27
3276)19875
19656
38934025)219000000
194670125
Remains 24329875

Thirdly 15
15
75
15
225
15
1125
225
3375
3
10125(21,6 Debth in [the hold

E. 4. Suppoſe I lend my neighbour a ſtack of hay 12 feet in length, breadth, and depth, and he returns me 2 ſtacks, each of whoſe ſides is 6 feet, how much will remain due?

$12 \times 12 \times 12 = 1728$ Solid feet borrowed
$6 \times 6 \times 6 \times 2 = 432$ Solid feet repaid
Anſwer 1296 Solid feet unpaid
For $1728 \div 432 = 4$; conſequently $\frac{3}{4}$ is ſtill unpaid.

E. 5.

E. 5. What dimentions must I give to a joiner, to make a cubical box, that will hold 2000 oranges, of $2\frac{1}{2}$ inches diameter each, supposing the oranges globular, keeping that form, and laid in rows exactly at the top of each other?

First $2,5 \times 2,5 \times 2,5 \times 2000 = 31250$ the solidity of the box

```
                31250(31,498 Inches, the side of the box
                27
         2791)  4250
                2791
       292036)  1459000
                1168144
     29663661)  290856000
                266972949
   2975616124)  23883051000
                23804928992
```

Case 3. To find two mean proportionals between two given numbers,

RULE. Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme, gives the lesser mean; multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

E. 6. What are the two mean proportionals between 6 and 384?

First 6)384(64 whose cube root is 4

```
                                   × 6 The lesser extreme
For, as 6 : 24 :: 96 : 384         24 Lesser mean
                  24                4
                 384               96 Greater mean
                192
              6)2304
                 384 Proof
```

Case 4. Having the dimensions and capacity of a solid, to find the dimensions of a similar solid, of a different capacity,

RULE. Like solids are in triplicate proportion to their homologous sides; therefore it will be, as the cube of a dimension: is to its given weight, so is the cube of any like dimension, to the weight sought.

E. 7. Suppose a cannon ball of 4 inches diameter weighs 18*lb.* I demand the diameter of another that weighs 141*lb.*

First $4 \times 4 \times 4 = 64$, cube of the diameter

```
           lb.   in.    lb.   in
Then as    18  :  64  :: 141 : 501,3 Cube of the diameter
```

501,333

$$\dot{5}0\dot{1},333(7,9+ \text{ Inches, the diameter required}$$

```
       501,333(7,9+ Inches, the diameter required
       343
16671)158333
      150039
        8294
```

E. 8. There is a ball or globe of marble, whose diameter is 6 inches, and its weight 11 pounds, what will be the diameter of another globe of the same marble, that weighs 500 pounds?

First $6\times6\times6=216$, cube of the diameter

```
          lb.   in.     lb.     in.
Then as   11 : 216 :: 500 : 9818,181 cube of the diameter
```

```
        9818,181)21,4 inches, the diameter sought
        8
  1261)1818
       1261
134836) 557181
        539344
         17837
```

LV. THE SINGLE RULE OF THREE,

IN DECIMALS.

RULE.

REDUCE the fractional parts into decimals of the higheſt name mentioned; then ſtate the queſtion, and proceed as taught in ſect. XII. and XIII.

EXAMPLE 1. If $2\frac{1}{2}$ pounds of tea coſt 1*l.* 5*s.* what will $14\frac{3}{4}$ come to at the ſame rate?

Firſt $2\frac{1}{2}=2,5$; and 1*l.* 5*s.* = 1,25; alſo $14\frac{3}{4}=14,75$.

```
Then, as 2,5 : 1,25 :: 14,75
                        1,25
                        7375
                        2950
                        1475
          ⎧ 5)18,4375
      2,5 ⎨
          ⎩ 5)  36875
   Anſwer     7,375=7l.
                   7s. 6d.
```

E. 2. Suppoſing the earth to be 81000000 miles diſtant from the ſun; I would know at what diſtance from him another body muſt be placed, ſo as to receive light and heat, double to that of the earth?

Firſt $81000000\times81000000=6561000000000000$; then reciprocally,

As

As 1 : 6561000000000000 :: 2 : 3280500000000000

```
Therefore 3280500000000000(57275649 miles, answer
            25
    107)   780
           749
   1142)    3150
            2284
  11447)    86600
            80129
 114545)    647100
            572725
1145506)    7437500
            6873036
11455124)   56446500
            45820496
114551289)1062590400
           1030961601
             31628799
```

Note. The effects, or degrees of light, heat and attraction, are reciprocally proportional to the squares of their distances from the centre, whence they are propogated.

E. 3. If the diameter of the earth is 7970 miles, of the moon 2170 miles, supposing them both to be exact spheres (as they are not;)what comparison is there between them in point of magnitude?

First $7970 \times 7970 \times 7970 = 506261573000$

And $2170 \times 2170 \times 2170 = 10218313000$

As 10218313000 : 506261573000 :: 1

10218313|000)506261573|000(49,5445 Times larger than the moon, answer.

Note. The quantity of matter contained in all spheres, is directly in proportion to the cubes of their diameters.

E. 4. Suppose a stone let go into an abyss, should be stopped at the end of the eleventh second after its delivery, what space would it have gone through;

First $11 \times 11 = 121$, and the square of 1 is 1;

Then, as 1 : 16,083 :: 121

```
     121
   16083
  32166
 16083
```

Answer 1946,043 Feet

Note. The velocity acquired by heavy bodies falling near the earth's surface, is 16,083 feet in the first second; and as 16,083 is to the square of one second, or 1, so is the given distance to the square of the seconds required. Or by multiplying 16,083 feet, the descent of an heavy body near the earth's surface, in one second of time, by as many of the odd numbers,

numbers, beginning from unity, as there are seconds in any given time, viz. by 1 for the first, 3 for the second, 5 for the third, 7 for the fourth, &c. the sum total will give the space it has passed, any where on this side the earth's centre.

E. 5. What is the difference between the depth of two wells, into each of which should a stone be dropped at the same instant, one will meet with the bottom at 6 seconds, the other at 10?

First $10 \times 10 = 100$; and $6 \times 6 = 36$, square of their descents

$$\text{Then as } 1 : 16{,}083 :: \left\{ \begin{array}{l} 100 : 1608{,}3 \\ 36 : \underline{579} \end{array} \right\} \text{their depths}$$

Answer 1029,3 difference

E. 6. In what time would a musquet ball, dropped from the top of St. Martin's steeple, in Birmingham, said to be 300 feet high be at the bottom?

First, as 16,083 : 1 :: 300

```
16,083)300,0000000(18,6532
      18,6532(4,318+ seconds ans.
      16
     ----
   83)265
      249
     ----
  861)1632
       861
      -----
 8628)77100
      69024
      -----
       8076
```

E. 7. A ball descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time; required the tower's height, and the whole time of descent?

First, the square root of 1 = 1, and the square root of 2 = 1,4142, from which take 1, and there remains ,4142; then, as ,4142 ; 1,4142 :: 1 : 3,414 sec. the descent

Now $3{,}414 \times 3{,}414 = 11{,}6554$

And, as [2]1 : 16,083 :: 11,6554

```
                 16,083
              ---------
                 349662
                932432
              6993240
             116554
             ----------
Answer, feet 187,4537982
```

LVI. THE DOUBLE RULE OF THREE

IN DECIMALS.

Example 1.

If 1000 men can dig a trench 500 feet long in 24 hours, what length of such a trench can 9800 men dig, in 10 hours?

```
                    men.      feet.      men.
Stated thus; as    1000   :   500   ::   9800
                    24h.  :    —    ::     10
                  ------                ------
                  24000                 98000
                                          500
               24|000)4900|000(2041,6 = 2041 feet
```

7 inches, the answer.

E. 2.

E. 2. When the buſhel of wheat was ſold at 10*s.* the 4*d.* loaf weighed $4\frac{1}{2}$ lb. What ſhould the 6*d.* loaf weigh, when the buſhel of wheat ſells at 15*s.*?

First reciprocally, as 10 s. : 4,5 lb :: 15 s.
4 : — :: 6

15
4
—
60

4,5
10
—
45,0
6
—
6|0)27|0(4,3 $=4\frac{1}{2}$ lb. Anſwer.

E. 3. A young hare ſtarts 5 rods, before a greyhound, and is not perceived by him till ſhe has been up 34 ſeconds; ſhe ſcuds away at the rate of 12 miles an hour, and the dog, on view, makes after her at the rate of 20; how long will the courſe hold, and what ground will he run, beginning with the outſetting of the dog?

First 34″=,009444 hours, and 5 rods=,015625 miles; then

As 1 *h.* : 12 *m.* :: ,009444 *h.* : ,113328 *m.*

,113328
,015625
—
,128953 Miles = 680,9 Feet, the hare had ſtarted

Now 20—12=8 Dog gained in running 20

Therefore, as 8 *m.* : 20 *m.* :: ,128953 *m.* : 2,57906 furlongs = $1702\frac{1}{4}$ feet, run by the greyhound.

Again, as 8 *m.* : 1 *h.* :: ,128953 *m.* : ,016119 *h.*=58″ ,0284 time run by the greyhound.

Note. It hath been found by experiment, that a pendulum 39,2 inches long, in our latitude, vibrates 60 times in one minute; and that the length of pendulums are to one another reciprocally as the ſquare of the number of their vibrations made in the ſame ſpace of time.

E. 4. What difference is there between the length of a pendulum that vibrates half a ſecond, or 120 times in a minute, and another that ſwings double ſeconds, or 30 times in a minute? First 60×60=3600; then reciprocally:

As 3600 : 39,2 :: 900=30^2
3600
—
2352
1176
—
9|00)1411,200
—
156,8 Length of the pendulum that vibrates double ſeconds, or 30 times in a minute.

Again:

As 3600 : 39,2 :: 14400=120^2
3600
—
2352
1176
—
144|00)1411,2|00(9,8 Length of [the pend. that [vib. half ſec.
1296
—
1152 156,8
1152 9,8
— —
.... 147 *In.* = 12 *f.* 3 *i.*

 [Anſwer,

E. 5. Obſerved, that while a ſtone was decending to meaſure the depth of a well, a ſtring and plumet, that from the point of ſuſpenſion, or the place where it was held, to the centre of oſciliation, or that part of the bob, which being divided by the circular line, ſtruck from the centre aforeſaid, would divide it into two equal parts of equal weight, meaſured juſt 18 inches, had made 8 vibrations; pray what was the depth, allowing the ſame as in Page 80 for the return of ſound to the ear?

First 60
60
in. 39,2 : 3600 :: in. 18
39,2
72
324
108
18 { 3)141120,0
6) 47040
7840 Seconds

7840,00000000(88,5437 vib. [in 1 minute.
64
168)1440
1344
1765)9600
8825
17704)77500
70816
177083)668400
531249
1770867)13715100
12396069
319031

Then 6|0)88,5437
1,475728 Vibrations in a ſecond

Alſo 1,475728)8,000000(5,421 Seconds in 8 vibrations

Now 5,421
5,421
5421
10842
21684
27105
Then, as 1 : 16,083 :: 29,387241
16,083
88161723
235097928
1763234460
29387241
472,634997003 Feet, ſound was returned through

Again, as 1150 : 1 :: 472,634997003
1
115|0)472,63499700|3(,410986953 Seconds, time ſound was returning.

∵ 5,421 — ,411 = 5,01 ſeconds, time of the bodies deſcent; and 5,01 × 5,01 = 25,1; alſo 25,1 × 16,083 = 403,674 feet, the depth of the well, Anſwer.

E. 6.

E. 6. *In Derbyshire, a wonder of the Peak,*
Is Eldon-hole, as poets often speak,
Whose depth exactly, none could e'er descry,
Tho' atheist Hobbs his utmost skill did try,
Who wrote de Mirabilibus Pecci.
And burlesque Cotton does strange things rehearse
In rustic words, and Hudibrastic verse,
How he this monstrous orifice did plumb,
But could not at the bottom of it come
With 16 *hundred yards of rope let loose;*
And tells a story of woman's goose:
Fabulous the one, so may the other be,
Erroneous too, without philosophy;
But I the depth have found exactly true
By gravity; a method something new.
As heavy bodies do accelerate.
In spaces known first to our NEWTON *great;*
Four pond'rous stones into the well let fall
In measur'd time, agreed in numbers all:
A pendulum sixty-one inches long,
By which the time I measured was not wrong,
Vibrated freely whilst that each stone fell,
Eight times; by which the depth I'd have you tell,
Allowing righly for th' approach of sound
That your own works may not themselves confound?

First 60 × 60=3600; then reciprocally,

In.		*sec.*		*in.*		*sec.*	
As 39,2	:	3600	::	61	:	2313,4426	∴ $\sqrt[2]{2313,4426}=48,09$

Then, as 48,09 : 1 :: 8 : ,1663; and ,1663×60 = 9,99, or 10″ nearly, the time the pendulum made 8 vibrations.

Now 10×10=100; then, as 21 : 16,083 :: 100 : 1608,3 feet, sound was returning. Again 1150 : 1 :: 1608,3 : 1,398 seconds, the time sound was ascending; ∴ 10 — 1,398 = 8,602 seconds, time of the bodies descent; and 8,602×8,602=74, nearly.

Also 74× 16,083 = 1200,142 feet = 400,473 yards, the depth of Eldon-hole.

LVII. FELLOWSHIP.

RULE.

DIVIDE the whole gain or loss by the whole stock, and multiply the quotient by each person's particular stock, and the several products, will be the respective gain or loss of each.

EXAMPLE 1. Three merchants, A, B, and C, freight a ship with 96 ton of wine, thus; A put on board 24, B 32, and C 40 ton; but the extremity of the weather obliged them to cast 12 ton overboard; how much must each merchant bear of this loss?

First 24+32+40=96, the whole stock;
And 121÷96=,125, the quotient.

$$\text{Then}\left.\begin{array}{l}24\text{ A's}\\32\text{ B's}\\40\text{ C's}\end{array}\right\}\text{Stock}\times{,}125=\left\{\begin{array}{l}3\text{ A's loss}\\4\text{ B's}\\5\text{ C's}\end{array}\right.$$

Proof 12 tons.

E. 2. Four men trade together, A puts in 200*l.* B 150*l.* C. 85*l.* and D 70*l.* they gain 60*l.*. what is the share of each?

First 200+150+85+70=505 the whole stock;

And 60÷505=,11881 the quotient; then

$$\left.\begin{array}{r} 200 \text{ A's} \\ 150 \text{ B's} \\ 85 \text{ C's} \\ 70 \text{ D's} \end{array}\right\}\text{Stock} \times ,11881 = \left\{\begin{array}{lcrrl} & & £. & s. & d. \\ 23,762 & = & 23 & 15 & 2,9 \text{ A's gain} \\ 17,8215 & = & 17 & 16 & 5,2 \text{ B's} \\ 10,09885 & = & 10 & 1 & 11,8 \text{ C's} \\ 8,3167 & = & 8 & 6 & 4,1 \text{ D's} \end{array}\right.$$

Proof - £.60 0 0

E. 3. *In honour of* CRISPIN, *the cordwainers, they*
Prepared a feast, to be jovial and gay;
Six tanners, eight curriers, at first took their place;
Sixteen cordwainers next, all with regular grace;
Then the coblers next, who were twenty and one,
At table sat down with their host merry John:
When dinner was over full bumpers did pass,
Some drank a full noggin, and some a wide glass;
Carousing and singing they pass the long day,
No sons of great BACCHUS *more jovial than they.*
At last for the reck'ning the tanners did call,
Whilst some of the coblers did nothing but bawl
For old hock, or stingo;—the landlord came in
With his scores round a trencher—to work did begin,
And found that Ten Pounds was the shot to defray,
Then tell me TYRO, *what each had to pay,*
When the tanners and th' other, agreed very true,
In proportion to pay, as five, four, three, and two?

It is plain by the question, that as often as each tanner paid 5*s.* the others paid 4*s.* 3*s.* and 2*s.* a-piece; which sum multiplied by the number of each trade or occupation, thus:

$$\begin{array}{rcrcrcl} & & & & s. & & £. \\ 5 & \times & 6 & = & 30 & = & 1,5 \\ 4 & \times & 8 & = & 32 & = & 1,6 \\ 3 & \times & 16 & = & 48 & = & 2,4 \\ 2 & \times & 21 & = & 42 & = & 2,1 \\ \hline & & & & 152 & = & 7,6 \end{array}$$

And 10*l.* ÷ 7,6 = 1,31578 the quotient.

$$\text{Then } \left.\begin{array}{r} 1,5 \\ 1,6 \\ 2,4 \\ 2,1 \end{array}\right\}\times 1,31578 = \left\{\begin{array}{l} 1,97367 \text{ the 6 tanners share} \\ 2,105248 \text{ the 8 curriers} \\ 3,157872 \text{ the 16 cordwainers} \\ 2,763138 \text{ the 21 coblers} \end{array}\right.$$

$$\begin{array}{lcrrl} & & s. & d. & \\ \text{Also } 6)1,97367(,328945 & = & 6 & 6\tfrac{3}{4} & + \text{ each tanner's share} \\ 8)2,105248(,263156 & = & 5 & 3 & + \text{ each currier's} \\ 16)3,157872(,197367 & = & 3 & 11\tfrac{1}{4} & + \text{ each cordwainer's} \\ 21)2,763138(,131578 & = & 2 & 7\tfrac{1}{2} & + \text{ each cobler's} \end{array}$$

LVIII.

LVIII. DOUBLE FELLOWSHIP.

WHEN the ſhares of partners are continued in company unequal times, they occaſion the name fellowſhip with time, or double fellowſhip; which is performed by the following

RULE. Divide the whole gain or loſs, by the firſt term or ſum of the products; the quotient is a common multiplier, by which multiplying the ſeveral products, you will have the ſeveral ſhares required.

EXAMPLE 1. Three merchants, A, B, and C, enter into partnerſhip thus; A puts into the ſtock 65*l.* for 8 months; B puts in 78*l.* for 12 months; and C puts in 84*l.* for 6 months with this they traffic, and gain 166*l.* 12*s.* it is required to find each man's ſhare of the gain, proportionable to his ſtock and time of employing it?

£.		mo.		pro.		
65	×	8	=	520	A's	Stock multiplied into his time
78	×	12	=	936	B's	
84	×	6	=	504	C's	
			Sum -	1960		

1960)166,6(,085 the common multiplier

Then	520 × ,085	=	44,2	=	44	4	0	A's	Gain
And	936 × ,085	=	79,56	=	79	11	$2\frac{1}{2}$	B's	
Alſo	504 × ,085	=	42,84	=	42	16	$9\frac{1}{2}$	C's	
				Proof £.	166	12	0		

E. 2. Four merchants trade after this manner; A puts in 100*l.* for 8 months; B puts in 80*l.* for 5 months, and then puts in 40*l.* more for 3 months longer; C puts in 176*l.* for 4 months, and then takes out 50*l.* for four months more; D puts in 230*l.* for 6 months, and then takes out the whole: they gained 212*l.* 10*s.* what is the gain of each merchant;

£.		mo.			products.
100	×	8	=		800 A's Stock and time
80	×	5	=	400	760 B's
120	×	3	=	+ 360	
176	×	4	=	704	1208 C's
126	×	4	=	+ 504	
230	×	6	=		1380 D's
				Sum - -	4148

4148)212,50(,05123 the common multiplier

Then

				£.	s.	d.	
Then	800 × ,05123	= 40,984	=	40	19	8	A's Share
And	760 × ,05123	= 38,9348	=	38	18	8¼	B's
Alſo	1208 × ,05123	= 61,4760	=	61	17	8½	C's
Likewiſe	1380 × ,05123	= 70,6974	=	70	13	11¼	D's
			Proof - £.	212	10	0	

E. 3. Three merchants, A, B, and C, trade together; A puts in 120*l.* for 8 months; B 250*l.* for 4 months; and C 100*l.* for 5 months; they gained 184*l.* 10*s.* what is each man's ſhare of the gain?

	£.				
Firſt	120 × 8	=	960	A's Stock and time
	250 × 4	=	1000	B's
	100 × 5	=	500	C's

Sum 2460)184,5(,075 the quotient

$$\text{Then}\quad\left.\begin{array}{r}960\\1000\\500\end{array}\right\}\times ,075=\left\{\begin{array}{ll}72 & \text{A's Gain}\\75 & \text{B's}\\37,5 & \text{C's}\end{array}\right.$$

Proof £. 184,5 = 184*l.* 10*s.*

E. 5. Four merchants, A, B, C and D, enter into partnerſhip thus; A put in 64*l.* 10*s.* for 4½ months; B put in 78*l.* 15*s.* for 6 months; C put in 112*l.* 14*s.* for 8¾ months; and D 125*l.* 5*s.* for 5¼ months; they gain 108*l.* 18*s.* 4½*d.* what is due to each in proportion to their ſtocks and time they were employed?

	£.		mo.			
Firſt	64,5	×	4,5	= 290,25	A's ſtock and time
	78,75	×	6	= 472,5	B's
	112,7	×	8,75	= 986,125	C's
	125,25	×	5,25	= 657,5625	D's

Sum - 2406,4375

And 2406,4375)108,91875(,045261 the quotient

$$\text{Then}\quad\left.\begin{array}{r}290,25\\472,5\\986,125\\657,5625\end{array}\right\}\times ,045261=\left\{\begin{array}{ll}13,1370 & \text{A's Gain}\\21,3859 & \text{B's}\\44,633 & \text{C's}\\29,762 & \text{D's}\end{array}\right.$$

Proof - - 108,9179 = 108*l.* 18*s.* 4½*d.*

Note. Queſtions of this kind ſeldom occur in buſineſs, therefore to enlarge on this head would be entirely uſeleſs.

LIX.

LIX. SIMPLE INTEREST.

INTEREST is the premium allowed for the loan of money, &c. See Section XVIII. Page 114.

There are ſeveral ways of computing ſimple intereſt; as by the ſingle and double rules of three, &c. But all computations relating to ſimple intereſt, are grounded upon arithmetical progreſſion; I ſhall from thence make uſe of ſuch general theorems, as will ſuit all caſes. In order to that, here are five letters to be obſerved, viz.

Let $\begin{cases} P = \text{any principal or ſum put to intereſt.} \\ I = \text{the intereſt.} \\ T = \text{the time of the principal's continuance at intereſt.} \\ A = \text{the amount, or principal and its intereſt.} \\ R = \text{the ratio, or rate per cent. per annum.} \end{cases}$

The ratio is the ſimple intereſt of 1*l.* for 1 year, at any given rate; and is thus found:

£. £. £.
Viz. 100 : 5 :: 1 : ,05 the ratio at 5 per cent. per annum
Or, 100 : 4 :: 1 : ,04 the ratio at 4 per cent. per annum.

And in this manner the ratios in the following table are found.

TABLE.

Rate.		*Ratio.*
3		,03
$3\frac{1}{2}$		,035
$3\frac{3}{4}$		,0375
4	=	,04
$4\frac{1}{4}$		,0425
$4\frac{1}{2}$		,045
5		,05

When the principal, time, and rate per cent. are given, to find the intereſt,

RULE. Multiply the principal, rate, and time continually into one another; the product is the intereſt ſought.

Or, if p = the principal, t = the time, r = the rate, and I = the intereſt.

THEOREM 1. $ptr = I$.

EXAMPLE 1. What is the intereſt of 241*l.* for 3 years, at 4 per cent. per annum?

By *Theorem* 241 = p.
3 = t.
723 = pt.
,04 = r.
£. 28,92 = ptr.
20
s. 18,40
12
d. 4,80
4
qrs. 3,2 Anſ. 28*l.* 18*s.* [$4\frac{3}{4}d.$ +

E. 2. What is the intereſt of 842*l.* 10*s.* for four years, at 5 per cent. per annum?

By the rule 842,5 Principal
,05 Ratio
42,125
4 No. of years
£. 168,500
20
s. 10,000
Anſwer 168*l.* 10*s.*

E. 3.

E. 3. What is the intereſt of 20,000*l.* for 7 years, at $4\frac{1}{2}$ per cent. per annum?

```
    20000 = p.
     ,045 = r.
   ------
   100000
   80000
   -------
  900,000 = p r.
        7 = t
  -------
£. 6300 = p t r.  Anſwer
```

E. 4. What is the intereſt of 482*l.* 17*s.* 6*d.* for $6\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?

```
      482,875 = p.
         ,045 = r.
     --------
      2414375
     1931500
    ---------
    21,729375 = p r.
          6,5 = t.
    ---------
    108646875
   130376250
   ----------
Anſwer 141,2409375 = 141l. 4s. 9¾d.
```

LX. *When the Intereſt required is for Days only.*

RULE.

MULTIPLY the intereſt of 1*l.* for 1 day, at the given rate, by the principal and number of days for the anſwer.

The intereſt of 1*l.* for one day is thus found. viz.

	d.		*£.*		*d.*		
As	365	:	,05	::	1	:	,0001369863, &c. the intereſt of 1*l.* for one day, at 5 per cent.

And in this manner the following table is made.

TABLE.

Per cent.		*Decimals.*
5	=	,0001369863
$4\frac{1}{2}$	=	,00012328767
4	=	,00010958904
$3\frac{1}{2}$	=	,00009589041
3	=	,00008219178

EXAMPLE 1. What is the intereſt of 547*l.* 15*s.* for 320 days, at 5 per cent. per annum?

```
       ,0001369863 Ratio
            547,75 Principal
       -----------
           6849315
          9589041
         9589041
        5479452
       6849315
      ------------
      ,075034245825
                320 Number of days
      -------------
      1500684916500
     225102737475
     ------------
Anſwer 24,010958664000 = 24l. 0s. 2½d.
```

Again

Again thus: 547,75 Principal
320 Number of days

1095500
164325

*73|00)1752|80,00(24,0109=24*l.* 0*s.* 2½*d.* the anſwer as before.

E. 2. What is the intereſt of 150*l.* from the 18th day of January to the 11th of November, at 5 per cent. per annum?

$$\begin{array}{r} ,0001369863 = r \\ 150 = p. \\ \hline 68493150 \\ 1369863 \\ \hline ,0205479450 \\ 297 = t. \\ \hline 143835615\,0 \\ 184931505\,0 \\ 41095890\,0 \\ \hline \text{Anſwer}\quad 6,1027396650 = 6l.\ 2s.\ 0\tfrac{1}{2}d. \end{array}$$

Again thus: 150 Principal
297 Number of days

1050
1350
300

73|00)445|50(6,102 = 6*l.* 2*s.* 0½*d.* as before.

E. 3. What is the intereſt of 40*l.* for 50 days, at 3 per cent?

$$\begin{array}{r} ,00008219178 = r. \\ 40 = p. \\ \hline ,00328767120 = p\,r. \\ 50 = t. \\ \hline \text{Anſwer}\quad ,16438356000 = p\,t\,r = 3s.\ 3\tfrac{1}{4}d. \end{array}$$

When the principal, time, and rate per cent. are given, to find the amount,

RULE. Find the intereſt by theorem 1, which added to the principal will give the amount.

THEOREM 2. $ptr + p = A$.

EXAMPLE 1. What will 312*l.* 10*s.* amount to in 3 years, at 4 per cent. per annum?

* The reaſon of this contraction may be ſeen in Section XVIII. Page 124.

312,5=*p*.
,04=*r*.

12,500=*pr*.
3=*t*.

37,500=*ptr*.
312,5 =*p*.

Anſwer £. 350,000=*ptr*+*p*.

E. 2. What will 672*l*. amount to in $8\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?

672 Principal
,045

3360
2688

30,240
8,5 Time

151200
141920

257,0400=Intereſt
672 Principal

Anſ. 929,0400=929*l*. 0*s*. $9\frac{1}{2}$*d*.

E. 3. What will 500*l*. amount to in 6 years 120 days. at $4\frac{3}{4}$ per cent. per annum?

6,328767=*t*.
,0475=*r*.

31643835
44301369
25315068

,3006164325=*tr*.
500=*p*.

150,30821625=*ptr*
500 =*p*.

650,30821625=*ptr*+*p*=650*l*. 6*s*. $1\frac{3}{4}$*d*. Anſwer

When the rate, time, and intereſt, are given, to find the principal,

RULE. Divide the intereſt by the product of the rate and time, the quotient is the principal.

THEOREM 3. $\frac{I}{tr}=p$

EXAMPLE 1. What principal being put to intereſt for 2 years, will gain 60*l*. at 5 per cent. per annum?

Firſt 2 × ,05 =,10 the product of the ratio and time

Then 10)60,00(600*l*. the anſwer

E. 2. What principal being put to intereſt for 3 years, will gain 69*l*. 13*s*. 6*d*. at 5 per cent. per annum?

Firſt 3 × ,05 =,15 product of ratio and time

$$,15\left\{\begin{array}{l}5)69,675\\ \hline 3)13935\end{array}\right.$$

Anſwer 464,5 = 464*l*. 10*s*,

E. 3.

E. 3. What principal being put to intereſt for $4\frac{1}{2}$ years, will gain 58*l.* 14*s.* 6*d.* at 4 per cent. per annum?

Firſt $4,5 \times ,04 = ,18$ product of ratio and time

$$,18\left\{\begin{array}{r} 2)58,725 \\ \hline 9)293,625 \end{array}\right.$$

$$\overline{326,25} = 326l.\ 5s. \text{ the anſwer}$$

When the amount, rate, and time are given, to find the principal,

RULE. Multiply the rate by the time; add unity or 1, to the product for a diviſor, by which ſum divide the amount, the quotient will be the principal.

THEOREM 4. $\dfrac{a}{tr+1} = p.$

EXAMPLE 1. What principal will amount to 4700*l.* in 5 years, at $3\frac{1}{2}$ per cent. per annum?

$5 \times ,035 + 1 = 1,175$; then, $4700l. \div 1,175 = 4000l.$ the anſwer.

The work at length:

$$\begin{array}{r} ,035 \\ 5 \\ \hline 175 \\ +\ 1,000 \\ \hline 1,175)4700,000(4000 \text{ Anſwer as before} \\ 4700 \\ \hline 0 \end{array}$$

E. 2. What principal being put to intereſt will amount to 354*l.* 4*s.* $0\frac{1}{2}d.$ in 7 years, at $2\frac{1}{2}$ per cent. per annum?

$$\begin{array}{r} ,035 \\ 7 \\ \hline ,245 \\ +1,000 \\ \hline 1,245)354,202083(284,499 = \\ 2490 \quad 284l.\ 9s.\ 11\tfrac{3}{4}d. \\ \hline 10520 \quad \text{Anſwer} \\ 9960 \\ \hline 5602 \\ 4980 \\ \hline 6220 \\ 4980 \\ \hline 12408 \\ 11205 \\ \hline 12033 \\ 11205 \\ \hline 828 \end{array}$$

E. 3. What principal being put to intereſt will amount to 40*l.* in 3 years, at 5 per cent. per annum?

Firſt $,05 \times 3 + 1 = 1,15$; then

$$\begin{array}{r} 1,15)40,00000(34,7826 = 34l. \\ 345 \quad (15s.\ 7\tfrac{3}{4}d. \text{ Anſ.} \\ \hline 550 \\ 460 \\ \hline 900 \\ 805 \\ \hline 950 \\ 920 \\ \hline 300 \\ 230 \\ \hline 700 \\ 690 \\ \hline 10 \end{array}$$

When the principal, intereſt and rate are given, to find the time,

RULE. Divide the intereſt by the product of the principal and rate, the quotient is the time.

$$\textit{THEOREM 5.}\quad \frac{I}{pr} = t.$$

EXAMPLE 1. In what time will 200*l.* gain 60*l.* at 5 per cent, per annum?

$$\begin{array}{rl} & 200 = p. \\ & ,05 = r. \\ \hline pr = & 10,00(60 = I. \\ & \text{Anſwer } 6 \text{ Years} = t. \end{array}$$

E. 2. In what time will 260*l.* gain 64*l.* 7*s.* at $4\frac{1}{2}$ per cent, per annum? Firſt 260*l.* × ,045 = 11,7 the product of the principal and rate; and 64*l.* 7*s.* = 64,35 the intereſt.

Then 11,7)64,35(5,5=$5\frac{1}{2}$ years, the anſwer.

$$\begin{array}{r} 585 \\ \hline 585 \\ 585 \\ \hline \end{array}$$

E. 3. In what time will 500*l.* gain 130*l.* 9*s.* at $3\frac{1}{2}$ per cent. per annum? Firſt 500*l.* × ,035 = 17,5 product of the principal and rate; and 130*l.* 9*s.* = 130,45 the intereſt;.

Then 17,5)130,45(7,4 = 7 years 146 days, anſwer

When the amount, principal and rate are given, to find the time,

RULE. Divide the amount leſs the principal, by the product of the principal and rate, the quotient is the time.

$$\textit{THEOREM 6.}\quad \frac{a-p}{pr} = t.$$

EXAMPLE 1. In what time will 284*l.* amount to 354*l.* at 5 per cent. per annum?

In this example $a = 354$, $P = 284$, and $r = ,05$.

Then per theorem $\frac{354-284}{284 \times ,05} = \frac{70}{14,2} = 4,9295 = 4$ years, 339 days, the anſwer. The work at length:

$$\begin{array}{rl} 284 & 354 \\ ,05 & 284 \\ \hline 14,20 &)\ 70,00000(4,9295 = 4 \text{ years, } 339 \text{ days, anſwer} \\ & 568 \\ \hline & 1320 \\ & 1278 \\ \hline & 420 \\ & 284 \\ \hline & 1360 \\ & 1278 \\ \hline & 820 \\ & 710 \\ \hline & 110 \end{array}$$

E. 2.

E. 2. In what time will 336*l.* 5*s.* amount to 423*l.* 10*s.* at $4\frac{1}{2}$ per cent. per annum?

In this example, $a = 423{,}5$, $p = 336{,}25$, and $r = {,}045$

Then per theorem, $\dfrac{423{,}5 - 336{,}25}{336{,}25 \times {,}045} = \dfrac{87{,}25}{15{,}13125} = 5766 = 5$ years, 279 days, the anſwer.

The work at length,

```
                423,5
                336,25
    15,13125)87,25000(5,766 = 5 years, 279 days, anſ.
              7565625
              11593750             336,25
              10591875               ,045
               10018750            168125
                9078750           134500
                 9400000          15,13125
                 9078750
                  321250
```

When the principal, intereſt, and time are given, to find the rate per cent.

RULE. Divide the intereſt by the product of the principal and time the quotient is the rate.

$$\textit{THEOREM } 7\text{: } \frac{I}{pt} = r.$$

EXAMPLE 1. At what rate per cent will 260*l.* gain 64*l.* 7*s.* in $5\frac{1}{2}$ years?

In this example, $I = 64{,}35$, $p = 260$, and $t = 5{,}5$.

Then per theorem, $\dfrac{64{,}35}{260 \times 5{,}5} = \dfrac{64{,}35}{1430} = {,}045$, or $4\frac{1}{2}$ per cent. the anſwer.

The work at length,

```
              260
              5,5
             1300
             1300
           1430,0)64,35(,045 Rate = 4½ the anſwer.
                  572
                   715
                   715
                   ...
```

E. 2. At what rate per cent. will 216*l.* 10*s.* gain 43*l.* 6*s.* in 4 years?

216,5
4
866,)43,30(,05 = 5 per cent. anſwer.
4330
....

When the principal, amount, and time are given, to find the rate,

RULE. Take the difference between the amount and principal, and divide it by the product of the principal and time, the quotient is the rate.

THEOREM 8. $\frac{a-p}{pt}=r.$

EXAMPLE 1. At what rate per cent. will 142*l.* 5*s.* amount to 177*l.* 2*s.* $0\frac{1}{4}$*d.* in $3\frac{1}{2}$ years?

In this example, $a=177{,}1010416$, $p=142{,}25$, and $t=7$.

Then per theorem, $\frac{177{,}1010416-142{,}25}{142{,}25\times7}=\frac{34{,}8510416}{995{,}75}=$,03498 = 3*l.* 9*s.* $11\frac{1}{2}$*d.* per cent. the anſwer.

The work at length,

142,25 177,1010416
7 142,25
995,75) 34,8510416(,03|498 = 3*l.* 9*s.* $11\frac{1}{2}$*d.* per cent. the anſwer as above.
298725 20
497854 9,960
399300 12
985541 11,52
896175 4
893666 2,08
796600
97066

E. 2. At what rate per cent. will 260*l.* amount to 324*l.* 7*s.* in $5\frac{1}{2}$ years?

In this example $a=324{,}35$, $p=260$, and $t=5{,}5$.

Then per theorem, $\frac{324{,}35-260}{260\times5{,}5}=\frac{64{,}35}{1430}=$,045, or $4\frac{1}{2}$ per cent. the anſwer. The work at length,

260
5,5
1300
130
1430,0)64,350(,045, or $4\frac{1}{2}$ per cent. the anſ.
5720
7150
7150
....

E. 3.

E. 3. At what rate per cent. will 672*l.* 5*s.* amount to 847*l.* 17*s* 6*d.* in $5\frac{1}{2}$ years?

In this example, $a = 847{,}875$, $p = 672{,}25$, and $t = 5{,}5$.

Then per theorem $\dfrac{847{,}875 - 672{,}25}{672{,}25 \times 5{,}5} = \dfrac{175{,}62500}{3697{,}375} = {,}0475$, or $4\frac{3}{4}$ per cent. the anfwer.

LXI. *Of Annuities or Penfions in Arrears, &c.*

AN annuity is a yearly income arifing from money, &c. and is either paid for a term of years, or upon a life.

Annuities or penfions are faid to be in arrears, when they are payable or due either yearly, half-yearly, or quarterly, and are unpaid for any number of payments, and each payment at the it is due, fimple intereft is allowed, at a certain rate per cent.

When the annuity, rate, and time are given, to find the amount; that is, when U, R, T, are given, to find A.

Here U reprefents the annuity, penfion, or yearly rent, A, T and R, as before.

RULE. Multiply the time by itfelf, and that product by the annuity; from this fubtract the product of the annuity multiplied by the time, and divide the remainder by 2; multiply this quotient by the ratio of the rate per cent. and to this product add the time multiplied by the annuity, the fum will be the amount.

THEOREM 9. $\dfrac{ttu - tu}{2} \times r :+ tu = A$, the amount.

EXAMPLE 1. If an annuity of 50*l.* be forborne or unpaid 5 years, what will it amount to in that time, at 5 per cent.?

In this example, $u = 50$, $t = 5$, and $r = {,}05$.

$$\begin{array}{rl}
50 & = u. \\
5 & = t. \\
\hline
250 & = tu. \\
5 & \\
\hline
1250 & = ttu. \\
-250 & \\
\hline
2)1000 & \\
\hline
500 & \\
{,}05 & = r. \\
\hline
25{,}00 & \\
+250 & \\
\hline
\end{array}$$

Anfwer £.275,00

Note. :+ in the 9th theorem denotes that all its fucceeding terms muft be added to all its preceding terms.

E. 2.

E. 2. If a houſe be let upon a leaſe for 7 years, at 45*l.* per annum, I deſire to know the amount for that time, at 4 per cent. per annum?

In this example, $u=45$, $t=7$, and $r=,04$.

Then per theorem, $\frac{45\times7\times7-45\times7}{2}\times,04 : +45\times7=352,8=$ 352*l.* 16*s.* the anſwer.

At length thus,

$$\begin{array}{r} 45 \\ \underline{\times 7} \\ 315 \\ \underline{\times 7} \\ 2205 \\ \underline{-315} \\ \div 2)1890 \\ \overline{945} \\ \underline{\times,04} \\ 37,80 \\ \underline{+315} \end{array}$$

Anſwer $352,8=$ 352*l.* 16*s.* as above.

Note. When the annuity, &c. is to be paid half-yearly, or quarterly, then for half-yearly payments take half the ratio, half the annuity, &c. and twice the number of years; and for quartely payments, take a fourth part of the ratio, a fourth part of the annuity, and four times the number of years, and then proceed as before directed.

E. 3. If 125*l.* yearly rent, penſion, &c. be forborne, or unpaid 3 years, what will it amount to in that time, at 3 per cent. for each payment as it becomes due?

In this example, $u=125$, $t=3$, and $r=,03$.

Then per theorem, $\frac{125\times3\times3-125\times3}{2}\times,03 : +125\times3=386,25$ $=$ 386*l.* 5*s.* the anſwer.

E. 4. If a ſalary of 125*l.* payable every half-year, remain unpaid for 3 years, what will it amount to in that time, at 3 per cent. per annum?

In this example $u=62,5$, $t=6$, and $r=,015$, per Note.

Then per theorem, $\frac{62,5\times6\times6-62,5\times6}{2}\times,015 : +62,5\times6=$ $389,0625=$ 389*l.* 1*s.* 3*d.* the anſwer.

E. 5. If a ſalary of 125*l.* payable every quarter, was left unpaid for three years, what would it amount to in that time, at 3 per cent. per ann.?

In this example, $u=31,25$, $t=12$, $r=,0075$, per Note.

Then per theorem, $\frac{31,25\times12\times12-31,25\times12}{2}\times,0075 : +31,25$ $\times12=390,46875=$ 390*l.* 9*s.* $4\frac{1}{2}$*d.* the anſwer.

At

At length thus,

$$\begin{array}{r} 31,25 \\ \times\ 12 \\ \hline 375,00 \\ \times 12 \\ \hline 4500 \\ -375 \\ \hline 2)4125 \\ \hline 2062,5 \\ \times\ ,0075 \\ \hline 103125 \\ 144375 \\ \hline 15,16875 \\ +375 \\ \hline \end{array}$$

Anſwer 390,16875 = 390*l.* 9*s.* $4\frac{1}{2}$*d.* as before.

Note. It may be obſerved by comparing the anſwers of the three laſt examples, that the half-yearly payments are more advantageous than the yearly, and quarterly more than the half-yearly.

When the amount, rate and time are given, to find the annuity; or when A, R and T are given, to find U,

RULE. Multiply the ſquare of the time by the ratio, to which product add twice the time; and from that ſum ſubtract the time multiplied by the ratio, for a diviſor: multiply the amount of the annuity by 2 for a dividend; the quotient ariſing from this diviſion will be the annuity required.

EXAMPLE 1 If the amount of an annuity for 5 years, at 5 per cent, be 275*l.* what is the annuity?

$$\begin{array}{r} 5 \\ \times 5 \\ \hline 25 \\ \times ,05 \\ \hline 1,25 \\ +10 \\ \hline 11,25 \\ -25 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ 2 \\ \hline 10 \end{array} \qquad \begin{array}{r} 5 \\ ,05 \\ \hline ,25 \end{array} \qquad \begin{array}{r} 275 \\ 2 \\ \hline 550 \end{array}$$

11,00)550(50*l.* the Anſwer.
55
—
0

Note. When the payments are half-yearly, take 4 times the amount of the annuity; if quarterly, 8 times the amount, and proceed with the rate and time as before directed; ſee Page 248.

E. 2. If a salary payable yearly amounts to 352*l.* 16*s.* in 7 years, at 4 per cent. what is the salary?

In this example, $a = 352{,}8$, $t = 7$, and $r = {,}04$.

The work at length,

7	7	7	352,8
×7	,04	2	2
49	,28	14	705,6
×,04			
1,96			
+14			
15,96			
—,28			

15,68)705,60(45*l.* Answer.
6272
7840
7840
....

E. 3. The amount of a salary payable half-yearly, for five years, at 5 per cent. is 278*l.* 2*s.* 6*d.* what is the salary?

Theorem for half-yearly payments $\dfrac{4a}{ttr + 2t - tr} = U$, the annuity

In the above example, $a = 278{,}125$, $t = 10$, and $r = {,}025$.

The work at length,

10	10	10	278,125
10	2	,025	4
100	20	,250	1112500
,025			
500			
200			
2,500			
+20			
22,500			
—250			

22,250)1112,500(50*l.* Answer.
111250
.....0

E. 4.

E. 4. If the amount of an annuity, payable quarterly, be 1629*l.* 7*s.* 6*d.* for 6 years, at 3 per cent, what is the annuity?

24	24	24	1629,375
24	2	,0075	8
96	48	120	13035,000
48		168	
576		,1800	
,0075			
2880			
4032			
4,3200			
+48			
52,3200			
—,1800			
52,1400)13035,00(250*l.* Anſwer			

When the annuity, amount, and time are given, to find the rate of intereſt; or when U, A and T are given, to find R,

RULE. From twice the amount, ſubtract twice the annuity multiplied by the time, and divide the remainder by the ſquare of the time multiplied by the annuity, made leſs by the time multiplied by the annuity, and the quotient will be the ratio of the rate per cent.

Note. If the payments be half-yearly, the amount and annuity muſt each be multiplied by 4 (that is $4a - 4ut$ muſt be taken for a dividend); if quarterly, by 8 (viz. $8a - 8ut$); in every other reſpect proceed as before-mentioned. See page 248.

EXAMPLE 1. If an annuity of 50*l.* per annum amount to 275*l.* in 5 years, what is the rate per cent?

275	50	50	50
2	2	5	5
550	100	250	250
—500	5	5	
1000) 50,00	500	1250	
		—250	
		1000	

Anſwer ,05, or 5 per cent.

E. 2. If a ſalary of 250*l.* per annum, amounts to 1612*l.* 10*s.* in 6 years, what is the rate per cent?

		250
		6
	250	1500
	2	6
1612,5	500	9000
2	6	—1500
3225,0	3000	7500 Diviſor.
—3000		

7500) 225,00(,03, or 3 per cent, anſwer.
225,00
.....

E. 3. If a ſalary of 50*l.* per annum, payable half-yearly, amounts to 278*l.* 2*s.* 6*d.* in 5 years, what is the rate per cent?

			25
			10
278,125	25	25	250
4	4	10	10
1112,500	100	250	2500
—1000	10		—250
	1000		2250

2250) 112,500(,05, or 5 per cent, the anſwer.
11250
.....

E. 4.

E. 4. Suppose a pension of 250*l.* per annum, payable quarterly, amounts to 1629*l.* 7*s.* 6*d.* in 6 years, what is the rate per cent?

Theorem for quarterly payments, $\dfrac{8a-8ut}{utt-ut}=r.$

In this example, $a=1629,375$, $u=62,5$, and $t=24$.

Then per theorem, $\dfrac{1629,375\times 8-62,5\times 8\times 24.}{62,5\times 24\times 24-62,5\times 24}=\dfrac{1035,00}{34500}$

$=,03$, or 3 per cent. the answer.

When the annuity, amount, and rate are given, to find the time; or, when U, A and R, are given, to find T,

RULE. Divide 2 by the ratio, and subtract 1 from the quotient; then square the remainder, and divide the square by 4; to this quotient add twice the amount divided by the annuity, multiplied by the ratio, and extract the square root of the sum. Lastly, from this root subtract half the number found by dividing 2 by the ratio, and subtracting 1 from the quotient: this result will be the time.

THEOREM 12. First $\dfrac{2}{r}-1=x.$ Then $\sqrt{\dfrac{2a}{ur}+\dfrac{xx}{4}}-\dfrac{x}{2}=T$, the time.

EXAMPLE 1. In what time will an annuity of 50*l.* per annum amount to 275*l.* at 5 per cent?

In this example, $a=275$, $u=50$, and $r=,05$.

```
,05)2,00          39               50 = u.
                  39              ,05 = r.
    40           ----            -------
   — 1           351             2,50 = ur.
   ----          117
    39 = x.     -----
                4)1521
                -----
275 = a.        380,25 = xx ÷ 4    2,5|0)55|0,00(220
  2            +220                      50
 ----          -----                    ----
550 = 2a.       600,25(24,5               50
                4       19,5              50
               ----    -----             ----
              44)200    5,0 Years, answ.   0.
                 176
               -----
             485) 2425
                  2425
                 -----
```

If

If payments are half-yearly, the time will be equal to the number of half years; if quarterly, the time will be equal to the number of quarterly payments; with the ratio and annuity proceed as before.

E. 2. If an annuity of 250*l.* per annum, payable half-yearly, amounts to 1623*l.* 15*s.* at 3 per cent. what time was the payment forborne?

In this example, $a = 1623{,}75$, $u = 125$, and $r = {,}015$.

Then per theorem first, $\frac{2}{,015} - 1 = 132{,}8 = x$.

$$\text{Then } \sqrt{\frac{1623{,}75 \times 2}{125 \times {,}015} + \frac{132{,}8 \times 132{,}8}{4}} : - \frac{132{,}8}{2} =$$

$$\sqrt{\frac{3247{,}5}{1{,}875} + \frac{17507{,}68}{4}} : - 66{,}1 = \sqrt{6108{,}92} - 66{,}1 = 78.$$

1—66,1=12 Half years, or 6 years, the time required.

LXII. *Of the Present Worth of Annuities, &c.*

WHEN the annuity, rate and time are given, to find the present worth; or when U, T and R are given, to find P. Here P represents the present worth; U, T and R, as before.

RULE. Square the time, and multiply the product by the ratio; to this add twice the time; then from the sum subtract the time multiplied by the ratio, and let the remainder stand for a dividend. Next, multiply twice the ratio by the time, and add 2 to the product for a divisor; lastly, divide the one by the other, and multiply the quotient by the annuity, pension, &c. and the product will be the present worth.

THEOREM 13. $\frac{ttr - tr + 2t}{2tr + 2} : \times u = P.$

The same is to be observed here, for half-yearly and quartely payments, as before-mentioned.

EXAMPLE I. What is the present worth of 200*l.* per annum, to continue 6 years, at 5 per cent?

$6 = t.$	6	$6 = t.$
6	2	$,05 = r.$
36	$12 = 2t.$	$,30 = tr.$
,05		2
1,80		,60
—,30		+ 2,
1,50		Divisor $2{,}60 = tr \times 2 + 2$
+ 12		

2,60)13,50(5,1923
200

1038,4600 = 1038*l.* 9*s.* 2¼*d.* Answer.

E. 2.

E. 2. What is the present worth of a house, whose yearly rent is 75*l.* per annum, to continue 9 years, at 6 per cent?

9	9	9	9
9	,06	2	,06
81	,54	18	,54
,06			2
4,86			1,08
—,54			+ 2
4,32			3,08 Divisor
+ 18,			

3,08)22,32(7,24675 × 75 = 543,50625 = 543*l.* 10*s.* $1\frac{1}{2}$*d.* Ans.

E. 3. What is the present worth of 40*l.* per annum, payable half-yearly, at 5 per cent. and to continue 6 years?

12	12	12	12
12	,025	2	,025
144	,300	24	,300
,025			2
720			,600
288			+ 2
3,600			2,6 Divisor
—,300			
3,300			
+ 24			

2,6)27,30(10,5 × 20 = 210*l.* the answer.

E. 4. What is the present worth of 50*l.* per annum, payable quarterly, at 5 per cent. to continue 6 years?

24	24	24	24
24	,0125	2	,0125
96	120	48	120
48	48		48
576	24		24
,0125	,3000		,3000
2880			× 2
1152			,6
576			+ 2
7,2000			2,6 Divisor
—,3000			
6,9000			
+ 48			

2,6)54,9000(21,114 × 12,5 = 263,9250 = 263*l.* 18*s.* 6*d.* Ans.

When

When the present worth, time, and ratio are given, to find the annuity, rent, &c. Or when P T and R, are given, to find U,

RULE. Multiply the ratio by the time, and add 1 to the product for a dividend; then multiply the square of the time by the ratio; subtract the product of the time and ratio, and to the remainder add twice the time for a divisor; lastly, multiply the quotient of these two numbers by twice the present worth, the product will be the annuity, &c.

THEOREM 14. $\dfrac{tr+1}{ttr-tr+2t} : \times 2p = U$, the annuity, &c.

EXAMPLE 1. There is an annuity of 6 years to come, I desire to know the yearly value, when the present worth at 3 per cent. is 1366*l.* 10*s.* 6*d.*

In this example, $t=6$, $r=,03$, and $p=1366,525$, to find U.

Then per theorem, $\dfrac{6\times,03+1}{6\times 6\times,03-6\times,03+6\times 2} : \times$ $1366,525\times 2 = \dfrac{1,18}{12,9}\times 2733,05 = ,0914728\times 1755,05 =$ 250*l.* the answer.

Note. If the payments be half-yearly, take $\frac{1}{2}$ of the ratio, and multiply by four times the present worth; if quarterly $\frac{1}{4}$ of the ratio, and multiply by eight times the present worth, and proceed with the time and ratio as before.

E. 2. There is an annuity payable half-yearly, for 6 years to come; what is the yearly income, when the present worth, at 3 per cent. is 1376*l.* 5*s*?

In this example, $t=12$, $r=,015$, and $p=1376,25$.

Then per theorem, $\dfrac{12\times,015+1}{12\times 12\times,015-12\times,015+12\times 2} : \times$ $1376,25\times 4 = 250$*l.* 0*s.* 8½*d.* the answer.

E. 3. The present worth of an annuity, payable quarterly, for 6 years to come, at 3 per cent. is 1380*l.* 17*s.* 6*d.* I desire to know the annuity?

In this example, $t=24$, $r=,0075$, and $p=1380,875$.

Then per theorem, $\dfrac{24\times,0075+1}{24\times 24\times,0075-24\times,0075+24\times 2}$ $\times 1380,875\times 8 = \dfrac{1,18}{52,14}\times 11047 = 250$*l.* the answer.

When the annuity, present worth and time are given, to find the ratio; or when U, P and T are given, to find R, the ratio,

RULE.

RULE. Multiply the annuity by twice the time, from which product subtract twice the present worth, and reserve the remainder for a dividend; next multiply twice the present worth by the time, and reserve this product; again, multiply the annuity by the square of the time, from which subtract the annuity multiplied by the time, and let this remainder taken from the product last reserved, be your divisor. Lastly, the quotient arising from the division of these two numbers will be the ratio required.

THEOREM 15. $\frac{2ut-p\times 2}{2pt-ut-utt}=R.$

EXAMPLE. At what rate per cent. will an annuity of 30*l.* 10*s.* per annum, to continue 10 years, produce the present worth of 250*l.* 6*s.* 8*d.*?

$30,5=u.$	Then $2261,6)109,3$ } See division of repetends.
$\times 10=t.$	109
$305=tu.$	$2035,5)8840(,04343=R=$ 4*l.* 7*s.* 11*d.* ans.
$\times 10=t.$	8142
$3050=ttu.$	698
$-305=tu.$	610
$2745=ttu-tu.$	88
$-5006,6=2pt.$	81
Divis. $2261,6$	7
	6
	1
And $500,6=2p.$	
Also $610,0=2tu.$	
Dividend $109,3=2p-2tu.$	

When the payments are half-yearly, or quarterly, proceed with the annuity and time as before directed, and the quotient will be the answer (i. e.) if for half-yearly, the quotient will be half the ratio, and if for quarterly, a fourth part of the ratio.

When the annuity, present worth, and ratio are given, to find the time; or when U, P and R are given, to find T.

THEOREM 16. $\frac{2p}{ru}+\frac{xx}{4}+\frac{1}{2}x=T.$

EXAMPLE. In what time will 7*l.* per annum pay a debt of 120*l.* 8*s.* at 6*l.* per cent.?

In this example, $u=7$, $r=,06$, and $p=120,4$, to find T.

First - - - - - 240,8 = $2p$.

And - - - - - - 34,4 = $\frac{2p}{u}$.

From which take - $\underline{-33,3}$ = $\frac{2}{R}$.

To the remainder - 1,06 = $\frac{2}{r} - \frac{2p}{u}$

Add unity - - - 1 = $+1$

The sum is - - - 2,06 = $\frac{2}{r} - \frac{2p}{u} + 1 = x$ by substitution.

Then - - - - - - 1,03 = $\frac{1}{2}x$.

And - - - - - 1,067 = $\frac{1}{4}xx$.

Again - - - - - - ,42 = ru.

And - - - - - - - 573,3 = $\frac{2p}{ru}$.

Then - - - - - - 574,4 = $\frac{2p}{ru} + \frac{xx}{4}$

Square root of that - 23,96 = $\sqrt{\frac{2p}{ru} + \frac{xx}{4}}$.

To which add - - - 1,03 = $\frac{1}{2}x$.

The sum is - - - 24,99 = 25 Nearly the time, answer.

$$\left\{\begin{array}{r} 1,03 = \frac{1}{4}x. \\ \underline{103 = \frac{1}{2}x.} \\ 9)309 \\ 344 \\ \underline{10333} \\ 1,0677 = \frac{1}{4}xx, \end{array}\right.$$

When the payments are half-yearly, or quarterly, proceed with the annuity and ratio as before, the quotient will be the number of payments.

A TABLE, Shewing the INTEREST of any SUM of MONEY, from a MILLION to a POUND, for any Number of Days, at any rate of INTEREST.

Sum.	£.	s.	d.	q.	Sum.	£.	s.	d	q.	Sum.	£.	s.	d.	q.
1000000	2739	14	6	0,99	10000	27	7	11	1,37	100	0	5	5	3,15
900000	2465	15	0	3,29	9000	24	13	1	3,23	90	0	4	11	5,71
800000	2191	15	7	1,59	8000	21	18	4	1,10	80	0	4	4	2,41
700000	1917	16	1	3,89	7000	19	3	6	2,96	70	0	3	10	0,11
600000	1643	16	8	2,19	6000	18	8	9	0,82	60	0	3	3	1,81
500000	1369	17	3	0,49	5000	13	13	11	2,58	50	0	2	8	3,51
400000	1095	17	9	2,95	4000	10	19	2	0,55	40	0	2	2	1,21
300000	821	18	4	1,09	3000	8	4	4	2,41	30	0	1	7	2,90
200000	547	18	10	3,40	2000	5	9	7	0,27	20	0	1	1	0,60
100000	273	19	5	1,70	1000	2	14	9	2,14	10	0	0	6	2,30
90000	246	11	6	0,32	900	2	9	3	2,12	9	0	0	5	3,67
80000	219	3	6	0,96	800	2	3	10	0,11	8	0	0	5	1,40
70000	191	15	7	1,59	700	1	18	4	1,10	7	0	0	4	2,41
60000	164	7	8	0,22	600	1	12	10	2,80	6	0	0	3	3,76
50000	136	19	8	2,85	500	1	7	4	3,70	5	0	0	3	1,15
40000	109	11	9	1,48	400	1	1	11	0,50	4	0	0	2	2,52
30000	82	3	10	0,11	300	0	16	5	1,40	3	0	0	1	3,80
20000	54	15	10	2,74	200	0	10	11	2,30	2	0	0	1	1,26
10000	27	7	11	1,37	100	0	5	5	3,15	1	0	0	0	2,63

Note. The decimals in the foregoing table are 100th parts of a farthing

The defign of tables of intereft (both fimple and compound) is eafe and expidition in practical calculations; and the rules expreffed in words for anfwering queftions of intereft are tedious and intricate, and the reafon not eafily underftood; the operations themfelves are, for the moft part, very laborious; for which reafon I have thought proper to infert this and the following tables of fimple intereft, by the help of which all queftions relating thereto may be quickly refolved, that fall within the compafs of the tables.

Ufe of the preceding Table.

RULE. Multiply the fum by the number of days, and the product thereof by the rate of intereft per cent. then cut off the two laft figures to the right-hand, and enter the table with what remains to the left, againft which numbers collected, you have the intereft for the given fum.

EXAMPLE. What is the intereft of 100*l.* at 5 per cent for 365 days?

Number of days 365
× 100*l.* Sum
36500
× 5 Rate per cent.
1825|00

		£.	s.	d.	q. pts.	
Then, in the table, againft	1000 is	2	14	9	2,14	
	800	2	3	10	0,11	
	20	0	1	1	0,60	
	5	0	0	3	1,15	
	1825 *l.*	5	0	0	0,00	Anfwer.

And in the fame way may the intereft of any other given number of pounds be found for any given number of days.

Queftions where principal, annuity, amount, &c. are concerned, are likewife to be folved by the tables; for there are fimilar numbers in the tables analogous to thofe given; and therefore having three terms given, a proportion or analogy muft be made by the rule of three, between the numbers given in the queftion, and thofe in the proper table for the fame rate and time, in order to find the fourth term, which is either the thing itfelf which is fought, or it will fhew it by the table. And as 1 is commonly a term in the proportion, the queftion will generally be folved by multiplication or divifion.

If any thing is wanting to make the proportion, or to carry on the procefs, it muft be found from what is given in the queftion.

 TABLE

TABLE I. Of the AMOUNT of 1£. for YEARS, at SIMPLE INTEREST.

Years	3 *Per Cent.*	3½ *Per Cent.*	4 *Per Cent.*	4½ *Per Cent.*	5 *Per Cent.*
1	1,03	1,035	1,04	1,045	1,05
2	1,06	1,070	1,08	1,090	1,10
3	1,09	1,105	1,12	1,135	1,15
4	1,12	1,140	1,16	1,180	1,20
5	1,15	1,175	1,20	1,225	1,25
6	1,18	1,210	1,24	1,270	1,30
7	1,21	1,245	1,28	1,315	1,35
8	1,24	1,280	1,32	1,360	1,40
9	1,27	1,315	1,36	1,405	1,45
10	1,30	1,350	1,40	1,450	1,50
11	1,33	1,385	1,44	1,495	1,55
12	1,36	1,420	1,48	1,540	1,60
13	1,39	1,455	1,52	1,585	1,65
14	1,42	1,490	1,56	1,630	1,70
15	1,45	1,525	1,60	1,675	1,75
16	1,48	1,560	1,64	1,720	1,80
17	1,51	1,595	1,68	1,765	1,85
18	1,54	1,630	1,72	1,810	1,90
19	1,57	1,665	1,76	1,855	1,95
20	1,60	1,700	1,80	1,900	2,00
21	1,63	1,735	1,84	1,945	2,05
22	1,66	1,770	1,88	1,990	2,10
23	1,69	1,805	1,92	2,035	2,15
24	1,72	1,840	1,96	2,080	2,20
25	1,75	1,875	2,00	2,125	2,25
26	1,78	1,910	2,04	2,170	2,30
27	1,81	1,945	2,08	2,215	2,35
28	1,84	1,980	2,12	2,260	2,40
29	1,87	2,015	2,16	2,305	2,45
30	1,90	2,050	2,20	2,350	2,50
31	1,93	2,085	2,24	2,395	2,55
32	1,96	2,120	2,28	2,440	2,60
33	1,99	2,155	2,32	2,485	2,65
34	2,02	2,190	2,36	2,530	2,70
35	2,05	2,225	2,40	2,575	2,75
36	2,08	2,260	2,44	2,620	2,80
37	2,11	2,295	2,48	2,665	2,85
38	2,14	2,330	2,52	2,710	2,90
39	2,17	2,365	2,56	2,755	2,95
40	2,20	2,400	2,60	2,800	3,00
41	2,23	2,435	2,64	2,845	3,05
42	2,26	2,470	2,68	2,890	3,10
43	2,29	2,505	2,72	2,935	3,15
44	2,32	2,540	2,76	2,980	3,20
45	2,35	2,575	2,80	3,025	3,25
50	2,50	2,750	3,00	3,250	3,50

Use of the foregoing Table.

EXAMPLE 1. If 250*l.* be put to interest, what will it amount to in 21 years, at 4 per cent?

First by table 1, the amount of 1*l.* for 21 years, at 4 per cent. is 1,84; then say,

	Prin.		*Amount.*		*Prin.*
As	1	:	1,84	::	250
			250		
			9200		
			368		

Answer £.460,00 the amount required.

E. 2. What principal, put out for 21 years. will amount to 460*l*, at 4 per cent?

First, by table 1, the amount of 1*l.* is 1,84, for the given time and rate; then say,

	Amount.		*Prin.*		*Amount.*
As	1,84	:	1	::	460

1,84)460(250*l.* the principal required.
368
———
920
920
———
0

E. 3. At what rate of simple interest will 250*l.* amount to 460*l.* in 21 years?

	Prin.		*Amount.*		*Prin.*
By table 1st say, If	250	:	460	::	1
			1		

250)460(1,84 the amount of 1*l.* which being sought for against 21 years, you find in the column under 4 per cent. the rate of int. required.
250
———
2100
2000
———
1000
1000
———
0

TABLE

TABLE 2. The AMOUNT of 1£. ANNUITY for YEARS, at SIMPLE INTEREST.

Years.	3 *Per Cent.*	$3\frac{1}{2}$ *Per Cent.*	4 *Per Cent.*	$4\frac{1}{2}$ *Per Cent.*	5 *Per Cent.*
1	1,00	1,000	1,00	1,000	1,00
2	2,03	2,035	2,04	2,045	2,05
3	3,09	3,105	3,12	3,135	3,15
4	4,18	4,210	4,24	4,270	4,30
5	5,30	5,350	5,40	5,450	5,50
6	6,45	6,525	6,60	6,675	6,75
7	7,63	7,735	7,84	7,945	8,05
8	8,84	8,980	9,12	9,260	9,40
9	10,08	10,260	10,44	10,620	10,80
10	11,35	11,575	11,80	12,025	12,25
11	12,65	12,925	13,20	13,475	13,75
12	13,98	14,310	14,64	14,970	15,30
13	15,34	15,730	16,12	16,510	16,90
14	16,73	17,185	17,64	18,095	18,55
15	18,15	18,675	19,20	19,725	20,25
16	19,60	20,200	20,80	21,400	22,00
17	21,08	21,760	22,44	23,120	23,80
18	22,59	23,355	24,12	24,885	25,65
19	24,13	24,985	25,84	26,695	27,55
20	25,70	26,650	27,60	28,550	29,50
21	27,30	28,350	29,40	30,450	31,50
22	28,93	30,085	31,24	32,395	33,55
23	30,59	31,855	33,12	34,385	35,65
24	32,28	33,660	35,04	36,420	37,80
25	34,00	35,500	37,00	38,500	40,00
26	35,75	37,375	39,00	40,625	42,25
27	37,53	39,285	41,04	42,795	44,55
28	39,34	41,230	43,12	45,010	46,90
29	41,18	43,210	45,24	47,270	49,30
30	43,05	45,225	47,40	49,575	51.75
31	44,95	47,275	49,60	51,925	54,25
32	46,88	49,360	51,84	54,320	56,80
33	48,84	51,480	54,12	56,760	59,40
34	50,83	53,635	56,44	59,245	62,05
35	52,85	55,825	58,80	61,775	64,75
36	54,90	58,050	61,20	64,350	67,50
37	56,98	60,310	63,64	66,970	70,30
38	59,09	62,605	66,12	69,635	73,15
39	61,23	64,935	68,64	72,345	76,05
40	63,40	67,300	71,20	75,100	79,00
45	74,70	79,650	84,60	89,550	94,50
50	86,75	92,875	99,00	105,125	111,25
51	89,25	95,625	102,00	108,375	114,75
52	91,78	98,410	105,04	111,670	118,30
53	94,34	101,230	108,12	115,010	121,90
54	96,93	104,085	111,24	118,395	125,55

Use of the foregoing Table.

EXAMPLE I. If 320*l.* yearly rent be forborne for 12 years, what will be in arrear at that time, at $4\frac{1}{2}$ per cent?

By table 2. the amount of 1*l.* annuity for 12 years, is 14,97;

	Ann.		*Amount.*		*Ann.*
Then say,	If 1	:	14,97	::	320
			320		
			29940		
			4491		
		Answer	4790,4 = 4790*l.* 8*s.* the arrear:		

E. 2. In what time will 320*l.* yearly rent amount to 4790,4*l.* at $4\frac{1}{2}$ per cent?

	Rent.		*Amount.*		*Rent.*
As	320	:	4790,4	::	1
			1		

320)4790,4(14,97 the amount of 1*l.* annuity, which being found in the column under $4\frac{1}{2}$ per cent. in table 2d, stands over-against 12 years, the time sought.

E. 3. If 640*l.* yearly rent be forborne 24 years, what will be in arrear at that time, at 5 per cent?

The Amount of 1*l.* annuity for 24 years, in the table, is 37,8;

	Ann.		*Amount.*		*Ann.*
Then say,	If 1	:	37,8	::	640
			640		
			1512		
			2268		
		Answer	24192,0 the arrear sought.		

E. 4. In what time will 640*l.* yearly rent, amount to 24192*l.* at cent?

	Rent.		*Amount.*		*Rent.*
As	640	:	24192	::	1
			1		

640)24192(37,8 the amount of 1*l.* annuity; which being found in the column under 5 per cent. in the table, stand over-against 24 years, the time sought.

TABLE

TABLE. 3. The DISCOUNT of 1£. for DAYS, at the rate of 3, 3½, 4, 4½, and 5 per cent. per annum.

Days	3 Per Cent.	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	,0000822	,0000959	,0001096	,0001233	,0001370
2	,0001644	,0001917	,0002191	,0002465	,0002739
3	,0002465	,0002876	,0003287	,0003679	,0004108
4	,0003287	,0003834	,0004382	,0004929	,0005477
5	,0004108	,0004792	,0005477	,0006161	,0006845
6	,0004929	,0005750	,0006571	,0007392	,0008212
7	,0005750	,0006708	,0007665	,0008623	,0009580
8	,0006571	,0007666	,0008759	,0009853	,0010947
9	,0007392	,0008623	,0009853	,0011084	,0012314
10	,0008212	,0009580	,0010947	,0012314	,0013680
20	,0016411	,0019141	,0021870	,0024597	,0027322
30	,0024597	,0028685	,0032769	,0036850	,0040928
40	,0032769	,0038210	,0043644	,0049073	,0054496
50	,0040928	,0047716	,0054496	,0061266	,0068027
60	,0049073	,0057205	,0065234	,0073429	,0081522
70	,0057205	,0066676	,0076128	,0085563	,0004980
80	,0065324	,0076128	,0086909	,0097667	,0108401
90	,0073429	,0085563	,0097667	,0109741	,0121786
100	,0081522	,0094980	,0108401	,0121786	,0135135
110	,0089601	,0104379	,0119112	,0133802	,0148448
120	,0097667	,0113760	,0129800	,0145788	,0161725
130	,0105720	,0123123	,0140465	,0157746	,0174966
140	,0113760	,0132468	,0151006	,0169674	,0188172
150	,0121786	,0141796	,0161725	,0181574	,0201342
160	,0129780	,0151106	,0172321	,0193444	,0214477
170	,0137801	,0160399	,0182894	,0205286	,0227577
180	,0145788	,0169674	,0193444	,0217100	,0240642
190	,0153763	,0178932	,0203972	,0228885	,0253672
200	,0161725	,0188172	,0214477	,0240642	,0266667
210	,0169674	,0197395	,0224960	,0252370	,0279627
220	,0177610	,0206601	,0235420	,0264070	,0292553
230	,0185534	,0215789	,0245858	,0275743	,0305445
240	,0193444	,0224959	,0256273	,0287387	,0318302
250	,0201342	,0234114	,0266667	,0299003	,0331126
260	,0209227	,0243251	,0277038	,0310592	,0343915
270	,0217100	,0252370	,0287387	,0322153	,0356671
280	,0224960	,0261473	,0297714	,0333686	,0369393
290	,0232807	,0270558	,0308019	,0345192	,0382082
300	,0240642	,0279627	,0318302	,0356671	,0394737
310	,0248464	,0288679	,0328564	,0368122	,0407352
320	,0256273	,0297714	,0338804	,0379547	,0419948
330	,0264070	,0306732	,0349022	,0390444	,0432503
340	,0271855	,0315734	,0359218	,0402314	,0445026
350	,0279627	,0324718	,0369393	,0413657	,0457516
365	,0291262	,0338164	,0384615	,0430622	,0476191

The Ufe of Table 3, *of Difcount.*

Example 1. What is the difcount of 83*l.* 10*s.* for 200 days, at 4 per cent? In the table, under 4 per cent. and againft 200 days, is ,0214477
× 83,5 Principal fum

1072385
643431
1715816

Anfwer 1,79088295 = £. 1 s. 15 d. $9\frac{3}{4}$

E. 2. What is the difcount of 100*l.* for 1 year, at 5 per cent?

In the table under 5 per cent. and againft 365 days, is - - } ,047619
Which multiplied by the fum - 100

Anfwer 4,7619 = £. 4 s. 15 d. $2\frac{3}{4}$
Now the intereft of 100*l.* for 1 year, at 5 per cent. is 5 0 0

The difference of Difcount and Intereft is - 0 4 $9\frac{1}{4}$

Whence (as I obferved in fection XX.) it is evident, he who allows intereft for difcount, wrongs himfelf confiderably.

E. 3. What is the difcount of 8462*l.* at 3 per cent. for a year?

The difcount of 1*l.* for 365 days, at 3 per cent. in the table, is - - - - - - } ,0291262
Which multiplied by the principal fum - 8462

582524
1747572
1165048
2330096

Anfwer 246,4659044 = £. 246 s. 9 d. $3\frac{3}{4}$

Note. This table of difcount is perfectly true for all the days expreffed therein, and is fufficiently exact for any ufe. None but a table of difcount for every day, can be perfect; becaufe every day's difcount differs, being ftill lefs, as the number of days increafe.

☞ If the number of days be a mixed one, refolve them into pure numbers.

A new and concise method to cast up the Interest of any sum of money, at any rate per cent.

	£.		
For	3	per cent. divide by	12166
	$3\frac{1}{2}$		10428
	4		9125
	$4\frac{1}{2}$		8111
	5		7300
	6		6083
	7		5214

RULE 1. Reduce the principal money into pence.

2. Multiply those pence, by the days the money is at interest.

3. Divide the last product by the number in the Table standing against the rate of interest, and the quotient is the interest in pence.

EXAMPLE 1. What is the interest of 342*l.* 12*s.* 6*d.* for 40 days at 7*l.* per cent?

£.	s.	d.
342	12	6
20		
6852		
12		
82230		
40		

7*l.* per cent. ÷ by 5214)3289200 (630 d. = 2 £. 12 s. 6 d. Answer.

31284
16080
15642
4380

E. 2. What is the interest of 100*l.* for 365 days at $4\frac{1}{2}$ per cent?

£.
100
20
2000
12
24000
365
120000
144000
72000

$4\frac{1}{2}$*l.* per cent. ÷ by 8111)8760000 (1080 d. = 4 £. 10 s. Answer.

E. 3. What is the intereſt of 300*l.* for 100 days at 3 per cent?

$$
\begin{array}{r}
\pounds. \\
300 \\
20 \\
\hline
6000 \\
12 \\
\hline
72000 \\
100 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad d. \quad \pounds. \;\; s. \;\; d. \\
3l.\ \text{per cent} \div \text{by } 12166)\ 7200000\ (\ 591 = 2 \;\; 9 \;\; 3\tfrac{3}{4}\ \text{Anſwer.} \\
\qquad\qquad\qquad\qquad\quad 60830 \\
\qquad\qquad\qquad\qquad\quad 111700 \\
\qquad\qquad\qquad\qquad\quad 109494 \\
\qquad\qquad\qquad\qquad\quad\;\; 22060 \\
\qquad\qquad\qquad\qquad\quad\;\; 12166 \\
\qquad\qquad\qquad\qquad\quad\;\;\; 9894
\end{array}
$$

E. 4. What is the ietereſt of 10000*l.* for 48 days at 5 per Cent?

Firſt 10000*l.* = 2400000*d.* × 48 days = 115200000 ÷ 7300 = 15300*d.* = 63*l.* 15*s.* 0¾*d.* Anſwer.

Note. The conſtruction of the preceeding Table is thus,
As 100 : 365 :: 5 : or any other rate, to the fourth term;
Or as 100 : 73 :: 1, that is $\frac{365}{5}=73$, and $\frac{5}{5}=1$. Hence the ſecond and third terms will always admit of the ſame abbreviations.

See *Note*, Section XVIII. Page 124 in ſimple intereſt.

LXIII. COMPOUND INTEREST.

WHAT compound intereſt is, I have already ſhewn in ſection XIX. Page 125.

RULE. Multiply the principal by the amount of 1*l.* at the given rate per cent. as often as there are numbers of years; the laſt product is the amount, from which if you ſubtract the principal, the remainder will be the intereſt.

EXAMPLE 1. What is the compound interest of 221*l.* forborne 3 years, at 5 per cent. per annum?

	£.
Principal - - -	221
	1,05
	1105
	221
First year's amount	232,05
	1,05
	116025
	23205
Second year's amount	243,6525
	1,05
	12182625
	2436525
Third year's amount	255,835125
Principal -	221
Interest - - -	34,835125 =

(35*l.* 16*s.* 8¼*d.* Answer

E. 2. What is the compound interest of 320*l.* forborne 4 years, at 5 per cent. per annum?

Principal - - -	320
	1,05
	1600
	320
First year's amount -	336,00
	1,05
	1680
	336
2d year's amount	352,80
	1,05
	17640
	3528
3d year's amount	370,440
	1,05
	185220
	37044
4th year's amt.	388,9620
Principal	320
Interest -	68,9620 = 68*l.*

(19*s.* 2¾*d.* Answer.

The amount of 1*l.* for one year, at any given rate, may be found by the following proportion, thus:

As { 100 : 105 :: 1 : 1,05 the ratio at 5 per cent.
  { 100 : 106 :: 1 : 1,06 the ratio at 6 per cent, &c.

E. 3. What is the amount of 500*l.* for 6 years, at 5 per cent. per annum?

Principal - -	500
	1,05
	2500
	500
First year's amount -	525,00
	1,05
	2625
	525
Second year's amount	551,25
	1,05
	275625
	55125
3d year's amount	578,8125
	1,05
	28940625
	5788125
4th year's amount	607,753125
	607,753125
	1,05
	3038765625
	607753125
5th year's amt.	638,14078125
	1,05
	319070390625
	63814078125
6th y. amt.	670,0478203125 =

(670*l.* 0*s.* 11¼*d.* Ans.

These

These examples being so easy to be understood, I shall omit giving any more, and proceed to shew the construction and use of a set of tables, which are absolutely necessary for those that do not understand logarithms or algebra; and as nothing shall be wanting in this system to make it comple, I have inserted the six following Tables for the purposes of compound interest, and continued each Table for 45 years; whereby any question of compound interest for the rates contained therein may be expeditiously answered.

The Construction of the following Tables.

Table 1st is constructed by a continual multiplication of the amount of 1*l.* for a day, being the root of its amount for a year, extracted to the 365th power.

The amount of 1*l.* for a day, at 5 per cent. being 1,0001336, then 1,0001336 × 1,0001336 = 1,0002673, the amount for 2 days; and 1,0001336 × 1,0001336 × 1,0001336 = 1,0004011, the amount of 1*l.* at 5 per cent. for 3 days, compound interest, &c.

Table 2d is constructed by involving the amount of 1*l.* for a year, to the power of the number of years; thus, the amount of 1*l.* for 2 years, at 5 per cent. will be 1,05 × 1,05 = 1,1025; 1,05 × 1,05 × 1,05 = 1,157625 the amount of 1*l.* for 3 years, at 5 per cent, &c.

Table 3d is constructed thus, 1 ÷ 1,05 = ,952381, first year's present worth at 5 per cent. and ,952381 ÷ 1,05 = ,9070295, the second year's present worth; and ,90703 ÷ 1,05 = ,8638376, the third year's present worth; and after the same method are all the other years in the table found, to 45 years, inclusive.

The 4th Table is constructed thus, take the first year's amount, which is 1*l.* and multiply it by 1,05 + 1 = 2,05 = the second year's amount, which also multiplied by 1,05 + 1 = 3,1525 = third year's amount, &c.

The 5th Table is constructed thus, divide 1 by 1,05 = ,95238, the present worth for the first year, which ÷ 1,05 = ,90703, added to the first year's present worth = 1,85941, the second year's present worth; again ,90703 ÷ 1,05 and quotient, added to 1,85941 = 2,72324 = the third year's present worth, &c.

The 6th table is constructed thus, find the present worth of 1*l.* par annum in the 5th table, at the assigned rate and time, and divide unity or 1 thereby, the quotient will be the annuity that 1*l.* will purchase, at the same rate, for the same time.

TABLE. 1. The AMOUNT of ONE POUND for DAYS.

Days.	3 Per Cent.	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	1,0000809	1,0000942	1,0001074	1,0001206	1,0001336
2	1,0001619	1,0001885	1,0002149	1,0002412	1,0002673
3	1,0002429	1,0002827	1,0003224	1,0003618	1,0004011
4	1,0003240	1,0003770	1,0004299	1,0004834	1,0005348
5	1,0004050	1,0004713	1,0005374	1,0006031	1,0006685
6	1,0004860	1,0005656	1,0006449	1,0007238	1,0008023
7	1,0005670	1,0006600	1,0007524	1,0008445	1,0009361
8	1,0006480	1,0007542	1,0008600	1,0009652	1,0010699
9	1,0007291	1,0008486	1,0009675	1,0010859	1,0012037
10	1,0008101	1,0009429	1,0010751	1,0012066	1,0013376
20	1,0016209	1,0018867	1,0021513	1,0024148	1,0026770
30	1,0024324	1,0028315	1,0032288	1,0036243	1,0040182
40	1,0032445	1,0037771	1,0043074	1,0048354	1,0053611
50	1,0040573	1,0047236	1,0053871	1,0060479	1,0067059
60	1,0048708	1,0056710	1,0064680	1,0072618	1,0080525
70	1,0056849	1,0066193	1,0075501	1,0084773	1,0094009
80	1,0064996	1,0075685	1,0086333	1,0096942	1,0107511
90	1,0073151	1,0085186	1,0097177	1,0109125	1,0121031
100	1,0081311	1,0094696	1,0108033	1,0121324	1,0134569
110	1,0089479	1,0104214	1,0118900	1,0133537	1,0148125
120	1,0097653	1,0113742	1,0129779	1,0145765	1,0161699
130	1,0105834	1,0123279	1,0140670	1,0158007	1,0175291
140	1,0114021	1,0132825	1,0151572	1,0170265	1,0188902
150	1,0122215	1,0142379	1,0162487	1,0182537	1,0202531
160	1,0130415	1,0151943	1,0173412	1,0194824	1,0216178
170	1,0138623	1,0161516	1,0184350	1,0207126	1,0229843
180	1,0146837	1,0171098	1,0195299	1,0219442	1,0243527
190	1,0155057	1,0180689	1,0206261	1,0231774	1,0257228
200	1,0163284	1,0190288	1,0217233	1,0244120	1,0270949
210	1,0171518	1,0199897	1,0228218	1,0256481	1,0284687
220	1,0179759	1,0209515	1,0239215	1,0268858	1,0298444
230	1,0188006	1,0219142	1,0250233	1,0281249	1,0312219
240	1,0196260	1,0228778	1,0261243	1,0293655	1,0326013
250	1,0204520	1,0238424	1,0272275	1,0306076	1,0339825
260	1,0212788	1,0248078	1,0283319	1,0318512	1,0353656
270	1,0221062	1,0257741	1,0294375	1,0330963	1,0367505
280	1,0229342	1,0267414	1,0305443	1,0343429	1,0381373
290	1,0237630	1,0277096	1,0316522	1,0355910	1,0395259
300	1,0245924	1,0286786	1,0327614	1,0368406	1,0409164
310	1,0254225	1,0296486	1,0338717	1,0383917	1,0423087
320	1,0262532	1,0306195	1,0349832	1,0393444	1,0437029
330	1,0270847	1,0315914	1,0360960	1,0405985	1,0450990
340	1,0279168	1,0325641	1,0372099	1,0418542	1,0464969
350	1,0287495	1,0335378	1,0383250	1,0431114	1,0478967
365	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000

TABLE 2. The AMOUNT of ONE POUND for YEARS.

Yrs.	3 Per Cent.	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	1,0609000	1,0712250	1,0816000	1,0920250	1,1025000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576250
4	1,1255088	1,1475230	1,1698586	1,1925186	1,2155063
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762816
6	1,1940523	1,2292553	1,2653190	1,3022601	1,3400956
7	1,2298738	1,2722792	1,3159318	1,3608618	1,4071064
8	1,2667700	1,3168098	1,3685691	1,4221006	1,4774554
9	1,3047731	1,3628973	1,4233118	1,4860951	1,5513282
10	1,3439163	1,4105987	1,4802443	1,5529694	1,6288946
11	1,3842338	1,4599697	1,5394541	1,6228530	1,7103393
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8156491
14	1,5125897	1,6186945	1,7316764	1,8519449	1,9799316
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789282
16	1,6047064	1,7339860	1,8729812	2,0223701	2,1828746
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183
18	1,7024330	1,8574892	2,0258165	2,2084787	2,4066192
19	1,7535060	1,9225013	2,1068492	2,3078603	2,5269502
20	1,8061112	1,9897888	2,1911231	2,4117140	2,6532977
21	1,8602945	2,0594314	2,2787681	2,5202111	2,7859626
22	1,9161034	2,1315115	2,3699188	2,6336520	2,9252607
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715238
24	2,0327941	2,2833284	2,5633042	2,8760138	3,2251000
25	2,0937779	2,3632449	2,6658363	3,0054344	3,3863549
26	2,1565912	2,4459585	2,7724697	3,1406790	3,5556727
27	2,2212890	2,5315871	2,8833685	3,2820095	3,7334563
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356
30	2,4272624	2,8067937	3,2433975	3,7453181	4,3219424
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380395
32	2,5750827	3,0067075	3,5080587	4,0899810	4,7649415
33	2,6523352	3,1119423	3,6483811	4,2740301	5,0031885
34	2,7319053	3,2208603	3,7943163	4,4663615	5,2533480
35	2,8138624	3,3335904	3,9460889	4,6673478	5,5160154
36	2,8982783	3,4502661	4,1039325	4,8773784	5,7918161
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069
38	3,0747834	3,6960113	4,4388134	5,3262192	6,3854773
39	3,1670269	3,8253717	4,6163659	5,5658990	6,7047511
40	3,2620377	3,9592597	4,8010206	5,8163645	7,0399887
41	3,3598989	4,0978338	4,9930614	6,0781009	7,3919881
42	3,4606958	4,2412579	5,1927839	6,3516154	7,7615875
43	3,5645167	4,3897020	5,4004952	6,6374381	8,1496669
44	3,6714522	4,5433416	5,6165150	6,9361229	8,5571503
45	3,7815958	4,7023585	5,8411756	7,2482484	8,9850078

TABLE

TABLE 3. The PRESENT WORTH of ONE POUND for YEARS.

Yrs.	3 Per Cent.	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	,9708738	,9661836	,9615385	,9569378	,9523810
2	,9425959	,9335107	,9245562	,9157299	,9070295
3	,9151417	,9019427	,8889964	,8762966	,8638376
4	,8884870	,8714422	,8548042	,8385613	,8227025
5	,8626088	,8419732	,8219271	,8024511	,7835262
6	,8374843	,8135006	,7903145	,7618957	,7462154
7	,8130915	,7859910	,7599178	,7348285	,7106813
8	,7894092	,7594116	,7306902	,7031851	,6768394
9	,7664167	,7337710	,7025867	,6729044	,6446089
10	,7440939	,7089188	,6755642	,6439277	,6139133
11	,7224213	,6849457	,6495809	,6161987	,5846793
12	,7013799	,6617833	,6245971	,5896639	,5568374
13	,6809513	,6394040	,6005741	,5642716	,5303214
14	,6611178	,6177818	,5774751	,5399729	,5050679
15	,6418619	,5968906	,5552645	,5167204	,4810171
16	,6231669	,5767059	,5339082	,4944693	,4581115
17	,6050164	,5572038	,5133733	,4731764	,4362967
18	,5873946	,5383611	,4936281	,4528004	,4155207
19	,5702860	,5201557	,4746424	,4333018	,3957340
20	,5536758	,5025659	,4563870	,4146429	,3768895
21	,5375493	,4855709	,4388336	,3967874	,3589424
22	,5218925	,4691506	,4219554	,3797009	,3418499
23	,5066917	,4532856	,4057263	,3633501	,3255713
24	,4919337	,4379571	,3901215	,3477035	,3100679
25	,4776056	,4231470	,3751168	,3327306	,2953028
26	,4636947	,4088378	,3606892	,3184025	,2812407
27	,4501891	,3950123	,3468166	,3046914	,2678483
28	,4370768	,3816543	,3334775	,2915707	,2550936
29	,4243464	,3687482	,3206514	,2790150	,2429463
30	,4119868	,3562784	,3083187	,2670000	,2313775
31	,3999871	,3442304	,2964603	,2555024	,2203595
32	,3883370	,3325897	,2850579	,2444999	,2098662
33	,3770263	,3213427	,2740942	,2339712	,1998762
34	,3660449	,3104761	,2635521	,2238959	,1903548
35	,3553834	,2999769	,2534155	,2142544	,1812903
36	,3450324	,2898327	,2436687	,2050282	,1726574
37	,3349829	,2800316	,2342969	,1961992	,1644356
38	,3252262	,2705619	,2252854	,1877504	,1566054
39	,3157536	,2614125	,2166206	,1796655	,1491479
40	,3065568	,2525725	,2082890	,1719287	,1420457
41	,2976280	,2440314	,2002779	,1645251	,1352816
42	,2889592	,2357791	,1925749	,1574403	,1288396
43	,2805429	,2278059	,1851682	,1506605	,1227044
44	,2723718	,2201023	,1780464	,1441728	1168613
45	,2644386	,2126594	,1711984	,1379644	,1112965

TABLE 4. The AMOUNT of ONE POUND per annum, or ANNUITY, for YEARS.

Ys.	3 Per Cent.	$3\frac{1}{2}$ Per Cent.	4 Per Cent.	$4\frac{1}{2}$ Per Cent.	5 Per Cent.
1	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0300000	2,0350000	2,0400000	2,0450000	2,0500000
3	3,0909000	3,1062250	3,1216000	3,1370250	3,1525000
4	4,1836270	4,2149429	4,2464640	4,2781919	4,3101250
5	5,3091358	5,3624659	5,4163226	5,4707097	5,5256312
6	6,4684099	6,5501522	6,6329755	6,7168917	6,8019128
7	7,6624622	7,7794075	7,8982945	8,0191518	8,1420084
8	8,8923360	9,0516866	9,2142263	9,3800136	9,5491089
9	10,1591061	10,3684958	10,5827953	10,8021142	11,0265643
10	11,4638793	11,7313931	12,0061071	12,2882094	12,5778925
11	12,8077957	13,1419919	13,4863514	13,8411788	14,2067871
12	14,1920296	14,6019616	15,0258055	15,4640318	15,9171265
13	15,6177904	16,1130303	16,6268377	17,1599133	17,7129828
14	17,0863242	17,6769864	18,2919112	18,9321094	19,5986320
15	18,5989139	19,2956809	20,0235876	20,7840543	21,5785636
16	20,1568813	20,9710297	21,8245311	22,7193367	23,6574918
17	21,7615877	22,7050158	23,6975124	24,7417069	25,8403664
18	23,4144354	24,4996913	25,6454129	26,8550837	28,1323847
19	25.1168684	26,3571805	27,6712294	29,0635625	30,5390039
20	26,8703745	28,2796818	29,7780786	31,3714228	33,0659541
21	28,6764857	30,2694707	31,9692017	33,7831368	35,7192518
22	30,5367803	32,3289022	34,2479698	36,3033779	38,5052144
23	32,4528837	34,4604137	36,6178886	38,9370299	41,4304751
24	34,4264702	36,6665282	39,0826041	41,6891963	44,5019989
25	36,4592643	38,9498567	41,6459083	44,5652101	47,7270988
26	38,5530422	41,3131017	44,3117446	47,5706446	51,1134538
27	40,7096335	43,7590602	47,0842144	50,7113236	54,6691265
28	42,9309225	46,2906273	49,9675830	53,9933332	58,4025828
29	45,2188502	48,9107993	52,9662863	57,4230332	62,3227119
30	47,5754167	51,6226773	56,0849377	61,0070698	66,4388475
31	50,0026782	54,4294719	59,3283352	64,7523878	70,7607899
32	52,5027585	57,3345025	62,7014687	68,6662452	75,2988294
33	55,0778413	60,3412101	66,2095274	72,7562263	80,0637708
34	57,7301765	63,4531524	69,8579045	77,0302565	85,0669594
35	60,4620818	66,6740127	73,6522248	81,4966180	90,3203073
36	63,2759443	70,0076032	77,5983138	86,1639658	95,8363227
37	66,1742226	73,4578693	81,7022464	91,0413443	101,6281388
38	69,1594493	77,0288947	85,9703362	96,1382048	107,7095458
39	72,2342327	80,7249060	90,4091497	101,4644249	114,0950231
40	75,4012597	84,5502778	95,0255157	107,0303231	120,7997742
41	78,6632975	88,5095375	99,8265363	112,8466876	127,8397829
42	82,0231964	92,6073713	104,8195978	118,9247885	135,2317511
43	85,4838923	96,8486293	110,0123817	125,2764040	142,9933386
44	89,0484191	101,2383313	115,4128169	131,9138422	151,1430056
45	92,7198614	105,7816729	121,0293920	138,8499651	159,7001559

TABLE 5. The PRESENT WORTH of ONE POUND per ann. or ANNUITY for YEARS.

Yrs.	3 Per Cent.	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	0,9708738	0,9661836	0,9615385	0,9569378	0,9523809
2	1,9134697	1,8996943	1,8860947	1,8726678	1,8594103
3	2,8286114	2,8016379	2,7750910	2,7489644	2,7232480
4	3,7170984	3,6730792	3,6298952	3,5875257	3,5459505
5	4,5797072	4,5150524	4,4518223	4,3899767	4,3294767
6	5,4171914	5,3285530	5,2421369	5,1578725	5,0756921
7	6,2302829	6,1145439	6,6020547	5,8927009	5,7863734
8	7,0196922	6,8739555	6,7327448	6,5958861	6,4632128
9	7,7861089	7,6076865	7,4353314	7,2687905	7,1078217
10	8,5302028	8,3166053	8,1108955	7,9127182	7,7217349
11	9,5256241	9,0015510	8,7604763	8,5289169	8,3064142
12	9,9540040	9,6633343	9,3850733	9,1185808	8,8632516
13	10,6349553	10,3027385	9,9856473	9,6828524	9,3925730
14	11,2960731	10,9205203	10,5631223	10,2228253	9,8986409
15	11,9379351	11,5174109	11,1183868	10,7395457	10,3796500
16	12,5611020	12,0941168	11,6522949	11,2340151	10,8377695
17	13,1661185	12,6513206	12,1656680	11,7071914	11,2740662
18	13,7535131	13,1896812	12,6592961	12,1599918	11,6895869
19	14,3237991	13,7098374	13,1339385	12,5932936	12,0853208
20	14,8774748	14,2124033	13,5903253	13,0079365	12,4622103
21	15,4150241	14,6979742	14,0291589	13,4047239	12,8211527
22	15,9369166	15,1671248	14,4511142	13,7844248	13,1630026
23	16,4436084	15,6204105	14,8568405	14,1477749	13,4885739
24	16,9355421	16,0583676	15,2469619	14,4954784	13,7986418
25	17,4131477	16,4815146	15,6220787	14,8282089	14,0939445
26	17,8768420	16,8903523	15,9827678	15,1466115	14,3751853
27	18,3270315	17,2853645	16,3295844	15,4513028	14,6430336
28	18,7641082	17,6670188	16,6630618	15,7428735	14,8981272
29	19,1884546	18,0357670	16,9837132	16,0218885	15,1410735
30	19,6004413	18,3920454	17,2920318	16,2888885	15,3724510
31	20,0004285	18,7362758	17,5884921	16,5443909	15,5928104
32	20,3887655	19,0688656	17,8735500	16,7888909	15,8026766
33	20,7657918	19,3902082	18,1476441	17,0228621	16,0025491
34	21,1318367	19,7006842	18,4111916	17,2467580	16,1929039
35	21,4872200	20,0006612	18,6646116	17,4610124	16,3741942
36	21,8322525	20,2904938	18,9082803	17,6660406	16,5468516
37	22,1672354	20,5705254	19,1425771	17,8622398	16,7112872
38	22,4924616	20,8410874	19,3678625	18,0499902	16,8678926
39	22,8082151	21,1024999	19,5844831	18,2296557	17,0170406
40	23,1147719	21,3550723	19,7927721	18,4015844	17,1590862
41	23,4123999	21,5991037	19,9930500	18,5661095	17,2943678
42	23,7013592	21,8348828	20,1856250	18,7235498	17,4232074
43	23,9819021	22,0626887	20,3707931	18,8742103	17,5459118
44	24,2542739	22,2827910	20,5488395	19,0183831	17,6627732
45	24,5187125	22,4954503	20,7200378	19,1563474	17,7740697

TABLE

TABLE 6. The ANNUITY which 1£. will PURCHASE for any NUMBER of YEARS.

Yrs.	3 Per Cent	3½ Per Cent.	4 Per Cent.	4½ Per Cent.	5 Per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	,5226108	,5264005	,5301961	,5339976	,5378049
3	,3535304	,3569342	,3603485	,3637734	,3672086
4	,2690271	,2722511	,2754901	,2787437	,2820118
5	,2183546	,2214814	,2246271	,2277916	,2309748
6	,1845975	,1876682	,1907619	,1938784	,1970157
7	,1605064	,1635445	,1666096	,1697015	,1728198
8	,1424564	,1454767	,1485279	,1516097	,1547218
9	,1284339	,1314460	,1344930	,1375745	,1406901
10	,1172305	,1202414	,1232909	,1263788	,1295046
11	,1080775	,1110920	,1141490	,1172482	,1203890
12	,1004621	,1034840	,1065522	,1096662	,1128254
13	,0940295	,0970616	,1001437	,1032754	,1064558
14	,0885263	,0915707	,0946690	,0978203	,1010240
15	,0837666	,0868251	,0899411	,0931138	,0963423
16	,0796109	,0826848	,0858200	,0890154	,0922699
17	,0759525	,0790431	,0821985	,0854176	,0886991
18	,0727087	,0758168	,0789933	,0822369	,0855462
19	,0698139	,0729403	,0761386	,0794073	,0827450
20	,0672157	,0703610	,0735818	,0768761	,0802426
21	,0648718	,0680366	,0712801	,0746006	,0779961
22	,0627474	,0659321	,0691988	,0725457	,0759705
23	,0608139	,0640188	,0673091	,0706825	,0741368
24	,0590474	,0622728	,0655868	,0689870	,0724709
25	,0574279	,0606740	,0640121	,0674390	,0709545
26	,0559383	,0592054	,0625674	,0660214	,0695643
27	,0545642	,0578524	,0612385	,0647195	,0682919
28	,0532932	,0566027	,0600130	,0635208	,0671225
29	,0521147	,0554454	,0588799	,0624146	,0660455
30	,0510193	,0543713	,0578301	,0613915	,0650514
31	,0499989	,0533724	,0568554	,0604435	,0641321
32	,0490466	,0524415	,0559486	,0595632	,0632804
33	,0481561	,0515724	,0551036	,0587445	,0624900
34	,0473220	,0507597	,0543148	,0579819	,0617554
35	,0465393	,0499984	,0535773	,0572705	,0610717
36	,0458038	,0492842	,0528869	,0566058	,0604345
37	,0451116	,0486133	,0522396	,0559840	,0598398
38	,0444593	,0479821	,0516319	,0554017	,0592842
39	,0438439	,0473878	,0510608	,0548557	,0587646
40	,0432624	,0468273	,0505235	,0543431	,0582782
41	,0427124	,0462982	,0500174	,0538616	,0578223
42	,0421917	,0457983	,0495402	,0534087	,0573947
43	,0416981	,0453254	,0490899	,0529824	,0569933
44	,0412299	,0448777	,0486645	,0525807	,0566163
45	,0407852	,0444534	,0482625	,0522020	,0562617

Use of the foregoing Tables.

The Use of all these Tables depends on the following general RULE. Multiply the tabular number which stands against the given number of days, or years, and under the given rate of interest, by the principal sum, and the product will be the answer to the question.

EXAMPLE 1. What will 246*l.* amount to in 30 days, at 5 per cent, per annum? In the first table against 5 per cent. and against 30 days, stands 1,0040182 the tabular number

```
      1,0040182
            246
      ---------
       60241092
      40160728
     20080364
     ----------
Answer 246,9884772 = 246l. 19s. 9d.
```

E. 2. What will 246*l.* amount to in 30 years, at 5 per cent. per annum? In table 2, against 30 years, at 5 per cent: is 4,3219424

```
        4,3219424
              246
        ---------
        259316544
       172877696
       86438848
       ----------
Answer 1063,1978304 =
       (1063l. 3s. 11¼d.
```

E. 3. What is the present worth of an annuity of 246*l.* to continue 30 years, at 5 per cent. per annum? In table 3, against 30 years, at 5 per cent. is ,2313775

```
        ,2313775
             246
        --------
        13882650
        9255100
       4627550
      ----------
Answer 56,9188650 = 56l. 18s. 4½d. the present worth.
```

E. 4. What is the amount of an annuity of 246*l.* per annum, forborne or unpaid 30 years, at 5 per cent. per annum? In table 4, against 30 years at 5 per cent, is 66,4388475

```
        66,4388475
               246
        ----------
        3986330850
       2657553900
      1328776950
     -------------
Answer 16343,9564850 = 16343l. 19s. 1½d, the amount
                                       (required.
```

E. 5.

E. 5. What is the present worth of an annuity of 246*l.* to continue 30 years, at 5 per cent. per annum? In table 5, against 30 years at 5 per cent. is

```
 15,372451
       246
 ---------
  92234706
 61489804
30744902
----------
```

Answer 3781,622946 = 3781*l.* 12*s.* 5½*d.* the present worth.

E. 6. What is the annuity which 246*l.* will purchase, to continue 30 years, reckoning 5 per cent? In table 6, against 30 years, at 5 per cent. is ,0650514

```
      246
  -------
  3903084
 2602056
1301028
---------
```

Ans. 16,0036444=16*l.* 0*s.* 0¾*d.* the purchased annuity per annum.

If the amount of any sum be sought, for a number of days, which are not in the first table, and years which are not in the second; divide the given number of days or years into two such numbers as are in the table; then multiply the amount pertaining to each into each other, the product will be the amount for the time required.

E. 7. What will 523*l.* amount to, in 194 days, at 5 per cent. per annum?

In table 1, against 190 days, under 5 per cent. is -	1,0257228
And against 4 days, at the same rate, is - - - - -	1,0005348
The product is the amount of 1*l.* for 194 days, viz.	1,0262714
Which multiply by the principal sum, viz. -	523
The product is the answer	536,7399840 =
	(536*l.* 14*s.* 9½*d.*

E. 8. What is the amount of 150*l.* in 81 years, at 5 per cent?

In table 2, against 40 years, under 5 per cent. is	7,0399887
And against 41 years, at 5 per cent. is - -	7,3919881
The product is the amount of 1*l.* for 81 years, viz.	52,0395126
Which multiply by the principal sum, viz. -	150
	7805,9268900 =
	[7805*l.* 18*s.* 6¼*d.* Answer.

Note. The other tables of compound interest, cannot be extended in this manner.

Questions

Questions for exercise, to shew the extensive use of the Tables.

Quest. 1. A person having 12 years to run in a lease of an estate of 60*l.* per annum, for 40 years, would know what present money he must pay, in order to complete the lease by adding 28 years thereto, computeing at 5 per cent. compound interest? By table 5, the present value of 1*l.* per annum, at 5 per cent. for 40 years, is 17,1590862

By the same table, the value of 1*l.* per annum, at that rate, for 12 years to run,	8,8632516
Difference -	8,2958346
Multiply by -	60
Answer	£.497,7500760

Quest. 2. Which is the most advantageous, a term of 15 years in an estate of 100*l.* per annum; or the reversion of such an estate for ever, after the expiration of the said 15 years, computing at the rate of 5 per cent. per annum, compound interest?

A Freehold estate of 100*l.* per annum, at 5 per cent. is worth - -	£. 2000	
In table 5, the present value of the same estate, at the same rate for 15 years, is	1037,965	
The difference is -	962,035	val. of rever.

Hence it appears that the first term of 15 years is better than the reversion for ever afterwards, by 75,930=75*l.* 18*s.* 7*d.* the answer.

Quest. 3. What annuity, to continue 14 years, may be purchased with 1000*l.* due at the end of 5 years; the annuity to commence presently, at 5 per cent? By table 3, the present worth of 1000*l.* due 5 years hence, at 5 per cent. may be found equal to 783,5262; and by table 6, it may be found that the annuity which 783,5262 will purchase for 14 years, at the rate of 5 per cent. is 79,1518=79*l.* 3*s.* 0¼*d.* per annum, the answer.

Quest. 4. For a lease of certain profits for 7 years, A offers to pay 150*l.* gratuity, and 300*l.* per annum; B offers 400*l.* gratuity, and 250*l.* per annum; C bids 650*l.* gratuity, purchase without any yearly rent; *query*, which is the best offer, and what is the difference, computing at 5 per cent? By table 5, the present worth of 300*l.* per annum, for 7 years, at 4 per cent. is — — 1800,6164

To which add — —	150
The value of A's offer —	1950,6164
The present worth of 250*l.* per annum, for 7 years	1500,5136
Add - - - - -	400
The value of B's offer -	1900,5136

The

The present worth of 200*l.* per annnm, for 7 years		1200,4109
Add	— —	650,
The value of C's offer	—	1800,4109
D's offer	— —	1850

Hence it appears that A's is the best offer; and that rejecting the decimals, he bids 50*l.* more than B, 100*l.* more than C, and 150*l.* more than D.

Quest. 5. What annuity is sufficient to pay off a debt of 50 millions, in 30 years, at 4*l.* per cent. compound interest?

In table 6, against 30 years, under 4 per cent. is —	,0578301
Which multiply by the debt — — —	50000000
The annuity sought — — — —	£.2891505
So that supposing the national debt to be 50 millions, the interest at 4 per cent. would be — —	2000000

Then it would require a sinking fund of 2891505*l.* per annum to clear the whole debt in 30 years.

Quest. 6. A son previous to his marriage, is minded to have 50*l.* a year freehold settled on his family; and to have immediate possession of it, offers his father in lieu an anuuity for his life, valued at 12 years purchase, discounting at 4 per cent. thereon; whereas he is content the estate should be valued at a discount of 3 per cent. and consequently will be worth $33\frac{1}{2}$ years purchase; pray what had the father for his life?

First $33,3 \times 50 = 1666,6 = 1666$*l.* 13*s.* 4*d.* nearly the value of the annuity. Then per table 6, 1*l.* for 12 years, at 4 per cent. will purchase ,1065522 per annum.

$\therefore 1666,6 \times ,1065522 = 177,587 = 177$*l.* 11*s.* $8\frac{3}{4}$*d.* the answer,

LXIV. *Concerning Divisors.*

IT being often necessary in arithmetical calculations, to find such multipliers, or numbers, which may be divided by any number of given divisors, without any remainder, or remainders; by which means many pleasant questions, not reducible to any other rule in common arithmetic, may be solved.

To find the least number that can be divided by any number of divisors, with a remainder.

RULE. Multiply all the prime numbers, and the root of such as are square or cube numbers, continually; the product will be the number required.

Note. A prime number is such as hath no measure but itself and unity, and consequently cannot be produced by the multiplication of two or more integers; as, 1, 2, 3, 5, 7, 11, &c. are prime numbers.

Composite numbers are such as are divisible by some numbers besides unity; as 8 is divisible by 4 and 2, &c.

A number

A number that will divide ſeveral numbers exactly, is called a common meaſure, as 3 is a common meaſure to 12 and 15.

EXAMPLE 1. Required the three leaſt numbers, which divided by 20, ſhall leave 19 for a remainder; but if divided by 19, ſhall leave 18; if divided by 18, ſhall leave 17; and ſo on, always leaving one leſs than the diviſor, to unity?

Firſt 1, 2, 3, 5, 7, 11, 13, 17, and 19, are prime numbers:

Alſo $\sqrt[2]{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt[2]{9} = 3$, and $\sqrt[4]{16} = 2$, and all the reſt are compoſite numbers. Therefore,

$1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 \times 13 \times 2 \times 17 \times 19 = 232792560$, the leaſt number that can be divided by the given diviſors without a remainder.

Then the number $232792560 - 1 = 232792559$, the firſt number.

And $\overline{232792560 \times 2} - 1 = 465585119$, the ſecond number.

Alſo $\overline{232792560 \times 3} - 1 = 698377679$, the third number.

And after this manner may the other numbers be found.

E. 2. What is the leaſt number that can be divided by the nine digits, without a remainder?

The given diviſors are 1, 2, 3 4, 5, 6, 7, 8, 9.

Now $\sqrt{4} = 2$; 6 may be cancelled, being compoſed of 2×3; and 3, 5 and 7 are prime numbers; and $\sqrt[3]{8} = 2$. Alſo $\sqrt{9} = 3$.

Then per rule, $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 = 2520$, the number required.

E. 3. Required the leaſt number which being divided by 7, 6, 5, 4, 3 and 2, ſhall leave 6, 5, 4, 3, 2 and 1 reſpectively?

Firſt the diviſors are 7, 6, 5, 4, 3, 2.

Now $\sqrt{4} = 2$, and 3, 5 and 7 are prime numbers, 6 may be cancelled being a compoſite number.

Then per rule, $2 \times 3 \times 2 \times 5 \times 7 = 420$, the leaſt number that can be divided by the given diviſors without a remainder. Therefore $420 - 1 = 419$, the number required.

E. 4. John the gardener counting ſome apples into a baſket, found that when he counted them in by two at a time, three at a time, and four at a time, there remained one; but when he counted them in by five at a time, there remained none; *quere*, the number of apples?

Firſt 2, 3 and 4, are the diviſors; now 2 and 3 are prime numbers, and $\sqrt{4} = 2$.

Then per rule, $2 \times 3 \times 2 = 12$; then $12 + 1 = 13$, which divided by 2, 3 and 4, leaves 1, according to the queſtion; but divided by 5 will leave 3, which is 2 ſhort of 5. ∴ To twice 12 add 1, and the ſum will be 25, the number ſought.

E. 5.

E. 5. *A country girl to town did go,*
Some walnuts there to ſell;
A gentleman ſhe chanc'd to meet,
And thus it her befell:
My pretty maid, ſays he to ſhe,
What number have you here?
I can't tell, Sir, ſays ſhe to him,
But this I'll make appear;
I told them o'er, e'er I came out,
By ſix's, fives, four's, three's, two's,
And every time I numbered them,
One remained overplus:
I told them o'er by ſeven's at laſt,
And there were no remains;
If you can find the number out,
Pray take them for your pains.

Firſt, the leaſt number that can be divided by 1, 2, 3, 4, 5, 6, without a remainder, will (per rule) be $1\times2\times3\times2\times5=60$. Then $60+1=61$, which divided by 2, 3, 4, 5, 6, will leave 1 according to the queſtion; but divided by 7, will leave 5; $\therefore\ \overline{60\times5}+1=301$, the leaſt number which admits of the conditions of the queſtion. Then to find the next leaſt number which admits of the ſame conditions, by proceeding as above we ſhall find to be $\overline{60\times12}+1=721$. Alſo $721-301=420$, the common difference of all the numbers anſwering the conditions of the queſtion. Therefore 301, 721, 1141, 1561, 1981, &c. ad infinitum, will anſwer the conditions of this queſtion.

LXV. DUODECIMALS;

OR,

CROSS MULTIPLICATION.

THIS rule is called duodecimals, becauſe the unit, or integer, is divided into 12 equal parts; and hence this way of computation is chiefly uſed amongſt workmen in caſting up the contents of ſuperficial and ſolid works, the lineary dimenſions being generally taken in feet, inches, and parts.

RULE. 1. Under the multiplicand, write the correſpondent denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning with the loweſt, by the feet in the multiplier; placing each reſult under its reſpective term, remembering to carry an unit for every 12 from each lower denomination to its next ſuperior.

3. Work in the ſame manner with the inches and parts, ſetting the reſult of each term one place more to the right-hand; having thus finiſhed multiplication, the ſum of all will be the product required.

Note. In multiplying feet, inches, and parts; if feet be multiplied by feet, the product is feet; and feet multiplied by inches, the product is inches; aud parts multiplied by feet, the product is parts; parts multiplied by inches, the product is ſeconds; and parts multiplied by parts, the product is thirds.

 EXAMPLE

EXAMPLE 1. What is the product of 8 feet 9 inches and 6 parts, by 5 feet 6 inches and 3 parts?

By the rule, First method.

	F.	I.	P.		
	8	9	6		
	5	6	3		
	43	11	6		
	4	4	9	0	
		2	2	4	6
Anſ.	48	6	5	4	6

Second method.

	8	9	6			
	5	6	3			
		2	2	4	6	
	4	4	9	0		
	43	11	6			
Anſ.	48	6	5	4	6	as before.

Third method.

	8	9	6			
	5	6	3			
	40	45	30			
		48	54	36		
			24	27	18	
Anſ.	48	6	5	4	6	as before.

Fourth method.

	F.	I.	P.			
	8	9	6			
	5	6	3			
	40					
	3	9				
	0	2	6			
	4	0	0			
	0	4	6			
	0	0	3			
	0	2	0			
	0	0	2	3		
	0	0	0	1	6	
Anſ.	48	6	5	4	6	as before.

Fifth method, by practice.

In.		F.	I.	P.		
4 =	$\frac{1}{3}$)	8	9	6		
				5		
		43	11	6		
2 =	$\frac{1}{2}$)	2	11	2		
3″ =	$\frac{1}{8}$)	1	5	7		
		0	2	2	4	6
Anſwer		48	6	5	4	6

The above example is worked by five different methods, to ſhew the conciſeneſs of each.

E. 2. Multiply 12 feet 8 inches by 5 feet?

	F.	I.
	12	8
	5	
Anſwer	63	4

E. 3. Multiply 97 feet 8 inches by 8 feet 9 inches.

	F.	I.	
	97	8	
	8	9	
	781	4	
	73	3	0
Anſwer	854	7	0

E. 4. Multiply 57 feet 3 inches by 28 feet 6 inches.

	57	3	
	28	6	
	456		
	114		
	28	7	6
	7	0	0
Anſ.	1631	7	6

Note.

Note. As this kind of arithmetic is useful to persons concerned in building, measuring, &c. I thought proper to insert a few promiscuous examples, with an intention to give them a clear insight into this useful rule.

Questions for exercise in Duodecimals.

Quest. 1. If a floor be 53 feet 6 inches long, and 47 feet 9 inches broad, how many squares are contained in that floor?

	F.	I.	
	53	6	
	47	9	
	371		
	212		
	40	1	6
	23	6	0
Answer	25,54	7	6 = 25 Squares, 54 feet 7 inches 6 parts.

Note. The reason of cutting off two figures, is, because there are 100 square feet in one square rod of 10 feet long, which is the same as dividing by 100.

Quest. 2. If a house within the walls, be 44 feet 6 inches long, and 18 feet 3 inches broad; how many squares of roofing will cover that house?

	F.	I.		
	44	6		
	18	3		
	352			
	44			
	11	1	6	
	9	0	0	
Add	812	1	6	Flat*
	406	0	0	Half-flat
Ans.	12,18	1	6 = 12 sq. 18ft.	

Quest. 3. If a pane of glass be 4 feet 8¾ inches long, and 1 foot 4 inches and 1 quarter broad, how many feet of glass does it contain?

	F.	I.	P.		
	4	8	9		
	1	4	3		
	4	8	9		
	1	6	11	0	
		1	2	2	3
Ans.	6	4	10	2	3

Quest. 4. If there be 8 panes of glass, each 4 feet 7 inches and three-quarters long, and 1 foot 5¼ inches broad; how many feet of glass are contained in the said 8 panes?

	F.	I.	P.		
	4	7	9		
	1	5	3		
	4	7	9		
	1	11	2	9	
		1	1	11	3
	6	8	1	8	3
					8 No. of Panes.
Answer	53	5	1	6	0

*In Tyling and Roofing, it is customary to reckon the flat and half-flat of any building within the walls, to be the depth or width of the roof of that building, when the said roof is a true pitch; that is, when the rafters are three-fourths of the breadth of the building.

Quest. 5. If there are 16 panes of glass, each 4 feet $5\frac{1}{2}$ inches long, and 1 foot $4\frac{3}{4}$ inches broad; how many feet of glass are contained in them?

F.	I.	P.		
4	5	6		
1	4	9		
4	5	6		
1	5	10	0	
	3	4	1	6
6	2	8	1	6
				4×4=16
24	10	8	6	0
				4
99	6	19	0	0 Answer.

Quest. 6. If a room be painted, whose height being girt over the mouldings, is 16 feet 6 inches, and the compass of the room be 67 feet 9 inches, how many yards are there in that room?

F.	I.	
67	9	
16	6	
414	0	
67		
33	10	6
9)1117	10	6
Answer 124	1=124 *yards* 1 *foot*.	

Note. The inches and parts in this kind of measure are generally neglected.

Quest. 7. If a room of wainscot be 16 feet 3 inches high, girt over the mouldings, and the compass of the room is 137 feet 6 inches, how many yards does it contain?

	F.	I.		
	137	6		
	16	3		
	830	0		
	137			
	34	4	6	
The square feet in 1 yard =	9)2234	4	6	
Answer	248	2	4	6=248 *yds.* 2 *ft.* 4 *in.* 6 *pts.*

Quest. 8. If a piece of timber be 21 inches broad, 21 inches deep, and 15 feet long, how many solid feet are contained therein?

F.	I.		
1	9 Breadth		
1	9 Depth		
1	9		
1	3	9	
3	0	9	
		5×3=15 Length	
15	3	9	
		3	
45	11	3 Ans. Solid content.	

Quest. 9. If a piece of timber be 25 inches broad, 7 inches deep, and 25 feet long; how many solid feet are contained therein?

F.	I.	
2	1 Breadth	
0	7 Depth	
1	2	7
		5×5=25 Length
6	0	11
		5
Ans. 30	4	7 Solid content.

GEO-

GEOMETRY.

PART IV.

GEOMETRY originally ſignifies the art of meaſuring the earth, or any diſtances or dimenſions thereon. But now it is uſed for the ſcience of quantity, abſtractedly conſidered, without any regard to matter.

It very probably had its firſt riſe in Egypt, where the Nile annually overflowing the country, and covering it with mud, obliged men to diſtinguiſh their lands one from another, by the conſideration of their figure; and to be able alſo to meaſure the quantity of it, and to know how to plot it, and to lay it out again in its juſt dimenſions, figure and proportion: after which it is likely a further contemplation of thoſe draughts and figures helped them to diſcover many excellent properties belonging to them, which ſpeculation has continually been improving to this day.

Before I proceed, I ſhall firſt explain the following uſeful terms:

1. *Axiom*, is a principal in any art, ſo evident, that it needs nothing but the light of reaſon to demonſtrate it.

2. *Conſtruction*, is the drawing of lines, and framing of figures, or preparing the propoſition for a demonſtration.

3. *Corollary*, is a conſequent truth gained from a preceding demonſtration.

4. *Definition*, is the unfolding or explicating of the nature and affection of a thing in a few words.

5. *Demonſtration*, is the proving of a thing by definitions and axioms, and ſo from ſeveral arguments drawing a concluſion, that it has that affection the propoſition did aſſert.

6. *Hypotheſis*, is when a thing is ſuppoſed, or given to be ſo.

7. *Lemma*, is the demonſtration of ſome premiſe, in order to ſhorten a following demonſtration.

8. *Problem*, is when ſomething is propoſed to be done.

9. *Propoſition*, is uſed promiſcuouſly, either for a problem or theorem.

10. *Poſtulate*, is a grantable requeſt, or ſuch a demand as cannot reaſonably be denied.

11. *Scholium*, is a ſhort critical expoſition, gained from a former demonſtration.

12. *Theorem*, is when ſomething is propoſed to be done.

GEOMETRICAL

GEOMETRICAL DEFINITIONS.

1. A Geometrical Point is ſo infinitely ſmall, as to be void of length, breadth, and thickneſs; and therefore a point may be ſaid to have no parts.

2. A Line, is called a quantity of one dimenſion, becauſe it may have any ſuppoſed length, but no breadth or thickneſs.

3. A Superfices, is a figure which hath length and breadth, and is bounded by lines either ſtraight or circular.

4. All three-ſided figures are called Triangles, but admit of ſeveral diſtinctions; as an Equilateral, when the ſides are equal: Iſoſeles, when only two ſides are equal: Scalene, when the three ſides are unequal; and Right-angled, when it has one right angle.

5. All four-ſided figures are called Quadrilaterals, but are divided into ſquares, parallelograms, rhombus's, and rhomboides. A ſquare is that where all the angles are right, and the lines equal: a Parallelogram, or oblong ſquare, is a figure that hath all its angles right, and its two oppoſite ſides equal: a Rhombus, is that which hath its four ſides equal, but no right angle.

6. A Circle, is a plane bounded by one curved line, called the circumference, to which all right lines drawn from a certain point within the figure, called its center, are equal.

7. The Diameter of a circle, is a right line drawn through the center, terminated at each end by the circumference, and divides the circle into two equal parts, each of which is called a ſemi-circle; half the diameter is called the Radius.

8. The circumference of every circle is divided into 360 equal parts called Degrees; each degree into 60 equal parts, called Minutes; and each minute into 60 equal parts, called Seconds, &c. Any part of the the circumference is called an Arch.

9. The Chord of an arch, is a right line joining the extremities of an arch, and by which the circles are divided into two unequal parts, called Segments.

10. A Sector, is a figure comprehended under two radiuſes of a circle, and the arch included between thoſe radiuſes.

11. A Polygon, is a figure contained under ſeveral ſides; and called a regular polygon, if the ſides and angles are regular amongſt themſelves, but if they are not, it is called an irregular polygon.

A polygon has different names, according to its number of ſides, viz. if it has five ſides, it is called a pentagon; if ſix, a hexagon; if ſeven, a heptagon; if eight, an octagon; if nine, a nonagon; if ten, a decagon; if eleven, an undecagon; and if 12, a duodecagon.

12. The Altitude, or height of any figure, is the perpendicular, let fall from its ſummit to its baſe, or line on which the figure is ſuppoſed to ſtand.

13. The Area of any figure, is the ſuperficial content of it.

LXVI.

LXVI. PROBLEMS.

PROBLEM 1. *Upon a given right line,* A B, *to erect a perpendicular.* Plate 1st, fig. 1.

1. ON each side of the point D, take any two equal distances, D *e* and D *n*.

2. From *e* and *n*, with any radius greater than D*e* or D*n*, describe the two arches cutting each other in *c*.

3. Through the points D, *c*, draw the line D, *c*, and it will be the perpendicular required.

PROB. 2. *From a given point* C, *above the given line* A B, *to let fall a perpendicular* C D. Fig. 2.

From the point C, with any radius, describe the arch *a c*, intersecting AB in *a c*; from the points *a* and *c*, with the same radius, describe two arches cutting each other in *b*; lay a ruler from C to *b*, and draw CD, and it will be the perpendicular required.

PROB. 3. *To divide a given line* A B, *into two equal parts.* Fig. 3.

From the points A and B, with any distance greater that half A B, describe the two arches cutting each other in *a* and *c*; through the points *a* and *c* draw the line *a c*, and it will divide the line A B as required.

PROB. 4. *To erect a perpendicalar on the extremity* A, *of a given right line* A B. Fig. 4.

From the point A describe the arch *a d*; and with the same opening of the compasses, from *a* make the intersection *b*, and on *b*, the intersection *c*; then from *b* and *c* make the intersection *e*, and draw *e* A, the perpendicular required.

Another method, Fig. 5. Take any point *e*, and with the distance *e*C, describe the arch *m* C *n*, cutting A C in *m* and C; through the center *e*, and the point *m*, draw the line *m e n*, cutting the arch *m* C *n* in *n*; then through the points *n* C, draw the line *n* C, and it will be the perpendicular required.

PROB. 5. *To divide an angle* ABC, *into two equal parts.* Fig. 6.

From the point B, with any radius, describe the arch *a b* cutting the sides in *a* and *b*; on which points, with the same radius, describe the arches cutting each other in *e*; then draw the line B *e*, and it will bisect the angle, as required.

PROB. 6. *At the end* B *of a given right line* A B, *to make an angle equal to a given angle* C D G. Fig. 7.

From the angular point D, describe at pleasure the arch *a b*; and with the same radius upon the point B, describe the arch *c d*, on which make *c e* = *a b*, and through the points B, *e*, draw the line E B, and it will make the angle A B E = C D G.

PROB.

PROB. 7. *To find the center of a circle.* Fig. 8.

Draw any chord A B, and bisect it with the chord C D; then bisect C D with the chord E F, and their intersection O, will be the center required.

PROB. 8. *To bring three points, not lying in a straight line, into the circumference of a circle.* Fig. 9.

Let A, B, and C, be the three points; upon A and B, with the same radius, make the intersections *a* and *b*, and draw the line *a b*: on the points B and C, make the intersections *d*, *e*, and draw *d e*, and it will intersect *a b* in I, the center of the circle, that runneth upon the three given points.

Note. By this problem may the center to any arch, or circle, be found.

PROB. 9. *To draw a tangent to a given circle, when the point* A *is without the circle.* Fig. 10.

From the center O, draw O A, and bisect it in *a*; and from the point *a*, with the radius *a* A, or *a* O, describe the semi-circle A B O, cutting the given circle in B; then through the points A and B, draw the line A B, and it will be the tangent required.

PROB. 10. *Between two given right lines* A *and* B, *to find a mean proportional.* Fig. 11.

Draw any right line, in which take *e b* equal to A, and *b a* equal to B; bisect *a e* in *o*, and with *o a* or *o e*, as radius, describe the semi-circle *a d e*; then from the point *b*, draw *b d* perpendicular to *a e*, and it will be the mean proportional required.

PROB. 11. *Upon a given right line* A B, *to make an equilateral triangle.* Fig. 12.

From the points A and B, with a radius equal to A B, describe arches cutting in C; then draw A C and B C, and the figure A C B is the triangle required.

Note. We have a problem directing us how to draw parallel lines, but now we have a parallel ruler, which solves this problem with accuracy and expedition; I would, therefore, advise the practitioner to make use of that instrument, before the problem.

PROB. 12. *Upon a given right line* A B, *to describe a square.* Fig. 13.

On the point B, erect the perpendicular B C = A B; with the extent A B on the points A C, describe the arches intersecting in D; draw A D and C D, and it is done.

PROB. 13. *To inscribe a circle in a given triangle* A B C. Fig. 14.

Bisect any two of the angles, as A and B, with the right lines A D and B D, meeting each other in D; then from the point of intersection D, let fall the perpendicular D E, and it will be the radius of the circle required.

PROB.

PROB. 14. *To make a triangle, whose three sides shall be equal to three given lines,* A, B, C. Fig. 15.

Draw a line D E, equal to the line A; and on the point D, with a radius equal to B, describe an arch; then on the point E, with a radius equal to C, describe another arch, cutting the former in F; lastly, draw the line D F and E F, and D F E will be the triangle required.

PROB. 15. *To make an angle of any proposed number of degrees.* Fig. 16

Take the first 60 degrees from the scale of chords, and from the point A; with this radius describe the arch *a b*, and take the chord of the proposed number of degrees from the same scale, and apply it from *a* to *b*; then from the point A, draw the lines A *a* and A *b*, and they will form the angle required. In this example $a\ b = 60°$.

Note. Angles greater than 90°, are usually taken at twice.

PROB. 16. *About a given triangle* A B C, *to circumscribe a circle.* Fig. 17.

Bisect the two sides A B and B C, with the perpendiculars *n o* and *b o*, then from the point of intersection *o*, with the distance *o* A or *o* B, describe the circle A C B, and it is done.

PROB. 17. *On a given line* A B, *to make a regular pentagon.* Fig. 18.

On the points A and B, with the distance A B, describe two circles cutting each other in *m* and *n*; draw the line *m n*, and from the point *n*, with the same radius as before, describe the arch *r* A *o* B *s*, cutting the two circles in the points *r* and *s*, and the line *m n* in the point *o*; draw the lines *r o* and *s o*, and produce them till they meet the circumferences in E and C; then from the points E and C, with the radius A B, describe arches crossing in D. Lastly, join the points A E, E D, D C, and C B, and A E D C B will be the pentagon required.

PROB. 18. *To draw a helex, or spiral line, with a pair of compasses.* Fig. 19.

Let the two centers be *a* and *o*, through which draw a right line what length you please, set one foot of the compasses in *a*, and extend the other to *o*, and draw the first semi-circle; remove that point of the compasses from *a* to *o*, and extend the other to join the semi-circle now drawn, and draw another semi-circle; remove the point of the compasses from *o* again to *a*, and extend the other point to the last semi-circle, and join it, and draw another semi-circle; do thus as long as you please, and you will have a spirial line, rolling in several circles, as per figure.

PROB. 19. *To reduce a circle to a square.* Fig. 20.

Divide the diameter A B, into 14 equal parts, and at 11 of those parts erect the perpendicular C D, and draw A D, so is A D the side of the square, nearly equal in content to the given circle.

Note. This problem is grounded upon *Archimedes's* proportion of the diameter of a circle to the circumference, being as 7 to 22; and although this proportion is not true, yet is is the nearest in whole numbers, and may serve very well for common purposes.

Prob. 20. *To reduce a ſquare to a circle.* Fig. 21.

Divide the ſide of the given ſquare into 11 equal parts; at $5\frac{1}{2}$ of thoſe parts, draw the ſemi-circle A B C, and at 8 of thoſe parts, on the ſide of the ſquare, erect the perpendicular D B, draw A B continued to the ſide of the ſquare at E, ſo is A E the diameter of the circle, nearly equal in content to the given ſquare.

Prob. 21. *Two points within a circle being given, to deſcribe the arch of another circle, which ſhall paſs through thoſe two points, and alſo divide the circumference of the given circle into two equal parts.* Plate 1. Fig. 22.

Let the two points be *e* and *c*, within the circle. Firſt, through either of them (as through *e*) draw the right-line *e* D, paſſing through the center of the circle at O. Then at right angles thereto, draw the line A C. Laſtly, draw the line *e* A, and upon the point A, erect the perpendicular A G, cutting the line B D (produced) in the point G; ſo have you three points, *e*, *c*, G, through which (by problem 8th) you may draw the arch P *e c* N G, whoſe center will be at *k*. Now, if you lay a rule upon the points P and N, and it paſſes over the center of the given circle at O, the circle is truly drawn.

Prob. 22. *To divid a circle into any number of equal parts.* Fig 23.

1. Draw a circle of any radius, and draw the diameter A B; this divides the circle into two equal parts.

2. Erect the perpendicular F C, and that is the ſide of an hexagon, or the ſixth part of the circle = A D.

3. Set F C fron A to D, and from D to E, draw A E, for the ſide of an equilateral triangle.

4. Draw A C for the ſide of the ſquare inſcribed, or the fourth part of the circle.

5. Biſect F B in G, and draw C G: make G H=G C, and draw C H for the ſide of the pentagon, or fifth part of the circle.

6. Join E G for the ſide of an heptagon, or one-ſeventh part of the circle.

7. Biſect the arch A C in I, and draw A I for the ſide of an octagon, or one eighth part of the circle.

8. Divide the arch A D E, into three equal parts, in K, and draw A K, for the ninth part of the circle.

9. The line H F, is the ſide of a decagon, or a ten-ſided figure.

10. The line F L is the endecagon, or eleventh-ſided figure.

11. The line D C, is the twelfth part of the circle; and by doubling and tripling theſe lines, the circle may be geometrically divided into any number of equal parts at pleaſure.

Prob. 23. *In a given circle, to inſcribe any regular polygon.* Fig. 24.

1. Draw the diameter A B, on which make the equilateral triangle, A C B.

2. Divide the diameter A B into as many equal parts as the required polygon has ſides.

3. From

3. From the point C, through the ſecond diviſion of the diameter, draw the line C D.

4. Join the points A and D, and the line A D, will be the ſide of the polygon required; (in this conſtruction A D is the ſide of a heptagon) and ſo of any other.

Note. This conſtruction is the invention of *Renaldinus.* See his ſecond book, *De Reſol. &c. Comp. Mathem.* page 367.

Prob. 24. *To draw an oval, by having the two diameters given.* Fig. 25.

Divide each diameter into four equal parts, and through thoſe parts draw the lines *a b c d*, then ſet one foot of the compaſſes in *d*, and extend the other foot to F, and draw the arch E F G; with this extent of the compaſſes, ſet one foot in *b*, and draw the arch H I K. Laſtly, ſet one foot of the compaſſes in *a* and *c* ſeverally, and draw the arches G H and E K, and the oval is compleated.

Prob. 25. *To draw an oval by the help of a parallelogram, or two geometrical ſquares.* Fig. 26.

Firſt, draw the line AC, and make C F = C B; then draw D F parallel to A C; draw alſo A D and B E, and you will have two ſquares A B D E, and B C E F; then draw the diagonal lines A E and B F, and opening the compaſſes with the extent of A E or C E, place one foot in E, and draw the arch A C; then with the former extent, one foot placed in B, deſcribes the arch D F; then ſet one foot in *a*, and with the diſtance *a* A ſweep the arch D A; with the ſame extent from *c* ſweep the arch C F, and the oval is compleated.

Prob. 26. *Having a line equal to the length of an oval, to make thereof a true oval.* Fig. 27.

Let A B be the given line; divide it into three equal parts A *b* B; then from the point *b*, with the diſtance *b* B, deſcribe the circle B *a* C *c*; and upon the other diviſion at *a*, draw the circle A *b* G *c*; theſe two circles will interſect one another in the center of each, and alſo at the points *d c*, draw C *b e* and F *a d* parallel, alſo G *a c* and H *b d* parallel; then from *c*, with the diſtance G *c*, ſweep the arch G N C, and from *d* with the ſame extent, ſweep F K H, and you have a true oval.

Prob. 27. *To draw an oval from three circles.* Fig. 28.

Draw the line A B, and divide it into four equal parts, and on the three points *d*, *c*, *e*, deſcribe three circles; draw M G and O F parallel thereto, and alſo draw F N and L G parallel thereto; then on G, with the extent G L or G M, deſcribe the arch L M; and upon F, with the ſame extent, deſcribe O N. Laſtly, upon the point *d* deſcribe O A L, and upon *e* deſcribe M B N, and the oval is finiſhed.

Prob. 28. *To lay down an ellipſis by the line of ſines on the ſector, having the tranſverſe and conjugate diameters given.* Fig. 29.

Firſt, take A E or E B in your compaſſes; then open the ſector at 90, 90 on the line of ſines; and as the ſector now ſtands, take off the ſines 10, 20, 30, 40, 50, 60, 70, 80, and ſet them from E, each way towards A and B; draw lines through thoſe points in the tranſverſe

ſector on the ſines 90, 90, to the radius C E, and take in your compaſſes the ſine of 80, and ſet 10 to 80; take the ſine 70, and ſet from 70 to 20 on each ſide the conjugate diameter; the ſine 60, ſet from 30 to 60; the ſine 50, ſet from 40 to 50; the ſine 40, ſet from 50 to 40; the ſine 30, ſet from 60 to 30; the ſine 20, ſet from 70 to 20; the ſine 10, ſet from 80 to 10; ſo will the points 10, 20, 30, 40, 50, 60, 70, 80, C B D A, be in the ellipſis, through which points draw the curve, and you will have a true mathematical ellipſis.

PROB. 29. *Any angle being given, to find the number of degrees it contains.* Fig. 16.

1. Take 60° out of your line of chords, and ſet one foot of your compaſſes in A, with the other deſcribe the arch *a b*.

2. Take the diſtance *a b* in your compaſſes, and ſet one foot in the beginning of the line of chords, and the other will reach to 60 upon the ſame line, the meaſure of the angle required.

PROB. 30. *In a given circle, to inſcribe a polygon of any propoſed number of ſides.* Fig. 30.

1. Divide 360° by the number of ſides, and make an angle A *c* B, at the center, whoſe meaſure ſhall be equal to the degrees in the quotient.

2. Join the points A and B, and apply the chord A B to the circumference, the given number of times, and you will have the polygon required.

PROB. 31. *To deſcribe a lune in a quadrant.* Fig. 31.

Draw the triangle A B C, and on the center B, deſcribe the quadrantal arch A C; upon the middle of the hypothenuſe A C, draw the other ſemi-circle, and you will have the lune A F C D required.

LXVII. MENSURATION OF SUPERFICIES.

THE area of any plain ſurface is the ſpace contained within the bounds of that ſurface, without any regard to thickneſs, and is made up of ſome certain number of ſquares, according to the different meaſures the dimenſions are taken in, viz. a ſquare whoſe ſide is one inch, one foot, one yard, &c. is called the *meaſuring unit*, and the content of any figure is computed by the number of thoſe ſquares contained in that figure.

PROBLEM I. *To find the area of a parallelogram; whether it be a ſquare, a rectangle, a rhombus, or a rhomboides.*

RULE. Multiply the length by the perpendicular height, the product is the area or content.

EXAMPLE

EXAMPLE 1. What is the area of the ſquare SS, whoſe ſide is 5 feet 6 inches?

By decimals.

F.
5,5
5,5
275
275
30,25
12
3,00

	F.	I.
Anſwer	30	3

By duodecimals.

	F.	I.
	5	6
	5	6
	27	6
	2	9
Anſwer	30	3

By practice.

6 in. = $\frac{1}{2}$)	5	6
		5
	27	6
	2	9
Anſwer	30	3

E. 2. What is the area in acres of a ſquare, whoſe ſide is 35,25 chains?

35,25 = Length of the ſide
35,25
17625
7050
17625
10575
124,25625
4
1,02500
40
1,00

Anſwer 124 acres, 1 rood, 1 pole.

E. 3. Required the area of a ſquare, whoſe ſide is 9 feet 3 inches?

By deicmals.

F.
9,25
9,25
4625
1850
8325
85,5625
12
6,7500
12
9,00

By duodecimals.

	F.	I.	
	9	3	
	9	3	
	83	3	
	2	3	9
Anſwer	85	6	9

By practice.

In.	F.	I.	
3 = $\frac{1}{4}$)	9	3	
		9	
	83	3	
	2	3	9
Anſwer	85	6	9

Anſwer 85 feet, 6 inches, 9 ſeconds.

E. 4.

E. 4. What is the area of the rectangle B L, whose length is 18 feet 6 inches, and breadth 12 feet 6 inches?

By decimals.	By duodecimals.	
F.	F.	I.
18,5	18	6
12,5	12	6
925	222	0
370	9	3
185		
Answer	231	3 as before
231,25		
12		
3,00 Answer 231 feet 3 inches.		

B

L

E. 5. What is the superficial content of a parallelogram, whose length is 68 feet, and breadth 16 feet?

	F.
	68
	16
Answer	1088 feet.

E. 6. If one side of a parallelogram is 130 feet, and the other 50 feet, what is the superficial content?

	130
	50
Answer	6500 feet.

E. 7. How many feet are contained in a floor 45 feet 6 inches long, and 9 feet 3 inches broad?

By decimals.	By duodecimals.				By practice			
F.		F.	I.		In.	F.	I.	
45,5		45	6		$3 = \frac{1}{4}$)	45	6	
9,25		9	3				9	
2275		409	6			409	6	
910		11	4	6		11	4	6
4095								
420,875	Ans.	420	10	6	Ans.	420	10	6
12								
10,500								
12								
6,0 Answer 420 feet, 10 inches, 6 parts.								

E. 8. Required the superficial content of a rhombus, whose length is 12 feet 6 inches and perpendicular height 9 feet 3 inches?

By

By decimals.

12,5
9,25
625
250
1125
115,625
12
7,500
12
6,000

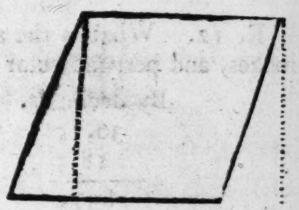
By cross multiplication.

12	6	
9	3	
112	6	
3	1	6
115	7	6

By practice.

$3 = \frac{1}{4}$)	12	6	
		9	
	112	6	
	3	1	6
	115	7	6

Answer 115 feet, 7 inches, 6 parts.



E. 9. What is the area of a rhombus, the length of whose side is 81 feet, and height 9 feet 6 inches?

By decimals.

81
9,5
405
729
769,5
12
6,0

By duodecimals.

F.	I.	
81	0	
9	6	
729	0	
40	6	0
769	6	0

By practice.

I.	F.	
$6 = \frac{1}{2}$)	81	
	9	
	729	
	40	6
	769	6

Answer 769 feet, 6 inches.

E. 10. How many square feet of paving are there in a court yard, in the form of a rhombus, whose length is 64 feet 6 inches, and perpendicular breadth 47 feet 8 inches?

By decimals.

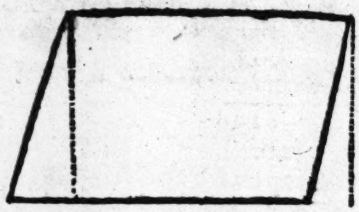
47,666
64,5
238330
190664
285996
3074,4570
12
5,4840
12
5,8080

By duodecimals.

F.	I.
64	6
47	8
448	
256	
23	6
43	0
3074	6 Answer.

E. 11. What is the area of a rhomboides, whose length is 26,5 feet, and perpendicular height 20,2 feet?

26,5



26,5
20,2
530
530
Anſwer 535,30=535*feet*, 3*inches*, 7″.

E. 12. What is the area of a rhomboides, whoſe length is 36 feet 9 inches, and perpendicular breadth 18 feet 6 inches?

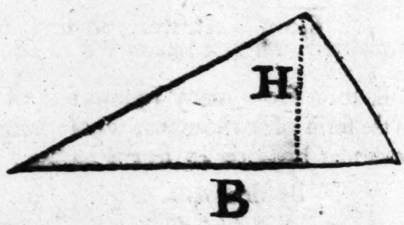
By decimals.	By duodecimals.
36,75	36 9
18,5	18 6
18375	648 0
29400	13 6
3675	18 4 6
Anſwer 679,875=679*ft*.10*in*.6″.	Anſwer 679 10 6 as before.

PROB. 2. *To find the area of a triangle.*

RULE. Multiply the baſe by half the perpendicular height, the product will be the area; or multiply the baſe by the perpendicular, and half the product will be the area.

EXAMPLE 1. What is the area of a triangle, whoſe baſe B is 36 feet, and perpendicular height H 16 feet?

	Or thus:
63	36
16	8
2)576	
Anſ. 288	Anſ. 288



E. 2. Required the area of a triangle, whoſe baſe is 6 feet 6 inches, and perpendicular height 4 feet 3 inches?

By decimals.	By duodecimals.	By practice.
	F. I.	*I.* *F. I.*
4,25	6 6	3 = ¼)6 6
6,5	4 3	4
2125	26 0	26 0
2550	1 7 6	1 7 6
2)27,625	2)27 7 6	2)27 7 6
13,8125	13 9 9	13 9 9
12		
9,7500		
12		
9,0000		

Anſwer 13 *feet*, 9 *inches*, 9″.

E. 3.

E. 3. How many acres are in a triangular field, whose base is 28, and perpendicular 21,30 chains?

21,30	Perpendicular
14	Half base
,29820	
4	
1,19280	
40	
7,71200	

Answer 1 *rood*, 7*poles*, ,712.

Note. The reason of pointing off 5 figures to the right, is the same as dividing by 100000, the number of square links in an acre.

PROB. 3. *To find the area of a triangle, whose three sides only are given.*

RULE. From half the sum of the three sides, subtract each side severally; multiply the half sum, and the three remainders continually together, and the square root of the product will be the area of the triangle.

E. 1. What is the area of a triangle, whose three sides measure 12, 18, and 24 feet respectively?

12	
18	
24	
2)54	Sum
27	Half sum
15	
405	
9	
3645	
3	
10935(104,57	feet, the area
1	
204)935	
816	
2085)11900	
10425	
20907)147500	
146349	
1151	Remains

27	
12	
15	First difference

27		27	
18		24	
9	Second diff.	3	Third diff.

E. 2. Required the area of a triangle, whose three sides measure 50, 60, and 70?

First, $\frac{50+60+70}{2} = \frac{180}{2} = 90.$

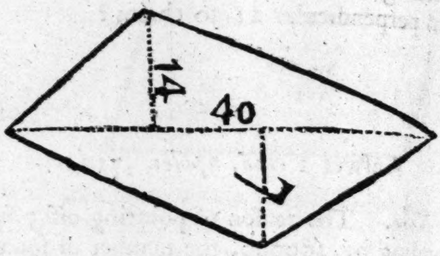
And $90 - \left\{\begin{array}{l} 50 = 40 \\ 60 = 30 \\ 70 = 20 \end{array}\right\}$ The three remainders.

Then $90 \times 40 \times 30 \times 20 = 2160000$. $\therefore \sqrt{2160000} = 1469,69$ the answer.

PROB. 4. *To find the area of a trapezium.*

RULE. Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and half the product will be the area.

E. 1. What is the area of a trapezium, whose diagonal is 40, and the two perpendiculars 14 and 7 feet?




```
    14
     7
    --
    21 Sum of the perpendiculars
    40 Diagonal
    --
  2)840
    ---
   420 Feet, answer.
```

E. 2. How many square yards of paving are there in a trapezium, whose diagonal line is 102 feet, and perpendiculars 30 feet and 24 feet?

```
         30
         24
         --
         54 Sum of the perpendiculars
        102 Diagonal
        ---
        108
       54
     ----
   2)5508
     ----
   9)2754
     ----
Answer 306 Yards.
```

PROB. 5. *To find the area of any irregular polygon.*

RULE. Divide it into triangles, in the manner you judge most convenient; then the sum of the areas of those triangles, calculated by Problem 2, will be the area of the irregular polygon.

E. 1. What is the area of an irregular polygon, whose two diagonals measure 49 and 39 feet, and three perpendiculars, 19, 9 and 14 feet respectively?

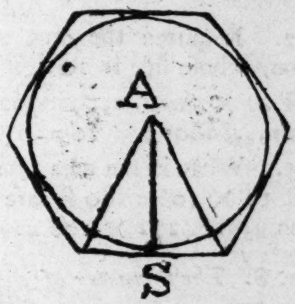
```
     19          39
      9          14
     --          --
     28       2)546
     49         ---
     --         273
    252
   112
   ----
 2)1372
   ----
    686 Area of the trapezium
    273 Area of the triangle
    ---
    959 Area of the polygon Ans.
```

PROB.

Prob. 6. *To find the area of a regular polygon.*

RULE. Multiply the whole perimeter, or ſum of the ſides, by half a perpendicular, let fall from the center, upon one of the ſides, and the product will be the area.

E. 1. What is the area of the regular hexagon, whoſe ſide S, is 20 feet, and perpendicular AS 16 feet?




```
     20
      6
    ---
    120 Perimeter
      8 Half perpendicular
    ---
Anſ. 960 Square feet.
```

E. 2. What is the area of a pentagon, whoſe ſide is 14,6 feet, and perpendicular 12,64 feet?

```
   14,6
      5
  -----
   73,0 Perimeter
   6,32 Half perpendicular
  -----
   1460
  2190
 4380
 -------
461,360 Square feet, anſwer.
```

E. 3. What is the area of an octagon, whoſe ſide is 9,5 and perpendicular 12?

```
   9,5
     8
  ----
  76,0 Perimeter
     6 Half perpendicular
  -----
 456,0 Anſwer
```

Prob. 7. *To find the area of a regular polygon, when the ſide only is given.*

RULE. Multiply the ſquare of the ſide of the polygon, by the multiplier ſtanding oppoſite to its name in the following table, and the product will be the anſwer; and if you multiply the ſide of any polygon, by the tabular perpendicular, it will give the radius of the inſcribed circle of ſuch polygon.

TABLE.

NAMES.	*No. of Sides*	*Angle at the Center.*			*Perpendiculars.*	*Multipliers.*
Trigon or equilateral △	3	120°	0′	0″	0,2886751	0,4330127
Tetragon or ſquare -	4	90	0	0	0,5000000	1,0000000
Pentagon - - - -	5	72	0	0	0,688191	1,7204774
Hexagon - - - -	6	60	0	0	0,866025	2,5980762
Heptagon - - - -	7	51	25	$42\frac{6}{7}$	1,038260	3,6339124
Octagon - - - -	8	45	0	0	1,2071068	4,8284272
Nonagon - - - -	9	40	0	0	1,373739	6,1818241
Decagon - - - -	10	36	0	0	1,5388418	7,6942087
Undecagon - - - -	11	32	43	$38\frac{2}{11}$	1,702844	9,3656412
Duodecagon - - -	12	30	0	0	1,866026	11,1961524

E. 1. What is the area of a pentagon, whose side is 10? First 10 × 10 = 100 square of the side. Then 1,7204774

100

Answer 172,0477400 the area.

E. 2. Required the area and radius of the inscribed circle of an hexagon, whose side is 20?

First 20 × 20 × 2,5980762 = 1039,23 the area.

Again, ,866025 × 20 = 17,32 the radius of the inscribed circle.

E. 3. What is the area of an octagon whose side is 10?

First 10 × 10 = 100 square of the side.

Then 4,8284272 × 100 = 482,84272 the area.

PROB. 8. *The diameter of a circle being given, to find the circumference; or the circumference given, to find the diameter.*

RULE 1. As 7 is to 22, so is the diameter to the circumference, Or, as 22 is to 7, so is the circumference to the diameter.

E. 1. If the diameter of a circle be 7, what is the circumference;

As 7 : 22 :: 7

7

7) 154

Answer 22 per rule.

E. 2. If the circumference of a circle be 22, what is the diameter?

As 22 : 7 :: 22

7

22) 154(7 Ans.

154

...

E. 3. If the diameter of a circle be 4, what is the circumference?

As 7 : 22 :: 4

4

7)88,00

Answer 12,57 the circumference.

E. 4. If the circumference of a circle be 12,57, what is the diameter? As 22 : 7 :: 12,57

7

22)87,99(3,999 = 4, nearly the diameter.

RULE

RULE 2. As 113 is to 355, so is the diameter to the circumference. Or, as 355 is to 113, so is the circumference to the diameter.

E. 1. If the diameter of a circle be 5 feet, what is the circumference?

```
As 113 : 355 :: 5
          5
113)1775(15,7 Feet the
    113    circumference.
    ---
    645
    565
    ---
     800
     791
     ---
       9
```

E. 2. What is the diameter of that circle, whose circumference is 15,7?

```
As 355 : 113 :: 15,7
         15,7
         791
        565
       113
355)1774,1(4,99=5,near-
    1420      (ly the diam.
    ----
     3541
     3195
     ----
      3460
      3195
      ----
       265
```

RULE 3. Multiply the diameter by 3,1416, and the product will be the circumference, Or, divide the circumference by 3,1416, and the quotient will be the diameter.

E. 1. If the diameter of a circle be 10, what is the circumference?

```
        3,1416
            10
        ------
Answer  31,416 = the circumference.
```

E. 2. If the circumference of a circle be 31,416, what is the diameter?

```
3,1416)31,4160(10 = the diameter.
       31416
       -----
       .....0
```

E. 3. If the diameter of a circle be 100 inches, what is the circumference?

```
        3,1416
           100
        ------
Answer  314,16 the circumference.
```

E. 4. The circumference of the earth is known to be 25020 miles, what is the diameter?

```
3,1416)25020,0000(7964 Miles, the diameter.
       219912
       ------
        302880
        282744
        ------
         201360
         188496
         ------
          128640
          125664
          ------
            2976
```

PROB.

Prob. 9. *To find the area of a circle.*

RULES. 1. Multiply half the circumference by half the diameter, and the product will be the area.

2. Multiply the square of the diameter by ,7854, the product will be the area.

3. Multiply the square of the circumference by ,079574, the product is the area.

4. Multiply the square of the semi-diameter by 3,1416, the product will be the area.

5. Multiply the circumference by the diameter, and a fourth part of the product will be the area.

Note. ,7854, and 3,1416, are areas of circles, whose diameters are 1 and 2, and ,079577, is the area of a circle, whose circumference is 1, likewise 452, and 1,273939, are squares of the diameters of circles. whose areas are 355, and 1, and 1,12831 is the diameter of a circle, whose area is equal to a square, whose side is 1.

E. 1. What is the area of a circle, whose diameter is 100 inches, and circumference 314,16?

By rule 2,
```
   100
   100
 -----
 10000
 ,7854
 -----
Answer 7854 area
```

Or thus, by rule 4, 50 × 50 = 2500. Then 3,1416 × 2500 = 7854, the area, as before.

By rule 1,
```
 2)314,16
   ------
   15708 = Half circumference
      50 = Half diameter
   ------
Answer 7854,000 Area, as before.
```

E. 2. What is the area of a circle, whose diameter is 17, and circumference 53,4072 inches?

By rule 1,
```
 2)53,4072
   -------
   26,7036 = Half circumference
       8,5 = Half diameter
   -------
Answer 226,98060 Inches.
```

E. 3. What is the area of a circle whose diameter is 3 feet?

By rule 2,
```
   ,7854
       9 = Square of the diameter
   -----
Answer 7,0686 = 7 Feet, 0 inches, 9 parts.
```

E. 4 If the diameter of a circle be 4 inches, what is the area?

By rule 2,
```
   ,7854
      16 = Square of the diameter
   -----
Answer 12,5664 Inches.
```

E. 5.

E. 5. What is the area of a circle, whose diameter is 4, and circumference 12,5664 inches?

By rule 1, 2)12,5664

6,2832 = Half the circumference.

2

Answer 12,5664 Inches, the area.

Note. In this example it may be observed that when the diameter is 4, the circumference and area are equal.

PROB. 10. *To find the length of any arch of a circle.*

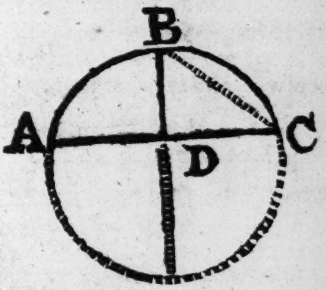
RULE 1. From 8 times the chord of half the arch, subtract the chord of the whole arch, and one-third of the remainder will be the length of the arch nearly.

E. 1. The chord of the whole arch A C is 60, and the versed sine D B of half the arch is 10; what is the length of the arch A B C?

30
30
900 = Square of D C, or half A C
100 = Square of B D

1000,00000000(31,6227 = B C, chord of half the arch.

```
9                         8
61)100                   252,9816
   61                    — 60
626)3900                3)192,9816
    3756                  64,3272 = Length of
6322)14400              [the arch required, ans.
     12644
63242)175600
      126484
63244)4911600
      4427129
       484471
```



E. 2. The chord of the whole arch is 48, and the versed sine of half the arch is 18, what is the length of the arch?

First, $\overline{18\times18+24\times24}=900$; and $\sqrt{900}=30=$ chord of half the arch.

Then $\frac{30\times8-48}{3}=64$, length of the arch required.

E. 3.

E. 3. The chord of the whole arch is 50,8, and the chord of half the arch is 30,6; what is the length of the arch?

$$\begin{array}{r} 30{,}6 \\ 8 \\ \hline 244{,}8 \\ -\ 50{,}8 \\ \hline 3)194{,}0 \\ \hline 64{,}6 \end{array}$$

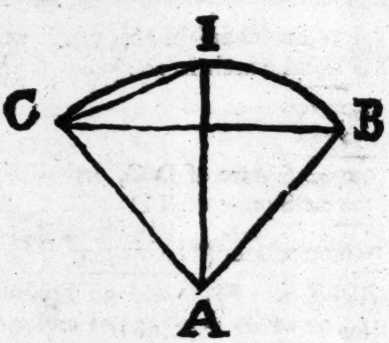
Answer 64,6 length of the arch.

PROB. 11. *To find the area of a sector, or that part of a circle which is bounded by any two radii and their included arch.*

RULE. Multiply the radius by half the arch of the sector, found by the last problem, and the product will be the area, or superficial content.

Note. A sector may be either less or greater than a semi-circle.

E. 1. The radius A B is, 25 and the length of the arch C B 21,5 required the area of the sector?



$$\begin{array}{r} 2)21{,}5 \\ \hline 10{,}75 \\ 25 \\ \hline 5375 \\ 2150 \\ \hline 268{,}75 \end{array}$$

10,75 = Half the arch

Answ. 268,75 the area.

E. 2. What is the area of a sector, whose radius is 45, and the length of the arch 49?

$$\begin{array}{r} 2)49 \\ \hline 24{,}5 \\ 45 \\ \hline 1225 \\ 980 \\ \hline 1102{,}5 \end{array}$$

24,5 = Half the arch

Answer 1102,5 the area.

PROB. 12. *To find the area of a segment of a circle.* (See Fig. to Prob. 11.)

RULE 1. Find the area of a sector, having the same arch with the segment, by the last problem.

2. Find the area of the triangle formed by the chord of the segment, and the radii of the sector.

3. The sum, or difference of these areas, according as the segment is greater or less than a semi-circle, will be the area required.

Note. If the segment is greater than a semi-circle, the area of the triangle must be added to that of the sector, and the product will be the area of the segment.

E. 1.

E. 1. If the ſemi-diameter A B, of a circle, be 24,5, the arch line 45,6, the chord upon which the triangle is formed 30,5, and the perpendicular of the triangle 16,2; what is the area of the ſegment?

24,5 = Semi-diameter
22,8 = Half the arch line
———
1960
490
490
———
558,60 Area of the ſector
247,05
———
311,55=Area of the ſegm. Anſ.

30,5 Chord
8,1 = Half perpendicular
———
305
2440
———
247,05 = Arch of the triangle.

E. 2. What is the area of a ſegment whoſe radius is 11,64, arch line 48, the chord upon which the triangle is formed 20,5, and the perpendicular of the triangle 5,53?

11,64 Radius
24 Half arch line
———
4656
2328
———
279,36 Area of the ſector
56,6825
———
anſ. 336,0425 Area of the ſegment.

2,765 = Half perpendicular
20,5 = Chord line
———
13825
5530
———
56,6825 = Area of the triangle

RULE 2. Firſt, add the ſquare of half the chord of the ſegment to the ſquare of its height, and multiply the ſquare root of the ſum by 4.

2. To $\frac{1}{3}$ of the number laſt found, add the whole chord of the ſegment, this ſum multiplied by $\frac{2}{5}$ of the height, will give the area.

E. 1. If the chord of a ſegment be 20, and its height or verſed ſine 5, what is the area of the ſegment?

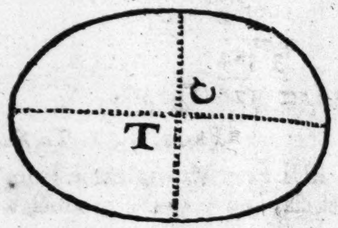
10
10
———
100 Square of half chord
25 Square of the height
———
125(11,18
1
———
21)25
21
———
221)400
221
———
2228)17900
17824
———
76

11,18
4
———
3)44,72
———
14,906
+20
———
34,906
2 = $\frac{2}{5}$ of the height
———
Anſ. 69,812 Area of the ſegment.

PROB. 13. *To find the area of an ellipsis, or oval.*

RULE. Multiply the transverse diameter by the conjugate, then multiply that product by ,7854, this last product is the area of the oval.

E. 1. What is the area of an ellipsis, whose transverse diameter T is 22, and conjugate C 16?




```
       22
       16
    -----
      352
    ,7854
   ------
    15708
   39270
  23562
  --------
Ans. 276,4608 the area.
```

E. 2. If the axis of an ellipsis be 36 and 26, what is the area?

```
       36
       26
    -----
      216
      72
    -----
      936
    ,7854
   ------
    47124
   23562
  70686
  --------
  735,1344 the area required.
```

E. 3. What is the area of an ellipsis, whose greatest diameter is 100, and least diameter 70?

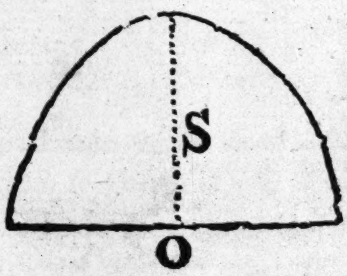
$,7854 \times 100 \times 70 = 5497,8$, the area.

PROB. 14. *To find the area of a parabola.*

RULE. Multiply the base, or greatest ordinate, by the height, or abscissa; and $\frac{2}{3}$ of the product will be the area.

E. 1. What is the area of the parabola, whose height S is 12, and the base, or greatest ordinate O, 36?

```
      36
      12
    -----
     432
       2
    -----
   3)864
Answer 288 the area required.
```



E. 2.

E. 2. What is the area of a parabola, whose base or greatest ordinate is 24, and the abscissa 8?

$$\begin{array}{r} 24 \\ 8 \\ \hline 192 \\ 2 \\ \hline 3)384 \\ \hline \end{array}$$

Answer 128 The area.

E. 3. The abscissa is 39,25, and the greatest ordinate, or base, 53,75; what is the area?

$$\frac{53{,}75 \times 39{,}25 \times 2}{3} = 1406{,}4583,$$

the area, answer.

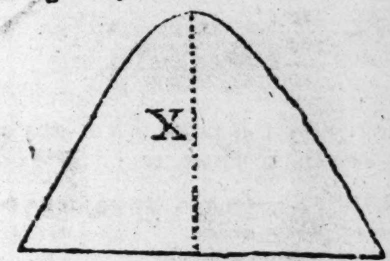
Prob. 15. *To find the area of an hyperbola.*

RULE. Multiply the base, or greatest ordinate, by the height, or abscissa, and $\frac{5}{8}$ of the product will be the area, nearly.

E. 1. What is the area of an hyperbola, whose base, or greatest ordinate O is 24, and the abscissa or height X, 10?

$$\begin{array}{r} 24 \\ 10 \\ \hline 240 \\ 5 \\ \hline 8)1200 \\ \hline \end{array}$$

Ans. 150 The area.



E. 2. Required the area of an hyperbola, whose base, or greatest ordinate, is 36, and the perpendicular height, or abscissa, 12?

$$\begin{array}{r} 36 \\ 12 \\ \hline 432 \\ 5 \\ \hline 8)2160 \\ \hline \end{array}$$

Answer 270 The area.

Note. The above rule is only an approximation, but will serve very well for common purposes.

Prob. 16. *To find the area of a spherical triangle.*

RULE. From the sum of the three angles, subtract 180 degrees; multiply the superficies of the whole sphere, or globe, by the remainder, this product divide by 720; the quotient is the area of the triangle.

E. 1. Suppose the angle at A = 36°; at B 148°; at C 32°, and the diameter of the globe 29; what is the area of the triangle A B C?

 First

First 36 + 148 + 32 = 216
And 216 — 180 = 36

Then 2642 surface of the [sphere*
36

15852
7926

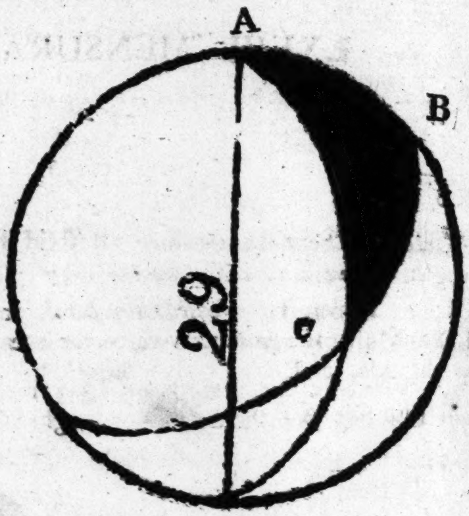
720)95112(132,1 = area [of the triangle
720

2311
2160

1512
1440

720
720

0



Note. By this problem you may find the number of miles or acres contained in the whole, or any part of the surface of the globe.

PROB. 17. *To find the areas of lunes, or the spaces included between the intersecting arches of two eccentric circles.* (See Plate 1, Fig. 31.)

RULE. Find the areas of the two segments from which the lune is formed, and their difference will be the area required.

EXAMPLE. Suppose the length of the chord A C is 40, the height E F 10, and E D 4; what is the area of the lune A F C D A?

First 400 = Square of half A C
100 = Square of E F

500 Then √500 = 22,36

22,36
4
3)89,44
29,81
+40
69,81
4 = ⅖ of the height
279,24 = Area of the segment [A C F A.

Again 400 = Square of half A C
16 = Square of E D
416 And √416 =
20,396
4
3)81,584
27,194
+40
67,194
1,6 = ⅖ of the height
403164
67194
107,5104 = Area of the segment [A C D A.

279,24
107,5104
Answ. 171,7296 The area of the lune required.

* The surface of a Sphere may be found by Problem 9, Sect. 70.

LXVIII.

LXVIII. MENSURATION

OF

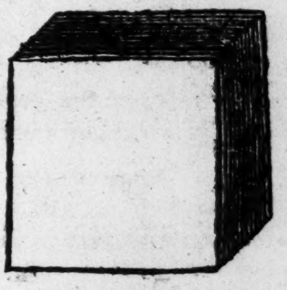
SOLIDS.

TEACHETH how to meaſure all ſolid bodies, which conſiſt of length, breadth, and thickneſs.

PROB. 1, *To find the ſolidity of a cube.*

RULE. Multiply the ſide of the cube into itſelf, and that product again by the ſide, and it will give the ſolidity.

E. 1. The ſide A B of the cube is 6,5, what is the ſolidity?




```
        6,5
        6,5
       ----
        325
       390
      -----
      42,25
        6,5
      -----
      21125
     25350
     ------
Anſwer 274,625 the ſolidity.
```

E. 2. The ſide of a cube is 12 inches, what is the ſolidity?

```
       12
       12
       --
      144
       12
     ----
Anſwer 1728 inches.
```

E. 3. What is the ſolidity of a cube, whoſe ſide is 21,5?

```
          21,5
          21,5
          ----
          1075
          215
         430
        ------
        462,25
          21,5
        ------
        231125
        46225
       92450
      --------
Anſw. 9938,375 the ſolidity.
```

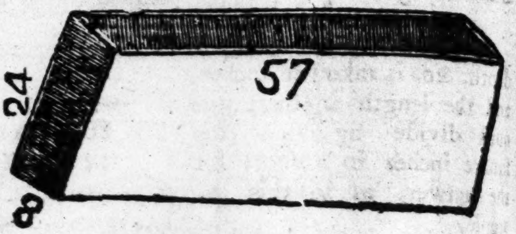
Note. If the anſwer be in inches, you muſt divide by 1728 (the ſolid inches in a foot) to bring them into feet.

PROB. 2. *To find the ſolidity of a parallelopipedon.*

RULE. Multiply the length by the breadth, and that product again by the depth or altitude, and it will give the ſolidity.

E. 1.

E. 1. Required the solidity of the parallelopipedon, whose length is 57 feet, breadth 24, and depth 8?



57 Length
24 Breadth
228
114
1368
8

Answ. 10944 the solidity.

E. 2. The length of a parallelopipedon is 12 feet, and each side of its square base 1 foot 6 inches; what is the solidity?

12 Length
1,5 Breadth
60
12
18,0
1,5 Depth.

Answ. 27,00 Feet, the solidity.

E. 3. What is the solid content of a block of marble, whose length is 8 feet, breadth 4, and depth $2\frac{1}{2}$ feet?

8
4
32
2,5
160
64

Answ. 80,0 Feet, the solidity.

PROB. 3. *To find the solidity of a prism.*

RULE. Multiply the area of the base into the height, and the product will be the solidity.

E. 1. What is the solidity of a square prism, whose length is 54 feet, and each of the equal sides 11 feet?

11
11
121 Area of the base
54 Length
484
605

Answer 6534 Feet, the solidity.

Note. To find the convex surface of a cube, parallelopipedon, or prism, you must find the area of each side and end separately, which areas added together, will give the whole surface.

E. 2.

E. 2. What is the ſolidity of a triangular priſm, each ſide of the baſe of which is 16 inches, the perpendicular of the baſe 10 inches, and the length of the ſolid 12 feet?

Note. When the breadth, depth, &c. is taken in inches, and the length in feet, you muſt divide by 144 (the ſquare inches in a foot) for the anſwer, as in this example.

16
5
———
80 Area of the baſe
12 Length
———
144)960(6,6 = 6 feet, $7\frac{1}{2}$ inches, anſw.
864
———
960
864
———
96
———

E. 3. What is the ſolidity of a priſm, whoſe baſe is a hexagon, ſuppoſing each of the equal ſides to be 12 inches, the perpendicular from the center to one of the ſides 10,5, and the length of the priſm 53 inches?

12
6
———
72 = Perimeter
5,25 = $\frac{1}{2}$ Perpendicular
———
360
144
360
———
378,00 = Area of the baſe
53 = Length
———
1134
1890
———
Anſw. 20034 the ſolidity.
———

PROB. 4. *To find the ſolidity of a cylinder.*

RULE. Multiply the area of the baſe by the height, and the product will be the ſolidity.

E. 1.

E. 1. What is the solidity of a cylinder, the diameter of whose base is 3 feet, and length 53?

,7854
9 = Square of the base
————
7,0686 = Area of the base
53
————
212058
354430
————
375,6358

E. 2. What is the solidity of a cylinder, whose height is 6 feet, and diameter of the end 2 feet? First ,7854 × 2 × 2 = 3,1416, the area of the base. ∵ 3,1416 × 6 = 18,8496, the solidity.

Note. To find the convex surface of a cylinder, multiply the periphery of the end by the height of the cylinder, the product will be the convex surface.

Example. What is the convex surface of a cylinder, whose length is 53 feet, and the diameter of its base 3 feet?

3,1416
3
————
9,4248 = Periphery of the base
53 = Length
————
282744
471240
————
Answer 499,5144 convex surface.

Prob. 5. *To find the solidity of a cone or pyramid.*

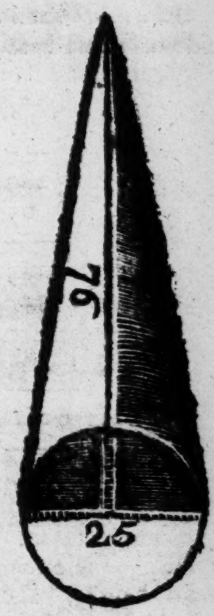
RULE. Multiply the area of the base by $\frac{1}{3}$ of the height, and the product will be the solidity.

E. 1. What is the solidity of a cone, whose diameter at the base is 10, and its altitude 60?

,7854
100 = Square of the diameter
————
78,5400 = Area of the base
20 = $\frac{1}{3}$ of the height.
————
Answer 1570,80 the solidity.

E. 2.

E. 2. What is the ſolidity of a cone, whoſe diameter is 25, and its perpendicular height 76?



,7854
625 = Square of the diameter

39270
15708
47124

490,8750 = Area of the baſe
25,3 = $\frac{1}{3}$ of the height

1472625
2454375
981750

Anſwer 12419,1375 The ſolidity.

E. 3. Required the ſolidity of a hexagonal pyramid, each of whoſe equal ſides of its baſe being 14, and the perpendicular height 67?

2,5980762 = Tabular multiplier
196 = Square of the diameter

155884572
233826858
25980762

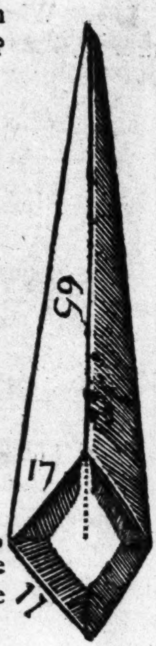
509,2229352 = Area of the baſe
67 = Height

35645605464
30553376112

3)34117,9366584

Anſwer 11372,6455528 The ſolidity.

E. 4. What is the solidity of a square pyramid, each side of whose base is 17, and the perpendicular height 65?



$$\begin{array}{r l}
17 & \\
17 & \\
\hline
289 & = \text{Area of the base} \\
65 & = \text{Height} \\
\hline
1445 & \\
1734 & \\
\hline
3)18785 & \\
\hline
\end{array}$$

Answer $6261\frac{2}{3}$ The solidity.

Note. To find the convex surface of a cone, or pyramid, multiply the circumference of the base or perimeter by the slant height, and half the product will be the surface required.

E. 1. What is the convex surface of a cone, whose base is 32 feet in circumference, and slant side 20 feet in length?

$$\begin{array}{r}
32 \\
10 \\
\hline
2)320 \\
\hline
\end{array}$$

Answer 160 The convex surface.

E. 2. What is the surface of a square pyramid, each of whose equal sides is 17, and the slant height 64?

$$\begin{array}{r}
17 \\
4 \\
\hline
68 \\
64 \\
\hline
272 \\
408 \\
\hline
\end{array}$$

Answer 4352 The convex surface.

PROB. 6. *To find the solidity of the frustum of a cone or pyramid.*

RULE 1. For the frustum of a cone, divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters; this quotient being multiplied by ,7854, and again by $\frac{1}{3}$ of the height, will give the solidity.

2. For the frustum of a pyramid: To the areas of the two ends of the frustum, add the square root of their products; this sum being multiplied by $\frac{1}{3}$ of the height, will give the solidity.

E. 1.

E. 1. What is the content of the fruſtum of a cone, the diameter of whoſe greater end D is 4 feet, that of the leſſer end *d* 2 feet, and the perpendicular height D*d* 9 feet?

4   2   4
4   2   2
16   4   2 = Difference of D and *d*.
4   2
64 = Cube of D   8 = Cube of *d*
− 8
2)56
28
3 = $\frac{1}{3}$ of D*d*
84

,7854
84
31416
62832

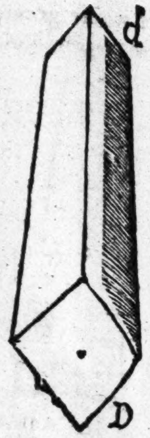
Anſwer 65,9736 the ſolidity.

E. 2. What is the content of the fruſtum of a cone, 18 feet high, the diameter of its ends being 10 and 6 feet?

10   6
10   6
100   36
10   6
1000 = Cube of the greater baſe   216 = Cube of the leſſer baſe
216
4) 784   ,7854   10
     1176   6
196   47124   4 = Difference of the ends
6 = $\frac{1}{3}$ of the height   54978
     7854
1176   7854

Anſwer 923,6304 = ſolidity required.

E. 3. What is the ſolidity of the fruſtum of a ſquare pyramid, one ſide of the greater end D being 8 inches, that of the leſſer end *d* 4 inches, and the height 60 inches?



8   4
8   4

64 = Area D   16 = Area *d*
16

384
64

. .
1024(32 = Square root of the products of the area of the two ends
9

62) 124
124

. . .

64
16
32

112
20 = $\frac{1}{3}$ of the height

2240 the solidity.

E. 4. Required the solidity of the frustum of a hexagonal pyramid, the side D of whose greater end is 3 feet, and *d* of the lesser end 2 feet, and the height 6 feet?

2,598 = Tabular multiplier
4 = Square of *d*

10,392 = Area of the polygon *d*.
2,598 = Tabular multiplier
9 = Square of D

23,382 = Area of the polygon D.
10,392

46764
210438
70146
23382

. . . . .
242,985744(15,588 = Square root of the product of the areas of the two ends.
1

25) 142
125

305) 1798
1525

3108) 27357
24864

31168) 249344
249344

10,392
23,382
15,588

49,362
2 = $\frac{1}{3}$ of the height

Answer 98,724 = solidity required.

Note.

Note. To find the convex ſurface of the fruſtum of a pyramid or cone, you muſt multiply the ſum of the perimeters or circumference of the ends, by the ſlant height, and half the product will be the ſurface required.

E. 1. How many ſquare feet are in the ſurface of a fruſtum of a ſquare pyramid, whoſe ſlant height is 10 feet, each ſide of the greater baſe being 3 feet 4 inches, and each ſide of the leſs 2 feet 2 inches?

f. *i.*
3 4
2 2
———
5 6
4
———
22 0 = Sum of perimeters
10
———
2)220
———
Anſw. 190 Feet, the content.

E. 2. How many ſquare feet are in the ſurface of a fruſtum of a cone, whoſe circumference of its ends are 64 and 16 feet, and ſlant ſide 14 feet?

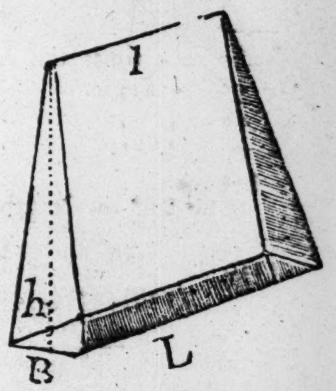
64
+ 16
———
80
14
———
2)1120
———
Anſwer 560 Feet the content.

PROB. 7. *To find the ſolidity of a cuneus or wedge.*

RULE. Multiply the ſum of twice the length of the baſe, and the length of the edge, by the product of the height of the wedge, and the breadth of the baſe, and $\frac{1}{6}$ of the laſt product will be the ſolidity.

E. 1. What is the ſolidity of a wedge, whoſe baſe is 4 inches long, and 2 inches broad; the length of the edge being 3 inches, and the perpendicular height 8 inches?

4
2
—
8 = Twice the length of baſe L
3 = Length of the edge *l*
—
11 = Sum.
8 = Height of the wedge *h*
2 = Breadth of the wedge B
—
16 = Product
11
—
6)176



Anſwer $29\frac{2}{6}$ inches the ſolidity.

When the length of the baſe is equal to that of the edge, the wedge is evidently equal to half a priſm of the ſame baſe and altitude, and may be meaſured by the following

RULE,

RULE. Multiply the area of the base by half the altitude of the wedge, and the product will give the solidity.

EXAMPLE. What is the solidity of a wedge whose base measures 15 feet by 8, and perpendicular height 6 feet?

15
8
———
120 = Area of the base
3 = ½ the altitude
———
Answer. 360 Feet the solidity.

PROB. 8. *To find the solidity of the prismoid.*

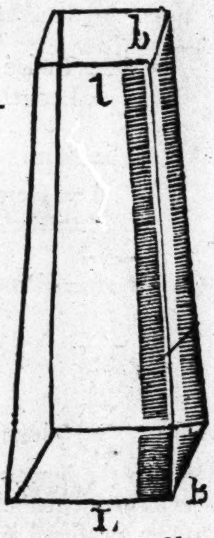
RULE. To the sum of the areas of the two ends, add four times the area of a section parallel to, and equally distant from both ends, and this last sum multiplied by ⅙ of the height will give the solidity.

EXAMPLE I. How many solid feet are there in a tree whose ends are rectangles, the length and breadth of the greater end measures 12 inches by 8, and the lesser end 8 inches by 6; and the length 60 inches?

12
8
———
96 = Area of the greater L B.
12
8
———
2)20
———
10 = length of the middle rectangle.
7
———
70
4
———
280 = 4 times the area of the middle rectangle.
96
48
———
424
10 = ⅙ of the length.
———
4240

8
6
———
48 = Area of the lesser base *l.b.*
8
6
———
2)14
———
7 = breadth of the middle rectangle.

1728)4240(2,45 solid feet the answer.
3456
———
7840
6912
———
9280
8640
———
640



Note.

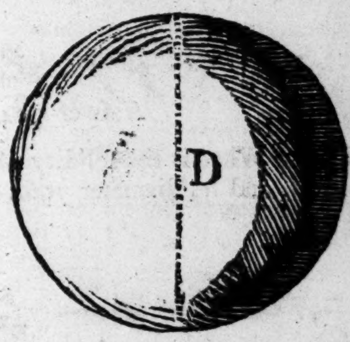
Note. The length of the middle rectangle is equal to half the ſum of the lengths of the rectangles of the two ends, and its breadth equal to half the ſum of the breadths of thoſe rectangles.

PROB. 9. *To find the convex ſurface of a ſphere or globe, or any ſegment or zone of it.*

RULE. Multiply the circumference of the ſphere by its diameter or height of the part required, and the product will be the convex ſurface.

Note. The height of the whole ſphere is its diameter.

E. 1. What is the convex ſurface of a globe, whoſe diameter D is 3 inches?



$$\begin{array}{r l} 3,1416 & \\ 3 & \\ \hline 9,4248 & = \text{Circumference} \\ 3 & = \text{Diameter} \\ \hline \end{array}$$

Anſ. 28,2744 the ſurface req.

E. 2. What is the convex ſurface of a globe, whoſe diameter is 9 inches?

$$\begin{array}{r l} 3,1416 & \\ 9 & \\ \hline 28,2744 & = \text{Circumference} \\ 9 & = \text{Diameter} \\ \hline \end{array}$$

Anſ. 254,4696 the ſurface.

E. 3. If the diameter of the earth be 7964 miles, what is the whole ſurface, ſuppoſing it to be a perfect ſphere?

$$\begin{array}{r l} 3,1416 & \\ 7964 & \\ \hline 125664 & \\ 188496 & \\ 282744 & \\ 219912 & \\ \hline 25019,7024 & = \text{Circumference} \\ 7964 & \\ \hline 1000788096 & \\ 1501182144 & \\ 2251773216 & \\ 1751379168 & \\ \hline \text{Anſwer } 199256909,9136 & \text{ſquare miles.} \end{array}$$

PROB. 10.

Prob. 10. *To find the ſolidity of a ſphere or globe.*

RULE. Multiply the cube of the diameter by ,5236, and the product will be the ſolidity.

E. 1. How many ſolid feet are there in a globe, whoſe diameter is 12 inches?

```
       12
       12
      ---
      144
       12
     ----
     1728 = Cube of the diameter
    ,5236
   ------
    10368
    5184
   3456
  8640
  ---------
Anſwer 904,7808 inches the ſolidity.
```

E. 2. What is the ſolidity of the earth, ſuppoſing it to be perfectly ſpherical, and its diameter 7964 miles?

```
             7964
             7964
            -----
            31856
           47784
          71676
         55748
        ---------
         63425296
             7964
       -----------
        253701184
       380551776
      570827664
     443977072
     ---------------
     505119057344 = Cube of the diameter
            ,5236
   -----------------
   3030714344064
  1515357172032
 1010238114688
2525595286720
-------------------
Anſwer 264480338425,3184 miles the ſolidity.
```

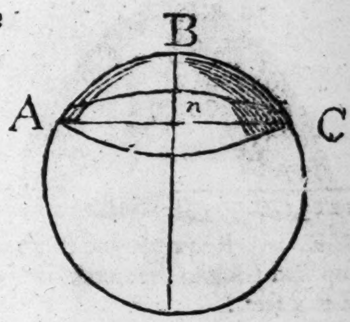
Prob. 11. *To find the ſolidity of the ſegment of a ſphere.*

RULE. To three times the ſquare of the radius of its baſe, add the ſquare of its height; this ſum multiplied by the height, and the product again by ,5236, will give the ſolidity.

E. 1.

E. 1. The radius *A n* of the baſe of the ſegment of ABC, is 12 inches, and the height *B n* 9 inches; what is its ſolidity?

12
12
———
144
3
———
432 = 3 times ſquare of *An*
81 = Square of *B n*
———
513
9 = height *B n*
———
4617
,5236
———
27702
13851
9234
23085
———
Anſw. 2417,4612 ſolid inches.



E. 2. What is the ſolidity of a ſegment of a ſphere, the baſe of whoſe diameter is 14 inches, and its height 5?

7
7
———
49
3
———
147
25 = Square of the height
———
172
5 = Height
———
860

,5236
860
———
314160
41888
———
Anſ. 450,2960 ſolid inches.

Prob. 12. *To find the ſolidity of a fruſtum or zone of a ſphere.*

RULE. To the ſum of the ſquares of the radii of the two ends, add $\frac{1}{3}$ of the ſquare of their diſtance, or the breadth of the zone; and this ſum multiplied by the ſaid breadth, and the product again by 1,5708, will give the ſolidity.

E. 1. What is the ſolid content of the fruſtum of the ſphere A B C D, whoſe greater diameter A B is 48 inches, the leſſer diameter D C 40 inches, and the diſtance of the ends 8 inches?

576 = Square of ½ A B
400 = Square of ½ D C

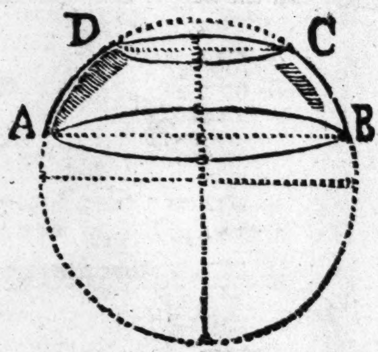
976
2,133 = ⅓ the ſquare of the dif. of the ends

997,33
8 = breadth or diſtance of the ends

7978,64
1,5708

6382912
55850480
3989320
797864

12532,847712 = Solid inches the anſwer.



E. 2. Required the ſolidity of the middle zone of a ſphere, whoſe top and bottom diameters are each 3 feet, and the breadth of the zone E F 4 feet?

2,25 = DF ^{2}
2,25

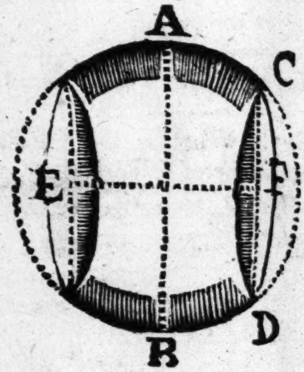
4,50
5,33 = ⅓ ſqu. of EF

9,83
4 = Breadth EF

39,32
1,5708

31456
275240
19660
3932

Anſwer 61,763856 ſolid feet



PROB. 13. *To find the ſolid content of a parabolic conoid.*

RULE. Multiply the area of the baſe by half the altitude, and the product will be the content.

E. 1. Required the ſolidity of a parabolic conoid, whoſe height is 24, and the diameter of its baſe 34?

24
34

136
102

1156 = Square of the baſe.

,7854
1156

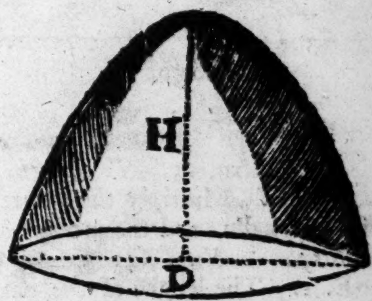
47124
39270
7854
7854

907,9224 = Areas of the baſe
12 = ½ the height

Anſwer 10895,0808 the ſolidity.

E. 2. What is the solidity of the paraboloid H D, whose height is 8 inches, and the diameter D of its circular base 24?

24
24
96
48
576 = Square of the diameter
,7854
2304
2880
4608
4032
452,3904 = Area of the base
4 = $\frac{1}{2}$ the height
1809,5616 Ans. the solidity.

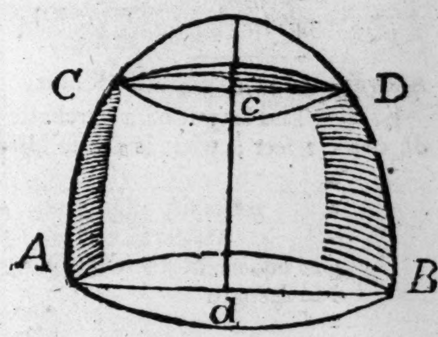


PROB. 14. *To find the solidity of the frustum of a parabolic conoid.*

RULE. Multiply the sum of the squares of the diameters of the two ends, by the height of the frustum, and the product again by ,3927, and it will give the solidity.

E. 1. What is the solidity of the parabolic frustum A B C D, the diameter A B of the greater end being 32, that of the lesser end C D 24 and the height *c d* 20?

32
32
64
96
A B^2 = 1024
C D^2 = 576
1600
20 = height *c d*
32000
,3927
224000
64000
288000
96000
Answer 12566,4000 the solidity.



E. 2.

E. 2. What is the solidity of the frustum of a parabolic conoid, the diameter of the greater end being 50, that of the lesser end 38, and the distance of the ends 16?

50	63104
50	,3927
2500 = □ of the greater base	441728
1444 = □ of the lesser base	126208
	567936
3944	189312
16 = height	
63104	Answer 24780,9408 the solidity.

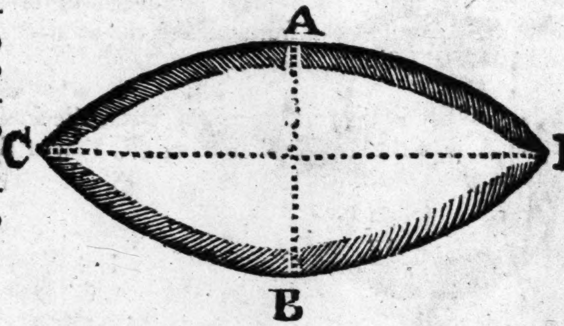
PROB. 15. *To find the solidity of a parabolic spindle.*

RULE. Multiply the square of the middle diameter by the length of the spindle, and the product again by ,41888, and it will give the solidity.——*Note.* ,4188 = $\frac{8}{15}$ of ,7854.

E. 1. The length of the parabolic spindle A B C D is 30, and the middle diameter A B 17; what is the solidity?

17
17
A B square = 289
C D = 30
8670
,41888

69360
69360
69360
8670
34680



Answer 3631,68960 the solidity.

E. 2. The length of a parabolic spindle is 6 feet, and the middle diameter 2 feet; what is the solidity?

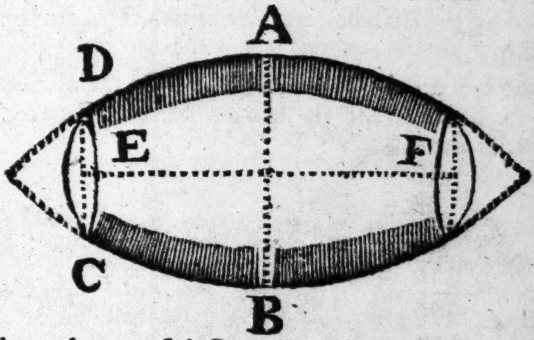
2	,41888
2	24
4 = Square of the diameter	167552
6 = Length	83776
24	Answer 10,05312 the solidity.

PROB. 16. *To find the solidity of a middle frustum of a parabolic spindle.*

RULE. Add 8 times the square of the middle diameter, 3 times the square of the less, and 4 times the product of those diameters into one sum; then this sum being multiplied by the length, and the product again by ,05236, will give the solidity.

E. 1.

E. 1. In the middle fruſtum A B E F of the parabolic ſpindle, the middle diameter A B is 72, the diameter of the end D C is 40, and the length E F 72; what is the ſolidity?



72
72
144
504
5184
8
41472 = 8 times the □ of A B
4800 = 3 times the □ of D C
11520 = 4 times the product of A B and D C
57792
72 = Length E F
115584
404544
4161024
,05236
24966144
12483072
8322048
20805120
217871,21664 Solidity, the anſwer.

E. 2. What is the ſolidity of the middle fruſtum of a parabolic ſpindle, the middle diameter being 16, the diameter at the ends 12, and the length 20?

16
16
256
8
2048 = 8 times the □ of the middle diameter
432 = 3 times the □ of the leſs diameter
768 = 4 times the product of the two ends.
3248
20 = Length
64960

64960
,05236
389760
194880
129920
324800
Anſwer 3401,30560 The ſolidity.

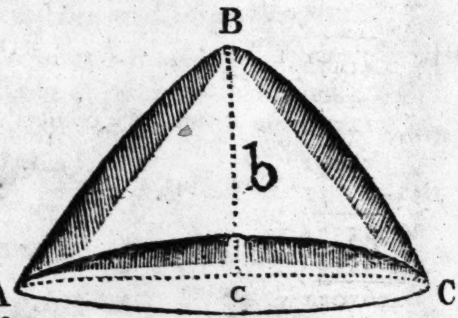
PROB.

PROB. 17. *To find the solidity of an hyperbolic conoid.*

RULE. To the square of the radius of the base, add the square of the middle diameter, between the base and the vertex; and this sum multiplied by the altitude, and the product again by ,5236, will give the solidity.

E. 1. In the hyperboloid A B C, the altitude *b* is 20, the radius *c* C of the base 24, and the middle diameter between the base and vertex 30; what is the solidity?

```
      30
      30
     ----
     900 = □ of the middle diameter
     576 = □ of the radius c C
    -----
    1476
      20 = Height
    -----
   29520
   ,5236
   ------
   177120
   88560
  59040
 147600
 ----------
 15456,6720 The solidity, answer.
```



E. 2. In an hyperbolic conoid, the altitude is 20, the radius of the base 32, and the middle diameter 48; what is the solidity?

```
  32          48
  32          48
  --          ---
  64          384
 96          192
 ----        ----
 1024        2304 = Square of the middle diameter
             1024 = Square of the radius
             ----
             3328
               20 = Height
            -----
            66560
            ,5236
           ------
           399360
          199680
         133120
        332800
        ----------
Answer 34850,8160 The solidity.
```

PROB. 18. *To find the solidity of the middle frustum of a circular spindle.*

RULE 1. Divide the square of half the length of the frustum by half the difference of the middle diameter, and that of either of the two ends, and

and $\frac{1}{2}$ this quotient added to $\frac{1}{4}$ of the ſaid difference, will give the radius of the circle.

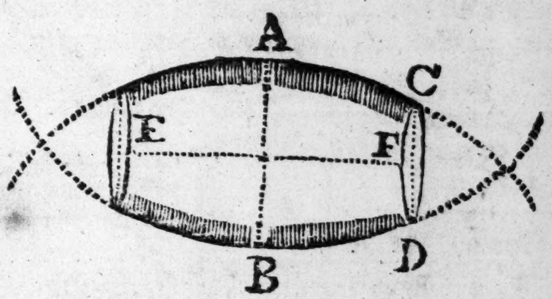
2. Find the central diſtance, by taking half the middle diameter from the radius of the circle.

3. From the ſquare of the radius, take the ſquare of the central diſtance, and the ſquare root of the remainder will give half the length of the ſpindle.

4. From the ſquare of half the length of the ſpindle, take $\frac{1}{3}$ of the ſquare of half the length of the fruſtum, and multiply the remainder into the ſaid half length.

5. Take this product from that of the generating area and central diſtance, and the remainder multiplied by 6,2832, will give the content of the fruſtum.

Example. What is the ſolidity of the fruſtum ABEF, whoſe middle diameter AB is 36, the diameter CD 16, and the length EF 40?



20
20
10)400 = □ of $\frac{1}{2}$ EF
2)40
20
+5
25 = Radius
18 = $\frac{1}{2}$ AB
7 = Central diſt.

50)10,0
,2 = Tabular verſed ſine

,111823 = Tabular ſegment
2500 = Square of the diameter

55911500
223646

279,557500 = Area of the ſegm. EAC
320

599,557500 = Generating area EACF
7 = Central diſtance

4196,9025

36 = AB
16 = CD
2)20
10 = $\frac{1}{2}$ Difference and
5 = $\frac{1}{4}$ Difference

40 = EF
8 = CF
320 = Area of EACF

25
25
125
50
625 = □ of the radius
49 = □ of central diſt.
576(24 = $\frac{1}{2}$ length of the [ſpindle.
4
44)176
176
...

24

24
24
90
48
576 = □ of $\frac{1}{2}$ the length of the ſpindle
133,3333 = $\frac{1}{3}$ □ of of $\frac{1}{2}$ E F
442,6667
20 = $\frac{1}{2}$ E F
8853,3340
4196,9025
4656,4315
6,2832
93128630
139692945
372514520
93128630
279385890

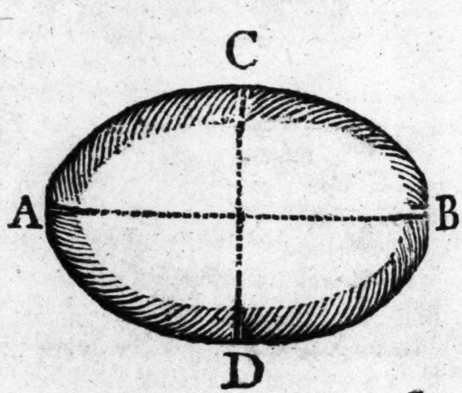
Anſwer 29257,29040080 The ſolidity.

PROB. 19. *To find the ſolidity of a ſpheroid.*

RULE. Multiply the ſquare of the revolving axe by the fixed axe, and this product again by ,5236, and it will give the ſolidity required.

E. 1. What is the ſolidity of a ſpheroid A B C D, the tranſverſe or fixed axe A B is 40, and the conjugate or revolving axe C D is 30; what is the ſolidity?

30
30
900 = □ of C D
40 = A B
36000
,5236
216000
108000
72000
180000
18849,6000 Solidity, Anſ.



E. 2.

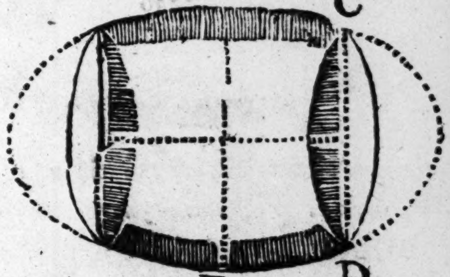
E. 2. What is the ſolidity of a ſpheriod, whoſe fixed axe is 90, and its revolving axe 50?

50	,5236
50	225000
2500	26180
90	10472
	10472
225000	
	Anſwer 117810,0000

PROB. 20. *To find the content of the middle fruſtum of a ſpheriod.*

RULE. To twice the ſquare of the middle diameter, add the ſquare of the diameter of either of the ends; this ſum multiplied by the length of the fruſtum, and the product again by ,2618, will give the ſolidity.

EXAMPLE. What is the ſolidity of the middle fruſtum of a ſpheriod, the diameter AB being 32, that of either of the ends 24, and the length 40?



32
32
64
96
1024
2
2048 = Twice the ſquare of A B
576 = Square of C D
2624
40 = Length
104960
,2618
839680
104960
629760
209920

Anſwer 27478,5280 the ſolidity.

PROB. 21. *To find the ſolidity of the ſegment of a ſpheroid.*

RULE 1. Divide the ſquare of the revolving axis by the ſquare of the fixed axe, and multiply the quotient by the difference between three time the fixed axe, and twice the height of the ſegment.

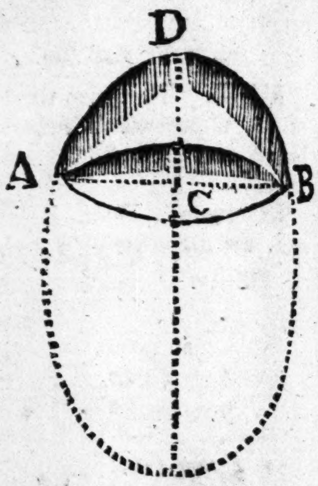
 2. Multiply

2. Multiply the product thus found by the ſquare of the height of the ſegment, and this product again by ,5236, and it will give the ſolidity required.

EXAMPLE. The axes of a ſpheroid are 50, and 30; what is the ſolidity of that ſegment, whoſe height is 5, and its baſe perpendicular to the fixed axe?

30
30
□ of fixed axe = 2500)900 = □ of the revolving axe
,36
140
1440
36
50,40
25 = □ of DC
25200
10080
1260,00

,5236
1260
314160
10472
5236
Anſwer 659,7360 the ſolidity.



PROB. 22. *To find the ſurface or ſolidity of the five regular bodies.*

RULE 1. To find the ſurface, multiply the ſquare of the ſide of the given body by the tabular ſuperficial multiplier againſt the given name, and the product will be the ſuperficies.

2. To find the ſolid content, multiply the tabular number againſt the given name by the cube of the ſide, and the product will be the ſolidity.

A TABLE of the Superficies and Solidity of each Body, whoſe Side is 1.

No. of Sides.	*Names.*	*Superficies.*	*Solidity.*
4	Tetaredron	1,732051	0,1178511
6	Heaxaedron	6,000000	1,0000000
8	Octaedron	3,464102	0,4714045
12	Dodecaedron	20,645729	7,663119
20	Icoſaedron	8,660254	2,181695

E. 1.

E. 1. The ſide of a tetraedron is 4; what is the ſuperficies?

1,732051 = Tabular multiplier
16 = Square of the ſide

27,712816 the ſuperficies.

E. 2. The ſide of a tetraedron is 4; what is the ſolidity?

,1178511 = Tabular multiplier
64 = Cube of the ſide

4714044
7071066

7,5424704 the ſolidity.

E. 3. What is the ſuperficies of an hexaedron, whoſe linear ſide is 3?

6 = Tabular multiplier
9 = Square of the ſide

54 the ſuperficies.

E. 4. The ſide of a hexaedron is 3; what is the ſolidity?

27 = Cube of the ſide
1 = Tabular multiplier

27 the ſolidity.

E. 5. What is the ſolidity of an octaedron, whoſe ſide is 4?

,4714045 = Tabular multiplier
64 = Cube of the ſide

18856180
28284270

30,1698880 the ſolidity

E. 6. What is the ſuperficies of an octaedron, whoſe ſide is 4?

3,464102 = Tabular multiplier
16 = Square of the ſide

55,425632 the ſuperficies.

E. 7. Required the ſuperficies of a dodecaedron, whoſe linear ſide is 3?

20,645729 = Tabular multiplier
9 = Square of the ſide

185,811561 the ſuperficies.

E. 8. The linear ſide of a dodecaedron is 3; what is the ſolidity?

7,663119 = Tabular multiplier
27 = Cube of the ſide

53641833
15326238

206,904213 the ſolidity.

E. 9. What is the ſuperficies of an icoſaedron, whoſe linear ſide is 3?

8,660254 = Tabular multiplier
9 = Square of the ſide

77,942286 the ſuperficies

E. 10. The linear ſide of an icoſaedron is 3; what is the ſolidity?

2,181695 = Tabular multiplier
27 = Cube of the ſide

15271865
4363390

58,905765 the ſolidity.

Note. The ſolidity of any irregular body may be determined by immerſing the ſame in a veſſel of water; for the ſolid content of the additional ſpace, occupied by the fluid on account of the immerſed body, will be equal to the ſolidity of that body.

Prob. 23. *To find the solidity of timber.*

The mensuration of timber (such as cylinders, pyramids, cones, &c. and their frustums (being very troublesome by the exact rules given in this section, an approximation has taken place, and the contents of such trees, or pieces of timber, are generally computed by the following

RULES 1. Multiply the square of the quarter girt (or $\frac{1}{4}$ of the circumference) in inches by the length in feet, and divide that product by 144; the quotient will be the content in feet.

2. Multiply the square of $\frac{1}{5}$ of the girt, or circumference, by twice the length; the product will be the content.

E. 1. What is the content of a tree, whose girt is 40 inches, and length 6 feet?

By Rule 1.	By Rule 2.
4)40	5)40
10 = $\frac{1}{4}$ of the girt	8 = $\frac{1}{5}$ of the girt
10	8
100	64
6 = Length in feet	12 = Twice length
144 { 12)600	144 { 12)768
12) 50	12) 64
Answer 4,16 feet.	Answer 5,33 feet.

Note. By the above example it appears, that the first rule is erroneous, by above $\frac{1}{5}$ part of the true content. The second rule differs from the truth only 1 foot in 190, and is full as easy in practice; therefore I think it ought to be brought into general use among the practitioners in this art, since the ease of the other method is the only argument alledged for employing it.

E. 2. How many feet of timber are there in a tree, whose girt is 48 inches, and length 9 feet?

By Rule 1.	By Rule 2.
4)48	5)48
12	9,6
12	9,6
144	576
9	864
144)1296(9 feet the Answer.	92,16
	18
	144)1658,88(11,52 feet Answ.

LXIX. ARTIFICERS WORK.

1. OF BRICKLAYERS WORK.

BRICKLAYERS compute, or value their work, at the rate of a brick and a half thick; and, if a wall be more or leſs than this ſtandard, it muſt be reduced to it by the following

RULE. Multiply the ſuperficial content of the wall in feet, by the number of half bricks in the thickneſs, aud $\frac{1}{3}$ of the product will be the content required.

E. 1. How many ſquare rods are there in a wall 52 feet long, 12 feet high, and $2\frac{1}{2}$ bricks thick?

```
         52 = Length
         12 = Height
272)624,00 (2,29
    544        5
    800  3)11,45              rods.  ft.  inch.
    544   ------ 3,816 = 3    221    11   The anſwer.
    2560
    2448
     112
```

Note: In practice it is uſual to divide the ſquare feet by 272, omitting the $\frac{1}{4}$ in favour of the workmen.

The uſual way to take the dimenſions of a building, is to meaſure half round its middle, on the outſide, and half round on the inſide; and this will give the true compaſs, in which the thickneſs of the wall is included.

When the height of the building is unequal, take ſeveral different altitudes, and their ſum being divided by the number you have taken, may be conſidered as the mean height.

To meaſure a chimney ſtanding by itſelf, without any party wall adjoining, girt it about for the length, and reckon the height of the ſtory for the breadth; but if it ſtands againſt a wall, you muſt meaſure it round to the wall for the girt, and take the height as before.

When the chimney is wrought upright from the mantle-tree to the ceiling, the thickneſs muſt always be the ſame with the jambs; and nothing is deducted for the vacancy between the floor and the mantle-tree, becauſe of the gathering of the breaſt and wings, to make room for the hearth in the next ſtory.

To meaſure chimney ſhafts, or that part which appears above the roof, girt them with a line, about the leaſt place for the length, and take the height for the breadth; and if they be 4 inches thick, ſet down the thickneſs at one brick-work; but if they be 9 inches thick, reckon it at a brick and a half, in conſideration of the plaſtering and ſcaffolding.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed. But this deduction is made only with regard to the materials; for the value of their workmanſhip is added to the bill, at the ſtated rate agreed on.

E. 2. A gentleman built a wall round his garden, which is 942 feet long, 8 feet high, and $2\frac{1}{2}$ bricks thick; how many rods doth it contain?

```
         942 = Length
           8 = Height
     ----
272)7536 ( 27,7
    544       5
    ----   ----
    2096 3)138,5
    1904  -----  rods.  ft.  inch.
    ----   46,16=46   43    6 The anſwer.
     1920
     1904
     ----
       16
```

E. 3. How many rods are there in a wall $82\frac{1}{2}$ feet long, $12\frac{1}{2}$ feet high, and 2 bricks thick?

```
          82,5
          12,5
         -----
          4125
         1650
         825
        ------
272)103125 ( 3,79
     816        4
    -----   -----
     2152 3)15,16
     1904   -----   rods.  ft.  inch.
    -----    5,05 = 5      13    7 The anſwer
      2485
      2448
      ----
        37
```

2. OF MASONS WORK.

To maſons work belongs all ſorts of ſtone work, and the meaſure made uſe of is a foot, either ſuperficial or ſolid.

RULE 1. For ſolid meaſure, multiply continually into one ſum the length, breadth, and thickneſs, and the product will be the ſolidity.

2. For ſuperficial meaſure, multiply the length and breadth of every part of the projection together, and the product will be the content.

E. 1. What is the ſolid content of a wall, whoſe length is 32 feet 6 inches, its height 5 feet 3 inches, and thickneſs 2 feet?

By

By Decimals.

$$\begin{array}{r} 32,5 \\ \underline{5,25} \\ 1625 \\ 650 \\ \underline{1625} \\ 170,625 \\ \underline{2} \\ \text{Answer } 341,250 \end{array}$$

By duodecimals.

$$\begin{array}{rrr} ft. & in. & \\ 32 & 6 & \\ \underline{5} & \underline{3} & \\ 162 & 6 & \\ \underline{8} & \underline{1} & \underline{6} \\ 170 & 7 & 6 \\ & & \underline{2} \\ \text{Anf. } 341 & 3 & 0 \end{array}$$ Same as before.

E. 2. Required the solid content of a wall, whose length is 107 feet, its height 24 feet 6 inches, and its thickness 4 feet?

$$\begin{array}{r} 107 \\ \underline{24,5} \\ 535 \\ 428 \\ \underline{214} \\ 2621,5 \\ \underline{4} \\ \text{Anf. } 10486,0 \text{ Feet.} \end{array}$$

E. 3. What is a marble slab worth, whose length is 8 feet 6 inches, and breadth 2 feet 6 inches, at 5*s*. per foot?

$$\begin{array}{r} 8,5 \\ \underline{2,5} \\ 425 \\ \underline{170} \\ 21,25 \end{array} = \begin{array}{rl} f. & in. \\ 21 & 3 \text{ content.} \end{array}$$

$$\begin{array}{lrrr} 3 = \tfrac{1}{4})5 & & & \\ & \underline{7} & \times 3 = 21 & \\ & 1 & 15 & \\ & & \underline{3} & \\ & 5 & 5 & \\ & 0 & 1 & 3 \\ £. & 5 & 6 & 3 \text{ Answer.} \end{array}$$

3. OF CARPENTERS AND JOINERS WORK.

Carpenters and joiners work is that of flooring, partitioning, roofing, &c. and is measured by the square of 100 feet.

RULE. Multiply the length by the breadth, and divide this product by 100 for the content.

E. 1. If a floor be 25 feet 3 inches long, and 12 feet 6 inches broad, how many squares will it contain?

$$\begin{array}{r} 25,25 \\ \underline{12,5} \\ 12625 \\ 5050 \\ \underline{2525} \\ \text{Answer } 3|15,625 = 3 \text{ squares, } 15 \text{ feet.} \end{array}$$

E. 2. How many oaken planks will floor a room, $60\frac{1}{2}$ feet long, and $32\frac{1}{2}$ wide; supposing the planks each 12 feet long, and 1 foot wide?

First $1 \times 12 = 12$, the area of one plank.

And $60,5 \times 32,5 = 1966,25$, the area of the room.

$\therefore$ 12)1966,25(163,854 Planks, the answer.

E. 3.

E. 3. If a houſe within the walls be 22 feet 6 inches long, and 10 feet 3 inches broad, how many ſquares of roofing will cover it?

$$\begin{array}{rl}
2)10,25 & = \text{breadth} \\
5,125 & = \tfrac{1}{2}\text{ breadth} \\
\hline
15,375 & \\
22,5 & = \text{length} \\
\hline
76875 & \\
30750 & \\
30750 & \\
\hline
\text{Anſwer } 3|45,9375 & = 3 \text{ ſquares, } 45 \text{ feet.}
\end{array}$$

E. 4. If a houſe meaſure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it coſt roofing, at 10*s.* 6*d.* per ſquare?

Firſt 30,5+15,25=45,75, width of the roof.

Then 52,66×45,75=2409 feet = 24 ſquares 9 feet, the content of the room.

$$\begin{array}{rl}
24|09 & \\
,525 & \text{of a pound} = 10s.\ 6d. \\
\hline
12045 & \\
4818 & \\
12045 & \\
\hline
\text{Anſwer } 12,64725 & = £.\ 12\ 12s.\ 11\tfrac{1}{4}d.
\end{array}$$

Note. In meaſuring joiners work, the ſtring is made to ply cloſe to every part of the work over which it paſſes.

4. OF SLATERS AND TILERS WORK.

Slaters and tilers work is meaſured by the ſquare of 100 *feet.*

RULE. Multiply the length in feet of the ridge, by the girt from eave to eave, and the product will be the content in feet, which muſt be reduced to ſquares as taught in carpenters work.

E. 1. The length of a ſlated roof is 50 feet 6 inches, and its girt 32 feet 3 inches; how many ſquares will it contain?

$$\begin{array}{rl}
50,5 & = \text{Length} \\
32,25 & = \text{Girt} \\
\hline
2525 & \\
1010 & \\
1010 & \\
1515 & \\
\hline
\text{Anſwer } 16|28,625 & = 16 \text{ ſquares, } 28 \text{ feet.}
\end{array}$$

Note. In ſlating it is common to reckon the breadth of the roof 2 or 3 inches broader than what it meaſures, becauſe the firſt row is almoſt covered by the ſecond: this is ſometimes practiſed in tiling.

E. 2.

E. 2. How many squares are contained in a roof, whose length is 70 feet, and depth 30 feet?

First $30 \times 2 = 60$; and $70 \times 60 = 4200$ feet $= 42$ squares, the answer.

5. PLASTERERS WORK.

Plasterers work is of two kinds; viz. plastering upon laths, called cieling, and plastering upon walls, called rendering, and is measured by the square yard, which is 9 square feet.

RULE. Divide the square feet by 9, and the quotient will be the number of square yards.

E. 1. If a cieling be 60 feet 9 inches long, and 22 feet 6 inches broad, how many yards does it contain?

By Decimals.

```
   60,75
    22,5
  ------
   30375
  12150
 12150
 --------
9)1366,875
 --------
  151,875
        9
  -------
    7,875
       12
  -------
   10,500
       12
      ---
      6,0
```

By Duodecimals.

```
        f.  in.
        60   9
        22   6
      ---------
      1320
        16   6
        30   4   6
      -------------
    9)1366  10   6
      -------------
Answer 151   7  10   6 As before.
```

Answer 151 *yards*, 7 *feet*, 10 *inches*, 6 *parts*.

E. 2. There is a quantity of partitioning, that measures 260 feet about, and 18 feet high, and is rendered between quarters; the lathing and plastering of which will be 8*d*. per yard, and the whitening 2*d*. per yard; what will the whole come to?

```
                 260
                  18
                ----
               9)4680
                ----
              ⅕)520 The whole content
                104
                ---                     £.  s.  d.
Plastered       416 at 8d. per yard = 13  17   4
Whitened        624 at 2d. per yard =  5   4   0
                                      ----------
                               Answer 19   1   4
```

Note. In meaſuring between quarters, there is commonly $\frac{1}{5}$ part of the whole area deducted; but when rendering between quarters is whitened or coloured, there is $\frac{1}{5}$ part to be added to the whole, for the ſides of the quarters and braces.

E. 3. What will plaſtering a cieling, at 10*d.* per yard come to, ſuppoſing the length 26 feet, and the breadth 15 feet?

Firſt $26 \times 15 = 390$, and $390 \div 9 = 43$ *yards*, 3 *feet*, the content.

By Multiplication.

F. *d.*
3 = $\frac{1}{3}$) 10
10 × 4 + 3 = 43
8 4
4
1 13 4 = 40
2 6 = 3
$3\frac{1}{3} = \frac{1}{3}$ of 10*d.*
1 16 $1\frac{1}{3}$

By the rule of Three.

yd. *d.* *yds.* *f.*
1 : 10 :: 43 3
9 9
9 390
10
9) 3900
12) 433 3 = $\frac{1}{3}$
2|0) 3|6 1
Anſwer £. 1 16 $1\frac{1}{3}$ Same as before.

6. PAINTERS WORK.

Painters work is meaſured the ſame as plaſterers, and in taking the dimenſions, the line muſt be forced into all the mouldings and corners.

E. 1. If a room be painted, whoſe height is 9 feet 6 inches, and its compaſs 40 feet 3 inches; how many yards does it contain?

By Decimals.

40,25
9,5
20125
36225
9) 382,375
Anſwer 42,486

By Duodecimals.

F. *I.*
40 3
9 6
362 3
20 1 6
9) 382 4 6
Anſwer 42 4 4 6 As before.

E. 2. How many ſquare yards are there in a room, whoſe height is 12 feet 6 inches, and the circumference 98 feet 9 inches?

98,75
12,5
49375
19750
9875
9) 1234,375
Anſwer 137,152 = 137 *yards*, 1 *foot*, 4 *inches*, 4 *parts.*

Note. Windows are done at ſo much a piece, and in carved mouldings, &c. it is cuſtomary to allow double the uſual meaſure.

7. GLAZIERS

7. GLAZIERS WORK.

Glaziers take their dimenſions in feet, inches, and parts, and eſtimate their work by the ſquare foot.

E. 1. If a pane of glaſs be 3 feet, 8 inches, and 3 quarters long, and 1 foot, 4 inches, 1 quarter broad; how many feet does it contain?

By Decimals.

	3,729
	1,354
	14916
	18645
	11187
	3729
Anſwer	5,049066

By Duodecimals.

	F.	*I.*	*P.*			
	3	8	9			
	1	4	3			
	3	8	9			
	1	2	11	0		
			11	2	3	
Anſwer	5	0	7	2	3	ſame as before.

Note. In taking the length and breadth of a window, the croſs bars between the panes are always included; and no allowance is ever made for round or oval windows, as the trouble of cutting them to thoſe ſhapes, is more than the value of the glaſs omitted.

Windows are ſometimes meaſured by taking the dimenſions of one pane, and multiplying it continually by the number of panes.

E. 2. There is a window with 16 panes of glaſs, each 3 feet, 7 inches, 3 quarters long, and 1 foot, 5 inches, 1 quarter broad; how many feet of glaſs are contained in the ſaid window?

By Decimals.

	3,643
	1,437
	25501
	10929
	14572
	3643
	5,234991
	4
	20,939964
	4
Anſwer	83,759856

By Duodecimals.

	F.	*I.*	*P.*			
	3	7	9			
	1	5	3			
	3	7	9			
	1	6	2	9		
			10	11	3	
	5	2	10	8	3	
					4	× 4 = 16
	20	11	6	9	0	
					4	
Anſwer	83	10	3	0	0	ſame as before.

8. PAVIOURS WORK.

Paviours work is done by the ſquare yard, and the content is found by dividing the area in feet by 9.

E. 1. If a ſtreet be $665\frac{1}{2}$ feet long, and $52\frac{3}{4}$ feet wide, how many ſquare yards are contained therein?

By Duodecimals.	By Decimals.
F. I.	665,5
665 6	52,75
52 9	33275
1330	46585
3325	13310
26	33275
499 1 6	9)35105,125
9)35105 1 6	Anſwer 3900,569 Same as before,
Anſwer 3900 5 1 6	

E. 2. There is a rectangular court-yard, whoſe length is 86 feet 3 inches, and breadth 40 feet 6 inches; how many ſquare yards are contained therein?

86,25	F. I.
40,5	86 3
	40 6
43125	3450 0
345000	43 1 6
9)3493,125	9)3493 1 6
	Anſwer 388 1 1 6 As before.
Anſwer 388,125	

VAULTED AND ARCHED ROOFS.

Vaulted Roofs are formed by arches ſpringing from the oppoſite walls, and meeting in a line at the top.

Domes are made by the arches ſpringing from a circular or polygon baſe, and meeting in a point at the top.

Saloons are formed by arches connecting the ſide walls to a flat roof, or cieling, in the middle.

Groins are formed by the interſection of vaults with each other.

PROB. 1. *To find the ſolid content of circular, elliptic, or gothic vaulted roofs.*

RULE. Multiply the area of one end by the length of the roof, and the product will give the ſolidity.

EXAMPLE. What is the ſolid content of a ſemi-circular vault, whoſe ſpan is 30 feet, and its length 100 feet?

,7854
900 = Square of 30
2)706,8600
353,4300 = Area of the end
100
Anſwer 35343 Feet the ſolidity.

PROB. 2.

PROB. 2. *To find the concave or convex surface of circular, elliptic, or gothic roofs.*

RULE. Multiply the length of the arch by the length of the vault, and the product will be the superficies.

EXAMPLE. What is the concave surface of a semi-circular vault, whose span is 30 feet, and its length 100?

3,1416
30

2)94,2480

47,1240 = Length of the arch
100

Answer 4712,4 Feet the concave surface.

PROB. 3. *To find the solid content of a dome; its height, and the dimensions of its base, being known.*

RULE. Multiply the area of the base by $\frac{2}{3}$ of the height, and the the product will be the solidity.

EXAMPLE. What is the solid content of a spherical dome, the diameter of whose circular base is 50 feet, and height 30?

,7854
2500 = Square of 50

3927000
15708

1963,5000
20 = $\frac{2}{3}$ of the height

Answer 39270,0 Feet the solidity.

PROB. 4. *To find the superficial content of a spherical dome.*

RULE. Multiply the area of the base by 2, and the product will be the superficial content.

EXAMPLE. What is the superficial content of an hexagonal spherical dome, each side of whose base being 20 feet?

2,598076 = Area of an hexagon, whose side is 1,
400 = Square of 20

1039,230400 = Area of the base
2

Answer 2078,460800 Feet the superficial content.

Elliptical domes are measured by the following Rule: Add the height to half the diameter of the base; this sum multiplied by 1,5708 will give the superficial content nearly.

PROB. 5.

PROB. 5. *To find the solid content of the vacuity formed by a groin arch, either circular or elliptical.*

RULE. Multiply the area of the base By the height, and the product again by ,904, and it will give the solidity.

E. 1. What is the solid content of the vacuity formed by a circular groin, one side of its square base being 10 feet, and the height 5?

10
10
———
100 = Area of the base
5 = Height
———
500
,904
———
2000
4500
———
Answ. 452,000 feet of the solidity.

E. 2. What is the solid content of the vacuity formed by an elliptical groin, one side of its square base being 40 feet, and the height 12?

40
40
———
1600
12
———
19200
,904
———
76800
1728000
———
Answer 17356,800

PROB. 6. *To find the concave superficies of a circular groin.*

RULE. Multiply the area of the base by 1,1416, and the product duct will be the superficies.

E. 1. What is the curve surface of a circular groin arch, one side of its square base being 10 feet?

10
10
———
100 = Area of the base
1,1416
———
Ans. 114,16 the superficies.

E. 2. What is the concave superficies of a circular groin arch, one side of its square base being 8 feet?

8
8
———
64 = Area of the base
1,1416
———
45664
68496
———
Ans. 73,0624 the superficies.

Note. Elliptical groins may be measured by the above Rule, the error being too small to be regarded in practice.

The general rule for measuring all arches is this: From the content of the whole, considered as solid, from the springing of the arch to the outside of it, deduct the vacuity contained between the said springing and the under side of it, and the remainder will be the content of the solid part.

Questions

Queſtions for exerciſe in Menſuration.

Queſt. 1. What difference is there between a floor 30 feet long, and 20 broad, and two others of half the dimenſions? and what will they all three come to at 2*l.* 10*s.* per ſquare?

$$\left.\begin{array}{ll}\text{Firſt} & 30\times20 \quad =600 \\ \text{And} & 15\times10\times2=\underline{300}\end{array}\right\}\ 300 \text{ the difference}$$

1|00)9|00 Sum

9 Squares, at 2*l.* 10*s.* = 2,5*l.*

Then 2,5×9=22,5=22*l.* 10*s.* the anſwer.

Queſt. 2. There is a ſtreet, whoſe length is 21,5 feet and the breadth 12,5 feet, is to be paved with ſtones, each 15 inches ſquare; how many will it take?

21,5
12,5
1075
430
215
1,5625)268,7500(172 Stones, anſ.
15625
112500
109375
31250
31250
0

1,25
1,25
625
250
125
1,5625 = Area of one ſtone.

Queſt. 3. What is the difference between a ſolid half foot, and half a foot ſolid?

Firſt 1728÷2=864 ſolid inches in a ſolid half foot.
And 6×6×6=216 ſolid inches in half a foot ſolid.
Difference 648 Solid inches, the anſwer.

Queſt. 4. Suppoſe the ball at the top of St. Paul's Church is 6 feet in diameter; what did the gilding of it come to, at $3\frac{1}{2}$*d.* per ſquare inch?

Firſt 6×12=72 inches; then 3,1416×72=226,1952, the circumference.

And 226,1952×72=16286,0544 inches, ſuperficial content.

∵ 16286,0544×3,5=57001,1904*d.* = 237*l.* 10*s.* $1\frac{1}{4}$*d.* Anſwer.

Queſt. 5. What is the diameter of a circle, whoſe area is 9 times as much as one of 21 inches diameter?

21
3 = Square root of 9
Anſwer 63 Inches.

Queſt. 6.

Quest. 6. The diameter of a circle is 63 inches; the diameter of another circle is required, whose area is 9 times less.

$\sqrt{9}=3)63$

Answer $\overline{21}$ Inches.

Quest. 7. The transverse diameter of an ellipsis is 57, and conjugate 41; the diameter of a circle is required, whose area is equal to that of the ellipsis.

First $57\times41=2337$, and $\sqrt{2337}=48,34$, the diameter required.

Quest. 8. Our satellite, the moon, is a globe, in diameter 2170 miles: I would know how many quarters of wheat she would contain, if hollow, 2150,425 solid inches being the bushel; and how much yard-wide stuff would make her a waistcoat, was she to be cloathed?

First $2170\times2170\times2170\times,5236=5350308686,8$ solid miles.

Then a mile $=1760\times1760\times1760=5451776000$ solid yards.

$\therefore 5350308686,8\times5451776000=29168684491287756800$ solid yards in the moon. Now in a solid yard are $36\times36\times36=46656$ solid inches. $\therefore 29168684491287756800\times46656=1360894143625521581260800$ solid inches in the moon: And a quarter $=2150,425\times8=17203,4$. Consequently, $17203,4)1360894143625521581260800(79106115281021285 40,06$ quarters of wheat, the moon would hold if hollow.

Again, $2170\times3,1416=6817,272$, the circumference of the moon.

Then $6817,272\times2170=14793480,24$ square miles the surface of the moon. And a mile $=1760\times1760=3097600$ square yards: Therefore $14793480,24\times3097600=45824284391424$ square yards of stuff. *Q. E. F.*

LXX. GAUGING.

GAUGING in general, is nothing more than the application of the foregoing rules in Mensuration of Solids, to particular vessels used by brewers, wine-merchants, &c. because the contents of all sorts of vessels, used for liquors, &c. are computed as though they were really solid bodies. But on account of the different capacities of gallons, bushels, and feet, it is necessary to find factors and divisors for each different denomination; because the dimensions are all taken in inches.

PROB. I. *To find divisors and gauge points for circles.*

RULE. Divide the solid capacities of each gallon, bushel, &c. by ,7854, the several quotients will be proper divisors for the square of any diameter, to reduce the area into ale, wine, and malt gallons, or bushels; and the square roots of those divisors will be the gauge points for circles.

EXAMPLES.

EXAMPLES.

Diviſors.	*Dividends.*	*Quotients.*	*Sq. Rts.*	
,7854)	282,0000(	359,05	18,95	Ale gallons
,7854)	231,0000(	294,12	17,15	Wine gallons
,7854)	268,8000(	342,24	18,5	Malt gallons
,7854)	2150,4200(	737,92	52,32	Malt buſhels
,7854)	227,0000(	289,	17,	Maſh tun gallons.

In like manner any other diviſor, or gauge point, may be found, when the ſolid capacity of the integer is given, whether it be a gallon, buſhel, or a foot, &c. and in this manner was the following table computed.

Prob. 2. *To find factors for circles.*

RULE. Divide ,7854 by the ſolid capacities of each gallon, buſhel, &c. and the quotients will be proper multipliers for the ſquare of the diameter of any circle, to reduce the area of that circle into ale, wine, malt gallons, or buſhels, &c.

EXAMPLES.

Diviſors.	*Dividends.*	*Quotients.*	
282,	),7854(	,002785	the multiplier for Ale gallons
231,	),7854(	,003389	—— Wine gallons
268,8	),7854(	,002922	—— Malt gallons
2150,42	),7854(	,000365	—— Malt buſhels
227,	),7854(	,00346	—— Maſh tun gallons

And in this manner were the other factors found for circles, in the following table.

Prob. 3. *To find factors for ſquares.*

RULE. Divide unity by the ſolid capacity of each gallon, buſhel foot, &c. and the quotients will be the proper factors or multipliers.

EXAMPLES.

282,	)1,000000(	,003546	factors for Ale gallons
231,	)1,000000(	,004329	—— Wine gallons
268,8	)1,000000(	,003720	—— Malt gallons
2150,42	)1,000000(	,000465	—— Malt buſhels
227,	)1,000000(	,0044	—— Maſh tun gallons.

Prob. 4. *To find gauge points for ſquares.*

RULE. Extract the ſquare roots of the ſolid capacities of each gallon, buſhel, &c. in inches, and it is done.

EXAMPLES.

$\sqrt{282}$	= 16,79	the ſquare gauge point for Ale gallons
$\sqrt{231}$	= 15,19	—— Wine gallons
$\sqrt{268,8}$	= 16,39	—— Malt gallons
$\sqrt{2150,42}$	= 46,37	—— Malt buſhels
$\sqrt{227}$	= 15	—— Maſh tun gallons.

A TABLE OF FACTORS.

Multipliers, Divisors, and Gauge Points, for Squares and Circles.						
	Factors for		*Divisors for*		*Gauge Pints for*	
	Squares.	*Circles.*	*Squares*	*Circles.*	*Squares*	*Circles.*
Inches the area of 1	1	,785398	1	1,27324	1	1,128
A superficial foot -	,006944	,005454	144	183,34	12	13,54
A solid foot - -	,000578	,000454	1728	2200,16	41,57	46,9
Ale gallon - - -	,003546	,002785	282	359,05	16,79	18,95
Wine gallon - - -	,004326	,003399	231	294,12	15,19	17,15
Malt or corn bushel	,000465	,000365	2150,42	2737,92	46,37	52,32
Malt gallon - - -	,003720	,002922	268,8	342,24	16,39	18,5
Mash tun gallon -	,004405	,00346	227	289	15,1	17
lb of hard soap, cold	,036845	,028939	27,14	34,56	5,21	5,88
lb of hot soap - -	,035714	,028050	28,0	35,65	5,29	5,97
lb of green soap -	,038956	,0306	25,67	32,68	5,06	5,72
lb of white soft soap	,039123	,030731	25,56	32,54	5,05	5,7
lb of tallow, net - -	,031844	,025101	31,4	39,98	5,6	6,32
lb of green starch -	,028736	,022565	34,8	44,32	5,9	6,66
lb of dry starch - -	,024813	,019491	40,3	51,3	6,35	7,16
lb of flint glass - -	,094697	,074405	10,56	13,44	3,25	3,69
lb of white glass -	,071123	,05586	14,06	17,9	3,74	4,34
lb of green glass -	,082102	,064516	12,18	15,5	3,48	3,94

PROB. 5. *To find the area of a square tun, back, or cooler, &c.*

RULE. Multiply one side of the square by itself, and that product multiply or divide by the factor, &c. for squares, and the product, or quotient, will be equal to the area of the same kind as the factor or divisor made use of.

EXAMPLE. What is the area in ale gallons of a square, each of whose equal sides is 30 inches?

By Division.

```
      30
      30
282)900(3,19 Area
    846
     540
     282
     2580
     2538
       42
```

By Multiplication.

Square factor = ,003546
Square of side = 900

Area 3,1914 as bef.

Note. In gauging all superficies, the areas are always understood to be one inch deep.

To

To find the same by the Rule.

Set the proper divisor on A to a side of the square on B; then against the other side of the square on A is the area on B, thus:

	A		B		A		B	
As	282	:	30	::	30	:	3,19,	the area in ale gallons as before.

Or thus: Set unity on C to the square gauge point on D, and against any side of a square on D is the area on C; thus:

	D		C		D		C	
As	16,79	:	1	::	30	:	3,19,	as before.

In like manner may the area of the square be found in any of the other denominations mentioned in the Table, by making use of the respective factors, &c. For if, instead of 282, I had divided by 231, 2150,42, the area, would have been found in wine gallons, or malt bushels.

PROB. 6. *To find the area of a parallelogram.*

RULE. Multiply the longest side by the shortest, and that product multiply or divide by the factors or divisors in the Table, and the product or quotient will be the area required.

EXAMPLE. The longest side of a parallelogram is 40 inches, and the shortest side 20 inches; what is the area in ale gallons?

By Division.

```
      40
      20
     ----
282)800(2,83 Gallons the area.
    564
    ----
    2360
    2256
    ----
     1040
      846
     ----
      194
```

By multiplication.

```
,003546 = Square factor
    800 = □ of the two sides
---------
2,836800 Gallons as before Ans.
```

The same by the Rule.

	A		B		A		B	
As	282	:	40	::	20	:	2,83,	the same as before.

PROB. 7. *To find the area of a rhombus.*

RULE. Multiply the perpendicular by one of the sides, and that product multiply or divide by the factors, &c. for squares, and the product or quotient will be the area required.

Example. The side of a rhombus is 37 inches, and the perpendicular 30 inches; what is its area in ale gallons?

37 = Side
30 = Perpendicular

282)1110(3,93 Area in ale gallons.
846
———
2640
2538
———
1020
846
———
174

The same by the Rule.

	A		B		A		B
As	282	:	37	::	30	:	3,93, area as before.

Prob. 8. *To find the area of a rhomboides.*

RULE. Multiply the longest side by the perpendicular, and that product multiply or divide by the proper factors in the table, and the product or quotient will be the area required.

Example. Required the area in ale gallons of a rhomboides whose longest side is 60 inches, and perpendicular 37 inches?

37
60
———
282)2220(7,87 Area in ale gallons.
1974
———
2460
2256
———
2040
1974
———
66

The same by the Rule.

	A		B		A		B
As	282	:	60	::	37	:	7,87, area as before.

Prob. 9. *To find the area of a plain triangle.*

RULE. Multiply half the longest side by the perpendicular, and that product multiply or divide by the factors in the table, and the product or quotient will be the area required.

Example.

EXAMPLE. The length of the base of a triangle is 50 inches, and its perpendicular height 30 inches; what is its area in ale gallons?

30 = Perpendicular
25 = Half base
———
282)750(2,65 Area in ale gallons.
564
———
1860
1692
———
1680
1410
———
270

The same by the Rule.

A B A B
As 282 : 30 :: 25 : 2,65 the area as before.

PROB. 10. *To find the area of a trapezium.*

RULE 1. Divide the trapezium into two triangles, then let fall a perpendicular from each of the angles upon the diagonal, which is a common base to both triangles.

2. Multiply half the diagonal by the perpendiculars, or half the sum of the perpendiculars by the whole diagonal, and that product multiply or divide by the factors in the table, and the product or quotient will be the area required.

EXAMPLE. Required the area in ale gallons of a trapezium whose diagonal is 60 inches, and the two perpendiculars 15 and 27 inches?

$\left.\begin{matrix}15\\27\end{matrix}\right\} =$ Perpendiculars
———
42
30 = $\frac{1}{2}$ Diagonal
———
282)1260(4,47 Area in ale gallons.
1128
———
1320
1128
———
1920
1974
———
46

Note. The are above found is not quite 4,47, but it is nearer to 4,47 than 4,46, and in the practice of gauging, the officers use but two decimal places of figures; therefore they take the nighest to the second place, whether it be more or less.

The

The same by the Rule.

A B A B
As 282 : 42 :: 30 : 4,47, the area as before.

PROB. 11. *To find a mean geometrical proportion between two given numbers.*

RULE. Multiply the two given numbers together, and extract the square root from their product, which will be the mean required.

E. 1. What is the mean proportional between 36 and 64?

```
     64
     36
   ----
    384
   192
   ----
   2304(48 the mean required.
   16
   ----
88) 704
    704
   ----
    ...
```

By the Rule.

Set one of the given numbers upon C to the same number upon D; then against the other given number upon C is the number sought upon D; thus:

C D C D
As 36 : 36 :: 64 : 48, mean as before.

E. 2. *by the Rule.* What is the mean between 42 and 30?

C D C D
As 42 : 42 :: 30 35,5 the mean required.

Note. This mean last found is a mean proportional between the sum of the perpendiculars and half the diagonal of the trapezium, in Prob. 10. by which the area may be found by the lines C and D, as follows; thus:

D C D C
As 16,79 : 1 :: 35,5 : 4,47, the area as before, found by the lines A and B.

And in like manner may the area of the parallelogram, rhombus, rhomboides, and triangle be found.

PROB. 12. *To find the area of any regular polygon.*

RULE. Divide it into triangles, then find the area of one triangle, as in Prob. 9. and because there are as many triangles as there are sides in the polygon, multiply the area of the triangle by the number of sides, and the product will be the area of the polygon.

EXAMPLE.

EXAMPLE. What is the area of in ale gallons of a pentagon, whoſe ſide is 50 inches, and the perpendicular 34,2 inches?

34,2 = Perpendicular
25 = ½ one of the ſides

1710
684

282)855,0(3,031 the area of the the triangle
5

Anſwer 15,155 Area of the polygon in ale gallons.

PROB. 13. *To find the area of a polygon, when the ſide only is given.*

RULE. Multiply the ſquare of the ſide of any regular polygon, mentioned in the following table, by the common factor belonging to that polygon, and the product will be the area in inches, ale gallons, wine gallons, or malt buſhels reſpectively.

A TABLE OF REGULAR POLYGONS.

No. of Sides.	*Names of the Polygons.*	*Area. Sq inches.*	*Area. Ale gallons*	*Area. Wine gals.*	*Area. Malt buſh.*
5	Pentagon	1,72	,006099	,007445	,00078
6	Hexagon	2,598	,009212	,011246	,001208
7	Heptagon	3,633	,012883	,015727	,001689
8	Octagon	4,828	,01712	,0209	,002245
9	Nonagon	6,183	,021925	,026726	,002875
10	Decagon	7,695	,027287	,033311	,003579
11	Undecagon	9,361	,033195	,040523	,004353
12	Duodecagon	11,196	,039702	,048467	,005205

EXAMPLE. Required the area of a pentagon in ale gallons, whoſe ſide is 50 inches?

,006099 = Tabular multiplier
2500 = Square of ſide

3049500
12198

Anſwer 15,247500 Area in ale gallons, ſame as before, nearly.

In like manner may the area of any other regular polygon mentioned in the table be found, in any of the denominations there mentioned.

PROB. 14.

Prob. 14. *To find the area of a circle.*

RULE. Square the diameter of the circle, and multiply or divide that ſquare by the factors in the table for circles (page 346) and the product, or quotient, will be the area required.

Example. The diameter of a circle is 80 inches; what is its area in ale gallons?

```
            80
            80
           ----
359,05)6400,00(17,82 Area in ale gallons
        35905
        -----
        280950
        251335
        ------
         296150
         287240
         ------
           89100
           71810
           -----
          17290
```

By the Rule.

D C D C

As 18,95 : 1 :: 80 : 17,82, Area as before.

The Rule being thus ſet, it is like a table, for againſt any diameter on D, is the area in ale gallons on C.

Prob. 15. *To find the area of an ellipſis, or oval.*

RULE. Multiply the tranſverſe and conjugate diameters together, and that product multiply or divide by the factors in the table for circles (p. 346), then that product, or quotient, will be the area of the ellipſis.

Example. The tranſverſe diameter of an ellipſis is 72 inches, and the conjugate 50 inches; what is its area in ale gallons?

```
             72 = Tranſverſe diameter
             50 = Conjugate diameter
            ----
359,05)3600,00(10,02 Area in ale gallons
         35905
         -----
          95000
          71810
          -----
          23190
```

By the Rule.

A B A B

As 359 : 72 :: 50 : 10,02 Area as before.

Note.

Note. If a mean proportion between the tranſverſe and conjugate diameters is found, the proportion will be the ſame as a circle, and may be found on the lines C and D, thus:

	C		D		C		D
As	72	:	72	::	50	:	60
	Tranſ. Diam.		Tranſ. Diam.		Conju. Diam.		Mean Dia.

LXXI. OF SOLID BODIES.

SOLIDS are comprehended under length, breadth and depth: now by having the area given at one inch deep, it will be eaſy to find the content of any ſolid body, at any depth; for if the content at one inch deep be multiplied by the whole depth, the product will be the ſolid content of the body.

PROB. I. *To find the content of a ſolid, whoſe baſes are either ſquares or parallelograms.*

RULE 1. Multiply the length of the baſe by the breadth, and that product by the depth; and this laſt product multiply or divide by the factors, &c. for ſquares in the Table of factors (Page 346,) and the product, or quotient, will be the content required.

2. Or find the area of the baſe by Problems 5 and 6, of the laſt Section, and multiply that area by the depth, and the product will be the content.

E. 1. There is a cube, each of whoſe equal ſides are 30 inches; what is the content in ale gallons?

```
          30
          30
         ---
         900
          30
282)27000(95,7 Content in ale gallons.
     2538
     ----
      1620
      1410
      ----
       2100
       1974
       ----
        126
```

The area of the baſe of this ſolid was found in PROBLEM 5. of the laſt Section to be 3,19 ale gallons.

∵ $3,19 \times 30 = 95,7$, the content in ale gallons, as before.

By the Rule.

	D		C		D		C	
As	16,79	:	30	::	30	:	95,7	the content, as before.

E. 2. Required the content of a prism in ale gallons, whose length is 45,6 inches, breadth 27,5, and depth 21,5 inches.

```
    45,6 = Length
    27,5 = Breadth
    ----
    2280
   3192
   912
  -------
  1254,00
    21,5 = Depth
    ----
    6270
   1254
  2508
  -------
282)26961,0(95,6 Content in ale gallons
    2538
    ----
     1581
     1410
     ----
      1710
      1692
      ----
        18
```

The proportion by the rule is the same as that for a square base, when there is a mean proportional found between the longest and shortest sides of the base.

1. *For the mean proportional by the Rule.*

	C		D		C		D	
As	27,5	:	27,5	::	45,6	:	35,4,	the mean.

2. *For the content by the Rule.*

	D		C		D		C	
As	16,79	:	21,5	::	35,4	:	95,6,	Content as before.

In like manner the content of any right-lined solid may be found, either regular or irregular, if by the foregoing rules you find the area of its base, and multiply that area by its depth.

PROB. 2. *To find the content of a cylinder, by having the depth and diameter given.*

RULE. Multiply the square of the diameter by the depth, and divide by the circular divisors; the quotient will be the content required.

EXAMPLE

EXAMPLE. There is a cylindrical veſſel, whoſe diameter is 45 inches, and the depth 12 inches; what is the content in ale gallons?

```
        45
        45
        ---
       225
      180
      ----
      2025
        12
      -----
359,05)24300,00(67,67 Content in ale gallons.
       215430
       ------
       275700
       251335
       ------
        243650
        215430
        ------
         282200
         251335
         ------
          30865
```

By the Rule:

	D		C		D		C	
As	18,95	:	12	::	45	:	67,67,	Content as before.

PROB. 3. *To find the content of a ſolid, that has two equal ellipſis for its baſes, called cylindroid, by having the diameters and depths given.*

RULE. Multiply the tranſverſe and conjugate diameters togethe and that product by the depths; and this laſt product divide by the circular diviſor in the table of factors, the quotient will be the content required.

EXAMPLE. What is the content of a cylindroid in ale gallons, the tranſverſe diameter of whoſe baſe is 72 inches, the conjugate 50 and depth 12 inches?

```
        72
        50
      ----
      3600
        12
      -----
359,05)43200,00(120,3 Content in ale gallons,
       35905
       -----
       72950
       71810
       ------
        114000
        107715
        ------
          6285
```


The

The proportion by the rule is the ſame as for a cylinder, when there is a mean proportional found between the tranſverſe and conjugate diameters.

1. *For the mean proportional by the Rule.*

C D C D
As 72 : 72 :: 50 : 60, the mean diameter.

2. *For the content by the Rule.*

D C D C
As 18,95 : 12 :: 60 : 120,3, Content as before.

PROB. 4. *To find the content of a pyramid or cone.*

RULE. Find the area of the baſe by the foregoing rules, which area multiply by $\frac{1}{3}$ part of the height, and the product will be the content.

E. 1: The ſide of the baſe of a ſquare pyramid is 27 inches, and the perpendicular altitude 45 inches; what is the content in ale gallons?

27 = Side of the baſe
27
189
54
282)729 (2,585 = Area
564 15 = $\frac{1}{3}$ of the height
1650 38,775 Content in ale gallons,
1410
2400
2256
1440
1410
30

D C D C
By the Rule. As 16,79 : 15 :: 27 : 38,77, Content as before.

E. 2. Required the content of a cone in ale gallons, the diameter of whoſe baſe is 38 inches, and altitude 45 inches.

38 = Diameter of the baſe
38
304
114
359,05)1444,00(4,022 = Area
15 = $\frac{1}{3}$ of the height
60,330 = Content in ale gallons.

D C D C
By the Rule, As 18,95 : 15 :: 38 : 60,33, Content as before.

PROB. 5. *To find the content of the frustum of a pyramid or cone.*

RULE. Find the area of each base by the foregoing rules, and a mean proportional between them; then multiply the sum of those three by $\frac{1}{3}$ part of the depth, and the product is the content required.

E. 1. What is the content in ale gallons of a square pyramid, one side of the greater base being 40 inches, that of the lesser base 30 inches, and the height 60 inches?

First $40 \times 40 = 1600$; and $1600 \div 282 = 5,67$, Greater area
Then $30 \times 30 = 900$; and $900 \div 282 = 3,18$, Lesser area
$\because \sqrt{5,67 \times 3,18} = \sqrt{18,0306} = 4,24$, Mean
Sum 13,09
$\frac{1}{3}$ of the height = 20
Content in ale gallons 261,80

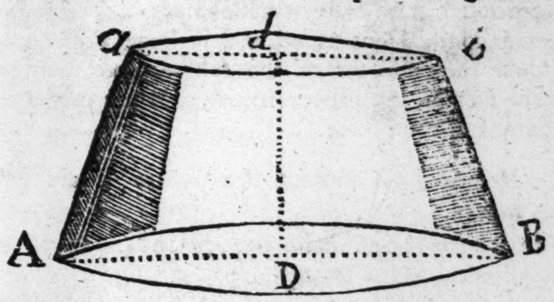
By the Rule.

	D		C		D		C	
Thus as	16,79	:	1	::	40	:	5,67,	Greater area
	16,79	:	1	::	30	:	3,18,	Lesser area

	C		D		C		D	
Then as	5,67	:	5,67	::	3,18	:	4,24,	Mean

Sum 13,09
$\frac{1}{3}$ of the height = 20
Answer 261,80 Ale gallons as before.

E. 2. What is the content in ale gallons of the lower frustum of a cone, whose diameter at the greater base A B is 38 inches, the diameter of the lesser base *a b* 20,2 inches, and the depth or height *D d* 21 inches?



First $38 \times 38 \div 359,05 = 4,02$ Greater area
And $20,2 \times 20,2 \div 359,05 = 1,13$ Lesser area
$\because \sqrt{4,02 \times 1,13} = \sqrt{4,5426} = 2,13$ Mean
7,28 Sum
7 = $\frac{1}{3}$ of the depth
Answer 50,96 Content in ale gallons.

By

By the Rule.

	D		C		D		C	
Thus as	18,94	:	1	::	38	:	4,02,	Greater area
	18,94	:	1	::	20,2	:	1,13,	Leſſer area.

	C		D		C		D	
Then as	4,02	:	4,02	::	1,13	:	2,13,	Mean
							7,28	Sum
							7	= $\frac{1}{3}$ of the height
						Anſwer	50,96	Same as before.

Prob. 6. *To find the content of a ſphere or globe.*

Every ſphere is two-thirds of its circumſcribing cylinder; therefore, if the cube of the diameter of a ſphere be multiplied by $\frac{2}{3}$ parts of the circular factors, &c. in Page 346, or if it be divided by 1, and half of the circular diviſors, then the product or quotient will be equal to the content of that ſphere. From hence may be found factors and diviſors for the cube of the diameter of any ſphere for all the denominations in Page 346.

1. FOR FACTORS.

	Factors for a cylinder.		*Factors for a ſquare.*	
$\frac{1}{2}$ of	,000365	=	,000243	Malt buſhels.
	,002785	=	,0018567	Ale gallons.
	,003399	=	,002267	Wine gallons.

2. FOR DIVISORS THE PROPORTION IS:

1	:	1,5	::	2737,9	:	4106,99,	Diviſor for malt buſhels.
1	:	1,5	::	359,05	:	538,58,	Diviſor for ale gallons.
1	:	1,5	::	294,12	:	441,18,	Diviſor for wine gallons.

And in this manner may factors and diviſors for a ſphere be found in all the different denominations mentioned in Page 346.

RULE. Cube the diameter of the ſphere, and multiply or divide that cube by the factors, &c. above found, and the product or quotient will be the content of the ſphere.

Example. Required the content of a ſphere in ale gallons, whoſe diameter is 22 inches.

Firſt $22 \times 22 \times 22 = 10648$; and 538,58)10648(19,76 Ale gallons, the content required.

Or $10648 \times ,0018567 = 19,76$ Ale gallons, as before.

The ſame by the lines D *and* E *on the Rule.*

	D		E		D		E	
As	1	:	,001856	::	22	:	19,76	the content, as before.

The ſame may be performed by the lines C and D on the Rule; for if the ſquare roots of the ſeveral diviſors be extracted, theſe roots will be the gauge points for a ſphere; thus:

The

The ſquare root of $\left\{\begin{array}{l}4106{,}99 = 64{,}1 \text{ the gauge point for malt buſhels.} \\ 538{,}58 = 23{,}2 \text{ ditto for ale gallons.} \\ 441{,}18 = 21{,}0 \text{ ditto for wine gallons.}\end{array}\right.$

D C D C

Then as 23,2 : 22 :: 22 : 19,76, Ale gallons as before

PROB. 7. *To find the content of the leſſer fruſtum of a ſphere, by having its diameter and altitude given.*

RULE. To three times the ſquare of half the diameter of its baſe, add the ſquare of its altitude; this ſum multiply by its altitude, and the product multiply or divide by the proper factors for a ſphere, in page 358, and the product or quotient will be the content required.

EXAMPLE. What is the content of the fruſtum of a ſphere in ale gallons, the diameter of whoſe baſe is 30 inches, and the altitude 9 inches?

225 = Square of ½ the diameter
3
———
675 = Three times the ſquare of ½ of the diameter
81 = Square of the altitude
———
756
9 = Altitude
———
538,58,6804,00(12,6 the content in ale gallons.

PROB. 8. *To find the content of the greater fruſtum of a ſphere.*

RULE. Multiply the ſquare of half the diameter by three times the altitude of the fruſtum, and to this ſum add the cube of the altitude: and this laſt ſum multiply or divide by the factors, &c. for a ſphere, in page 358; the product or quotient will be the content required.

EXAMPLE. Required the content of the fruſtum of a ſphere in ale gallons, the diameter of whoſe baſe is 30 inches, and altitude 25 inches?

225 = Square of ½ the diameter
75 = Three times the altitude
———
1125
1575
———
16875
15625 = Cube of the altitude
———
538,58) 32500,00(60,3 the content in ale gallons.

PROB. 9. *To gauge a maſh tun in the form of the fruſtum of a cone.*

RULE 1. With ſome convenient inſtrument, find the top and bottom diameters, and alſo the depth of the tun; then add the two diameters together, and take half that ſum for a mean diameter, which will be near enough the truth in practice.

2. To

2. To find the area at one inch deep, ſquare the mean diameter and divide that ſum by the circular diviſor for a maſh tun gallon in p. 346, and the quotient will be the area at one inch deep, which ſerves as a mean for the whole depth.

EXAMPLE. There is a maſh tun, whoſe top diameter is 70 inches, the bottom diameter 50,4 inches and depth 40 inches; what is the area at one inch deep?

```
                70 = Top diameter
              50,4 = Bottom diameter
             ------
       Sum   120,4
  Half ſum    60,2 = Mean diameter
            × 60,2
             ------
              1204
             36120
             ------
   289)3624,04(12,54 Mean area in gallons.
```

And in this manner was the table of maſh tun areas, in ſection 75, conſtructed.

By the Rule.

```
      D    C    D      C
As   17 : 1 :: 60,2 : 12,54, the area as before.
```

PROB. 10 *To gauge a ſquare maſh tun, whoſe ſides are every where ſtraight, from baſe to baſe.*

RULE 1. With ſome convenient inſtrument meaſure the length and breadth of the baſe, and alſo the depth of the tun.

2. Multiply the length of the baſe by the breadth, and the product multiply or divide by the factors, &c. for a ſquare maſh tun, in p. 346, and the product or quotient will be the area at one inch deep.

EXAMPLE. There is a maſh tun, whoſe baſe is a ſquare, each ſide meaſuring 50 inches, and the depth 20 inches; what is the area in maſh tun gallons?

```
           50
           50
          ----
   227)2500(11 Area in gallons.
        227
        ---
         230
         227
```

By the Rule.

```
      D     C    D    C
As  15,1 : 1 :: 50 : 11, Area as before.
```

PROB. 11.

PROB. 11: *To deduct the heat out of warm wort, in casting up the gauge.*

RULE 1. By subtraction: Set down the number of warm gallons twice? but the under number must be set one figure farther to the right hand, or, which is the same, remove the dot of the subtrahend one place farther to the left hand, and, and it is divided by 10; this tenth part subtracted from the number of warm gallons, will leave the quantity of wort to be charged.

2. By multiplication: Multiply the number of warm gallons by ,9, a decimal, and the product will be the neat gallons.

E. 1. Suppose a gaugue of warm wort were 35 gallons, what must be charged?

By Subtraction.	By Multiplication.
35	35
3,5	,9
Answer 31,5 Neat gallons.	Answer 31,5 Neat gallons.

If the warm worts are in circular bye-tubs, and the depth under diameters, then, instead of the gauge point 18,95, on the line D, make use of 20, to which set the depth on C, and against any diameter on D is the content in neat gallons on C.

E. 2. Suppose the diameter of a bye-tub is 30 inches, and the depth of warm wort in it 10 inches; how many neat gallons does it contain?

By the Sliding Rule.

	D	C		D	C	
As	20 :	10	::	30 :	22,5	Neat gallons, the content.

PROB. 12. *To find new gauge points.*

In business it sometimes happens, that when one of the given numbers is set to the gauge point, the other given number will fall off the rule: In this case there must be new gauge points found.—Thus, set unity on C to the old gauge point on D, and against the other 1 on C is the new gauge point on D.

E. 1. Suppose it were required to find a new gauge point to 18,95, the old ale gauge point?

Set unity on C to 18,95 on D, and against the other 1 on C is 59,92, the new gauge point on D for circular ale gallons.

E. 2. To find a new gauge point for wine measure.

Set unity on C to 17,15 on D, and against the other 1 on C is 54,22, on D, the new gauge point for circular wine gallons.

And in like manner may any other new gauge point be found.

Note. These second gauge points are the square roots of the divisors, multiplied by 10.—Thus, the circular divisor for ale gallons is 359,05, which multiplied by 10 = 3590,5, whose square root is 59,92, the new gauge point for ale gallons, the same as before found; and after the same manner may any other new gauge point be found.

Prob. 13. *To gauge and inch a mash tun, whose bases are equal squares or parallelograms.*

RULE 1. Find the area, at one inch deep, by the foregoing rules, and multiply that area by the depth for the content in gallons.

2. Reduce both the area and the whole content of the tun into quarters, bushels, and gallons.

3. To find the content upon every inch of the tun's depth, add the area of the first inch reduced, to itself, and the sum will be the content, at 2 inches deep; to this sum add the area of the first inch, and you will have the content, at 3 inches deep; and in this manner keep continually adding, till you arrive at the depth of the whole tun; which last will be equal to the content of the whole tun, if the work be right.

Example. The depth of a square mash tun is 30 inches, the length of the bases 119 inches, and the breadth 102 inches; what is the content of the tun, and also the content at every inch of its depth?

```
          119
          102
         ----
          238
        1190
        -----
227)12138(53,47 Area at 1 inch deep
             30 = Depth
          -------
       8)1604,10 = Content in gallons
          -------
       8)200,512 = 200  4,1 = Content in bushels
          -------
          25,064 = 25q. 0b. 4,1 = Content in quarters
8)53,47
   -----
    6,68 = 6 b. 5 g. 47 = Area at 1 inch in bushels.
```

	Q.	B.	G.	
Then per rule	0	6	5,47	= Content at 1 inch
	0	6	5,47	
	1	5	2,94	= Content at 2 inches
	0	6	5,47	
	2	4	0,44	= Content at 3 inches.

And in this manner proceed till you come to 30 inches, the whole depth of the tun.

The several contents thus found, are placed in the following table against their respective inches through the whole depth.

Note. In filling up the table, I have rejected the odd tenths, if they were under 5; and when they exceed 5, I have added one more to the even gallons.

TABLE.

TABLE.

Wet Inch.	Q.	B.	G.	Wet Inch	Q.	B.	G.	Wet Inch.	Q.	B.	G
1	0	6	5	11	9	1	4	21	17	4	3
2	1	5	3	12	10	0	2	22	18	3	1
3	2	4	1	13	10	6	7	23	19	1	6
4	3	2	6	14	11	5	5	24	20	0	3
5	4	1	4	15	12	4	2	25	20	7	1
6	5	0	1	16	13	3	0	26	21	5	6
7	5	6	6	17	14	1	5	27	22	4	4
8	6	5	4	18	15	0	3	28	23	3	1
9	7	4	1	19	15	7	0	29	24	1	7
10	8	2	7	20	16	5	6	30	25	0	4

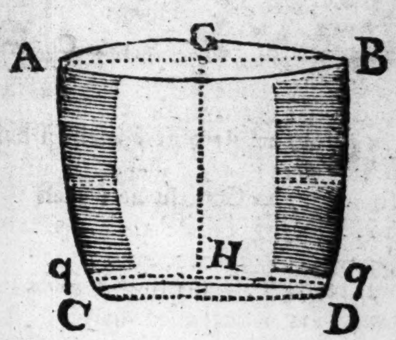
The USE of the TABLE.

EXAMPLE. Suppoſe the depth of the goods or grains were 20 inches in the maſh tun, how many quarters, &c. would it contain at that depth?

I ſeek for 20 in the column, under wet inches, and againſt it are 16 quarters, 5 buſhels, 6 gallons, the content.

PROB. 14. *To gauge and inch a copper with a riſing crown, and make an allowance for the ſame.*

EXAMPLE. Let the figure A B C D repreſent the copper to be gauged and inched.



RULE 1. Take a ſmall cord and faſten one end of it at A, and extend the other end to the oppoſite ſide of the copper at B, where make it faſt; then with ſome convenient inſtrument find the greateſt depth of it, that is, the neareſt diſtance from the thread to the bottom at C, and ſuppoſe its equal 30 inches: in like manner take the leaſt depth of the copper, which is the neareſt diſtance from the thread to the top of the crown at H, which ſuppoſe to be 27 inches; then 30—27 = 3 inches, the height of the crown.

2. To find the diameter at the bottom of the crown, measure A B the top diameter, which suppose 90 inches; then hold a thread with a plummet at the end of it, so that it may hang over D or C; then measure the distance between the edge of the copper and the place where the threads cut each other, which suppose 5 inches; then $5 + 5 = 10$, subtracted from the top diameter, 90, leave $80 = DC$, the diameter at the bottom of the crown.

3. To find the content of the liquor required to cover the crown, measure the diamater $q\,q$, which touches the top of the crown, and suppose it 82 inches; then by having the top and bottom diameters, and the altitude of the frustum of a cone, or that part $q\,q$ C D, the content of that part may be found by Prob. 6, Sect. 68, from which the content of the crown must be subtracted.

4. The content of the crown may be found by Prob. 11, Sect. 68, as the segment of a sphere; or thus: multiply the area of the bottom diameter D C by half the altitude of the crown, and the product will be nearly equal to the content of the crown; which subtracted from the part $q\,q$ C D, will leave the quantity of liquor required to cover the crown.

Ex. To find the quantity of liquor to cover the crown by the last rule:

$$\left.\begin{array}{l}\text{Diameter } q\,q = 82\\ \text{Diameter CD} = 80\\ \text{Mean Diameter} = 80{,}9\end{array}\right\}\text{ Areas }\left\{\begin{array}{l}18{,}727\\ 17{,}825\\ 18{,}23\end{array}\right.$$

Area of diameter D C = 17,825
½ the altitude = 1,5

89125
17825
26,7375

The content of $q\,q$ C D = 54,782
Content of the crown = 26,7375
Liquor to cover the crown 28,0445

To find the content of the copper, take mean diameters between every 6 or 10 inches, from the top downwards to the crown, viz. the part A B $q\,q$; as supposing in this example I found the first mean diameter at 5 inches, from the top 88 inches; at 15 inches, from the top 85,5 inches; and at 25 inches, from the top 82,5 inches, as they are set down in the second column of the following table: then find their respective areas, and set them down against their diameters in the third column, and the contents of the several parts of the depth in gallons in the fourth column; which several contents are reduced to barrels, &c. in the three last columns.

Parts of Depth.	*Diameter.*	*Area.*	*Content in Gallons.*	*Content in* B.	F.	G.
10	88,0	21,568	215,680	6	1	3,180
10	85,5	20,360	203,600	5	3	8,100
07	82,5	18,956	132,692	3	3	5,192
03	To cover the crown		28,044	0	3	2,044
50	Content of copper		580,016	17	0	2,516

Note. The two upper contents in gallons are found by removing the dot in the areas one place farther to the right hand; the last content is found by multiplying its correspondent area by the depth 7.

To find the content upon every inch of the copper's depth.

RULE. From the whole content of the copper reduced into barrels, &c. subtract the area of the first 10 inches, and the remainder will be the content, when one inch is dry; and so continue subtracting that area from the remainder, until 10 inches are dry, and you will gain the contents of the first 10 dry inches.—See the whole operation.

Whole	*B.*	*F.*	*G.*	*Contin.*	*B.*	*F.*	*G.*	*Contin.*	*B.*	*F.*	*G.*
Cont. =	17	0	2,516	10 =	10	2	7,336	20 =	4	2	7,736
Subtract.	0	2	4,568		0	2	3,360		0	2	1,956
1 =	16	1	5,948	11 =	10	0	3,976	21 =	4	0	5,780
	0	2	4,568		0	2	3,360		0	2	1,956
2 =	15	3	1,380	12 =	9	2	0,616	22 =	3	2	3,824
	0	2	4,568		0	2	3,360		0	2	1,956
3 =	15	0	5,312	13 =	8	3	5,756	23 =	3	0	1,868
	0	2	4,568		0	2	3,360		0	2	1,956
4 =	14	2	0,744	14 =	8	1	2,396	24 =	2	1	8,412
	0	2	4,568		0	2	3,360		0	2	1,956
5 =	13	3	4,676	15 =	7	2	7,536	25 =	1	3	6,456
	0	2	4,568		0	2	3,360		0	2	1,956
6 =	13	1	0,108	16 =	7	0	5,176	26 =	1	1	4,500
	0	2	4,568		0	2	3,360		0	2	1,956
7 =	12	2	4,040	17 =	6	2	0,816	27 =	0	3	2,544
	0	2	4,568		0	2	3,360	Crown	0	3	2,544
8 =	11	4	7,972	18 =	5	3	5,956	Remain.	0	0	0,000
	0	2	4,568		0	2	3,360				
9 =	11	3	3,404	19 =	5	1	2,596				
	0	2	4,568		0	2	3,360				
10 =	10	2	7,336	20 =	4	2	7,736				
	6	1	3,180		6	1	3,180				
					5	3	8,100				
	17	0	2,516		17	0	2,516				

Note. When I have finished the subtraction with the first area, which is at the 10th dry inch, I add the content of the first 10 inches to the remainder, in order to prove my work, whose sum, if right, will be equal to the content of the copper.

Then

Then I take the second area, and subtract it from the last remainder; and so continue to do till I come to the 20th dry inch; and then to prove my work, I add the contents of the first and second 10 inches to the last remainder, whose sum, if right, will be also equal to the whole content of the copper.

Lastly, I take the third area, and subtract it from the last remainder, and so proceed till I come to the 27th dry inch, whose remainder, if the work be right, will be equal to the quantity of liquor to cover the crown, as found before. Thus having finished the subtraction, I transfer the several contents to the nearest even gallon, as in the annexed table, and it is done.

Dry Inch.	*B*	*F.*	*G.*
Full.	17	0	2
1	16	1	6
2	15	3	1
3	15	0	5
4	14	2	1
5	13	3	5
6	13	1	0
7	12	2	4
8	11	3	8
9	10	1	3
10	10	2	7
11	10	0	4
12	9	2	1
13	8	3	6
14	8	1	2
15	7	2	8
16	7	0	4
17	6	2	1
18	5	3	6
19	5	1	3
20	4	2	8
21	4	0	6
22	3	2	4
23	3	0	2
24	2	1	8
25	1	3	6
26	1	1	4
27	0	3	3
To cover the crown	0	3	3
1 half inch	0	1	2,5
2 half inch	0	1	2
3 half inch	0	1	1,5

PROB. 15. *To gauge a back or cooler, and to find the content at every tenth of an inch.*

RULE 1. Find the area at one inch deep, by the foregoing rules, which reduce to barrels, firkins, and gallons; then find the one-tenth of that area by removing the dot one place farther to the left hand, which also reduce to barrels, &c.

2. To find the content at every tenth, add the area or content of the first tenth to itself, and you will have the content at two tenths; again, add the content of the first tenth to the content of the second tenth, and you will have the content of three tenths; and thus by continually adding the content of the first tenth for every tenth, you may tenth a back to what depth you please.

EXAMPLE. Suppose I found the length of a back 272 inches, and breadth 90,8 inches; what is the content at every tenth of the first 3 inches of the back's depth?

Length 272
Breadth 90,8

2176
24480

282)24697,6(87,58 = Area at 1 inch in gallons.
2256

2137
1974

1636 — *B.* 2, *F.* 2, *G.* 2,58 = Area at 1 inch reduced.
1410

2260
2256

Remains 4

The area of 1 inch being 87,58 gallons, the tenth part of that sum is 8,758 gallons, which reduced is *ob.* 1*f.* 2,58*g.* and so much will the back hold on every tenth of an inch of its depth.

To find the content at every tenth of an inch, till you come to the depth required, proceed as follows.

	B.	*F.*	*G.*		*B.*	*F.*	*G.*
1 Tenth =	0	1	0,258	6 Tenth =	1	2	1,548
	0	1	0,258		0	1	0,258
2 Tenth =	0	2	0,516	7 Tenth =	1	3	1,806
	0	1	0,258		0	1	0,258
3 Tenth =	0	3	0,774	8 Tenth =	2	0	2,064
	0	1	0,258		0	1	0,258
4 Tenth =	1	0	1,032	9 Tenth =	2	1	2,322
	0	1	0,258		0	1	0,258
5 Tenth =	1	1	1,290	Content at 1 inch =	2	2	2,580
	0	1	0,258				
6 Tenth =	1	2	1,548				

Thus, when I come to 1 inch of the back's depth, I find the content to be equal to that before found, which proves the work to be true; and in this manner I proceed to make the following table, to three inches of the back's depth, where all the contents are placed to the nearest even gallon.

TABLE

TABLE.

Wet Inch, and Tenths.	*B.*	*F.*	*G.*
,1	0	1	0
,2	0	2	1
,3	0	3	1
,4	1	0	1
,5	1	1	1
,6	1	2	2
,7	1	3	2
,8	2	0	2
,9	2	1	2
1,0	2	2	3
,1	2	3	3
,2	3	0	3
,3	3	1	3
,4	3	2	4
,5	3	3	4
,6	4	0	4
,7	4	1	4
,8	4	2	5
,9	4	3	5
2,0	5	0	5
,1	5	1	5
,2	5	2	6
,3	5	3	6
,4	6	0	6
,5	6	1	6
,6	6	2	7
,7	6	3	7
,8	7	0	7
,9	7	1	7
3,0	7	2	8

Note. In gauging you muſt take only even inches in the maſh tun: But the copper under, back, round, and ſquares, muſt be taken to the neareſt half inch, and the backs to the neareſt tenth.

Likewiſe the wet inches are taken in the maſh tun, under back and backs; but the dry inches in coppers, rounds, and ſquares.

PROB. 16. *To find the ſolidity of the ungulæ, or hoofs, of the fruſtum of a cone, having the length of the greateſt and leaſt diameters, and alſo the depth given in inches.*

RULE 1. *For the greater hoof.*

Multiply the product of the greater diameter and the fruſtum's height by the ſquare of the greater diameter, made leſs by the product of the leſſer diameter into a mean proportional between the two diameters; and this laſt product divide by three times the circular diviſors in Page 346, multiplied into the difference of the diameters, and the quotient will be the ſolidity of the greater hoof.

RULE 2. *For the leſſer hoof.*

Multiply the product of the leſſer diameter and height by the product of the greater diameter, into a mean proportional between the two diameters, made leſs by the ſquare of the leſs diameter; and this laſt product divide by three times the circular diviſors for ale, wine, malt, &c. in Page

Page 346, multiplied into the difference of the diameters, and the quotient will be the ſolidity of the leſſer hoof.

EXAMPLE. Let the following figure repreſent the fruſtum of a cone, whoſe greater diameter A B = 36 inches, the leſſer diameter D C = 30 inches, and the height h = 20 inches; what is the ſolidity of the greater hoof A C B, and alſo the leſſer hoof A D C, in ale gallons.

1. For the greater hoof A C B.

36 = A B   36   32 = Mean dia.
20 = h 36   30 = Leſs = D C

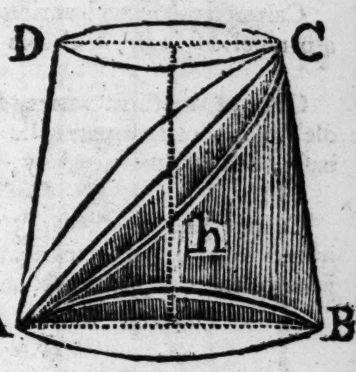
720 = 1ſt pro.   216   960 = 2d prod.
108

1296 = A B ſquare
— 960 = 2d product

336
× 720 = 1ſt product

6720
2352

6462)241920(37,43, the content in ale gallons.



2. For the leſſer hoof A D C.

30 = D C   36 = A B   30
20 = h 32 = Mean diameter   30

600 = 1ſt product   72   900 = □ of D C
108

1152
— 900

252
× 600

3×359×6=6462)151200(23,39 ale gallons
37,43
23,39

60,82 = Content in ale gallons of the whole fruſtum.

LXXII. CASK GAUGING.

IN order to have a right underſtanding in this matter, it is neceſſary for the gauger to have ſome idea of the conic ſections, becauſe caſks are generally compared to ſolids, generated by one or other of thoſe ſections; which would enable him the better to diſtinguiſh to which of theſe ſolids the caſk's curve bears the neareſt reſemblance.

Gaugers have reduced those different curvatures of casks to four varieties, &c.

Casks of the first variety are most curved, and are considered as the middle frustum of a spheroid; which is represented by the outer lines of the annexed figure.

Casks of the second variety, are supposed to be the middle frustum of a parabolic spindle, which is represented by the second lines.

Casks of the third variety, are supposed to be in the form of the middle frustum of two parabolic conoids, joined together at their greater bases, and is represented by the third lines.

Casks of the fourth variety, are formed of the frustums of two cones, joined together at their greatest bases, and are represented by the inner lines of the annexed figure.

Prob. 1. *To find the content of a cask in any of the four varieties, both by pen and rule.*

1. To find a mean diameter.

RULE. Multiply the difference between the head and bung diameters, when it is less than 6 inches.

By	,68	For the	1	Variety
	,62		2	
	,55		3	
	,5		4	

Or if the difference between the head and bung exceed 6 inches.

By	,7	For the	1	Variety
	,64		2	
	,57		3	
	,52		4	

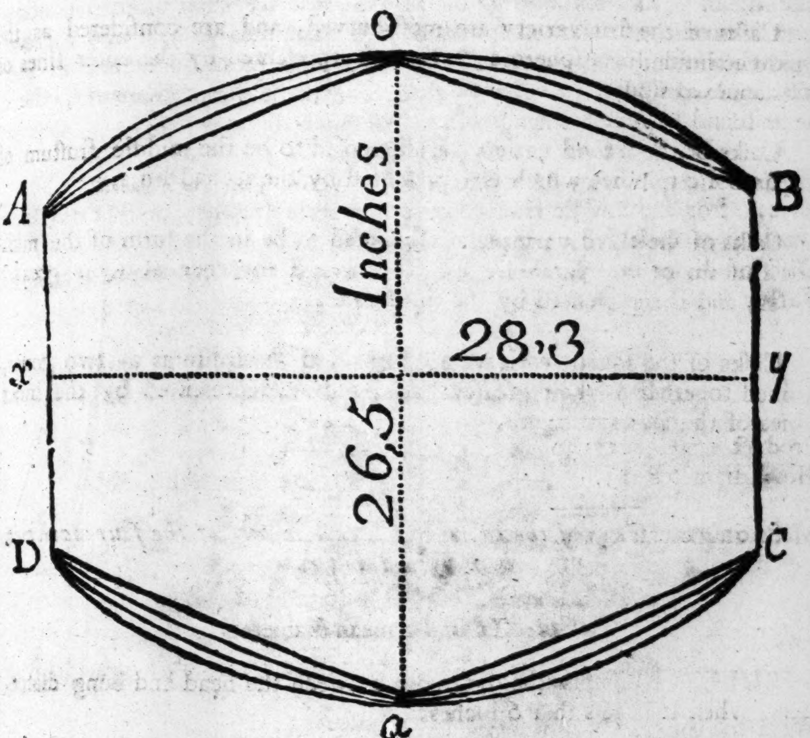
And add the product to the head diameter, then that sum will be a mean diameter.

2. To find the content.

RULE. Square the mean diameter and multiply that square by the length of the cask, and the product multiply or divide by the factors in Page 346, and the product or quotient will be equal to the content of the cask.

The

The Figure of the four Varieties of Casks.



E. 1. Let the above figure represent all the four varieties of casks, whose bung diameter *O a* is 26,5 inches, head diameter A D = B C 23 inches, and length *x y* 28,3 inches; required the content in ale gallons, supposing it to belong to all the four varieties.

For the Spheroid, or 1st Variety, by the Pen.

		25,4 = Mean diam. nearly
Bung diameter	26,5	25,4
Head diameter	23,0	1016
Difference -	3,5	1270
Factor - -	,68	508
	280	645,16 = ◻ of mean diameter
	210	28,3 = Length
Product -	2,380	193548
Head diam.	23,	516128
Mean diameter	25,38	129032
		359,05)18258,028(50,8 Content in ale gals.

By the Sliding Rule.

Find the difference of the diameters, which in this caſe is 3,5; look on the line of inches on the edge of the rule, and ſee what number ſtands againſt it on the line marked ſpheriod, and what number you find there, add to the head diameter, and the ſum will be a mean diameter; as in this example againſt 3,5 is 2,4 + 23 = 25,4, the mean diameter, the ſame as found before. Then,

D	C	D	C
As 18,95 :	28,3 ::	25,4 :	50,8, content in ale gallons.

E. 2. For the middle fruſtum of a parabolic ſpindle, or that of the 2d variety of casks, by the pen.

Diff. of diameters	3,5	25,2 = Mean diam. nearly
Factor	,62	25,2
	70	504
	210	1260
		504
Product	2,170	
Head diameter	23,	635,04
		28,3 = Length
Mean diameter	25,170	
		190512
		508032
		127008
		359,05)17971,632(50,0 cont. in ale gals.

By the Sliding Rule.

With the difference of the diameters, look on the line of inches, on the edge of the rule, and what number ſtands againſt it on the line marked 2d variety(add to the dead diameter, which ſum will be a mean diameter. In this example the difference is 3,5, againſt which on the line marked 2d variety, is 2,2 nearly, which added to the head diameter is 25,2, the mean diameter. Then,

D	C	D	C
As 18,95 :	28,3 ::	25,2 :	50, content as before.

E. 3. For the fruſtum of two parabolic conoids, or 3d variety of casks, by the pen.

Diff. of diameters	3,5	24,9
Factor	,55	24,9
	175	2241
	175	996
		498
Product	1,925	
Head diameter	23,	620,01
		28,3 = Length
Mean diameter	24,925	186003
		496008
		124002
		359.05)17546,283(48,8 cont. in ale galls.

By the Sliding Rule.

With the difference of the diameters, look on the line of inches on the edge of the rule, and what number you find againſt it on the line marked 3d variety, add to the head diameter, and that ſum will be a mean diameter. In this example the difference is 3,5, againſt which, on the line marked 3d variety, is 1,9, which added to the head diameter, is 24,9, the mean diameter. Then,

D C D C
As 18,95 : 28,3 :: 24,9 : 48,8, content as before.

E. 4. For the fruſtum of two cones, joined together at their greateſt baſes, called the 4th variety, by the pen.

Diff. in diameters	3,5	24,75
Factor	,5	24,75
Product	1,75	12375
Head diameter	23,	17325
		9900
Mean diameter	24,75	4950
		612,5625
		28,3
		18376875
		49005000
		12251250

359,05)17335,51875(48,281 content in ale gals.

By the Sliding Rule.

With the difference of diameters, look on the line of inches on the edge of the rule, and what number ſtands againſt it, on the line marked FC, add to the head diameter, whoſe ſum will be a mean diameter. In this example againſt 3,5, on the line FC, is 1,75, which added to the head diameter = 24,75, the mean diameter. Then,

D C D C
As 18,95 : 28,3 :: 24,75 : 48,3, content in ale gallons.

The content of the ſeveral varieties may be found in wine meaſure, by dividing by its proper diviſor, page 346; and the proportion by the rule for wine gallons would be, as the wine gauge point on D is to the length of the caſk on C, ſo is the mean diameter on D to the content in wine gallons on C.

Example. What is the content of the firſt variety in wine gallons, ſee Example 1, by the rule.

D C D C
As 17,15 : 28,3 :: 25,4 : 63,5, content in wine gallons.

And in this manner may the content of the other three varieties be found in wine gallons.

LXXIII.

LXXIII. ULLAGING OF CASKS.

SEVERAL writers on this ſubject have ſhewn how to ullage a caſk by a table of ſegments, calculated for a cylindrical cask; but, becauſe that requires you always to have that table ready at hand, and doth not always agree with the lines of ſegments, on the ſliding rule; I ſhall here omit it, and ſhew how to effect the ſame by pen and ſliding-rule.

Prob. 1. *To ullage a lying caſk by the pen, having the bung diameter, wet inches, and the content of the caſk given.*

RULE 1. Divide the wet or dry inches by the bung diameter, and if the quotient be under ,500, ſubtract a fourth part of what that quotient wants of ,500, from the quotient, and the remainder multiply by the content of the cask, and the product will be equal to the quantity of liquor in the cask.

2. When the quotient of the wet inches divided by the bung diameter exceeds ,500, then add a fourth part of that exceſs to the quotient, and that ſum multiplied by the content of the cask, will produce the content of the liquor in the cask; but if the dividend was dry inches, the product is what it wants to fill it up.

Example. There is a cask, whoſe bung diameter is 31 inches, (wet inches 21, dry inches 10) and content 75,37 gallons; what liquor is there in the cask, and how many gallons will fill it up?

31)21,000(,677 = more than ,500
,500

4),177(,044
,677 + Quotient

The area of ſegment ,721 × 75,37 = 54,34177 gallons of ale, in the cask.

2. For the vacuity, or what will fill it up.

,500
31)10,000(,322 = leſs than ,500

4),178(,0445 } ſubtract
,322

Area of the ſegment	-	,2775
Content of the cask	-	75,37
		19425
		8325
		13875
		19425
Wants to fill the cask	-	20,915175
+ What is in the cask	-	54,34177
Content of the cask	-	75,256945 ale gallons.

By

By the Sliding Rule.

1. As the bung diameter on the line of numbers on the little ſlider, marked N, is to 100 on the line of ſegment, marked SL, ſo is the wet or dry inches on the line of numbers N, to a ſegment upon SL; which reſerve.

2. As 100 upon A, is to the cask's content upon B, ſo is the reſerved ſegment upon A, to the quantity of liquor in the cask.

N SL N SL
As 31 : 100 :: 21 : 73,8; which reſerve.
A B A B
And as 100 : 75,37 :: 73,8 : 56,6, ullage of liquor in the cask.

To find the vacuity by the rule, you muſt work in all reſpects as you did for the ullage, only, inſtead of the wet inches, you muſt make uſe of the dry inches: Thus,

N SL N SL
As 31 : 100 :: 10 : 26,2 which reſerve.
A B A B
Then as 100 : 75,37 :: 26,2 : 19,7, the vacuity of cask.
In the cask — — 55,6

Content of the cask = 75,3, nearly the ſame as before.

PROB. 2. *To find the content of the ullage of a ſtanding caſk, by the pen.*

RULE. Divide the wet or dry inches by the length of the cask, and if the quotient exceeds ,500, add to the ſaid quotient one tenth part of the exceſs; but if it be under ,500, ſubtract one tenth part of what it wants of ,500; then let this ſum or difference be multiplied by the content of the cask, and the product will be equal to the quantity of liquor therein, if the dividend was the wet inches; but if it was the dry inches, it gives the vacuity, or what it wants to fill it up.

EXAMPLE. Let us ſuppoſe a ſpheroidal cask poſited as above, the length 32,5 inches, the bung 27, the head 23, the content of this cask will be 59,95 ale gallons: then let the wet inches be 8,5, I demand how much liquor there is in the cask, and alſo the vacuity?

,5000
32,5)8,50000(,2615 = under ,500

10),2385(,02385 = $\frac{1}{10}$ of the wants of ,5000
,2615 = the wet quotient

Difference 23765
The Content 59,95
118825
213885
213885
118825

14,2471175 = the content of the liquor.

$$
\begin{array}{l}
32,5)24,00000(,7384 \\
\quad ,5000 \\
\hline
10),2384(,02384 \\
\quad +,73840 \text{ Dry quotient} \\
\hline
\quad ,76224 \\
\quad 59,95 = \text{Content} \\
\hline
\quad 381120 \\
\quad 686016 \\
\quad 686016 \\
\quad 381120 \\
\hline
45,6962880 = \text{Vacuity} \\
14,2471175 = \text{Ullage} \\
\hline
59,9434055 = \text{Content of the cask.}
\end{array}
$$

By the Sliding Rule.

	N		SL		N		SL	
As	32,5	:	100	::	8,5	:	24;	which reserve. Then,
	A		B		A		B	
As	100	:	59,95	::	24	:	14,3,	content of the liquor.

For the Vacuity.

	N		SL		N		SL	
As	32,5	:	100	::	24	:	76;	which reserve. Again,
	A		B		A		B	
As	100	:	59,95	::	76	:	45,6	the vacuity
						+	14,3	the ullage
							59,9	Content of the cask.

Note. The difference between the sum of the separate parts thus found, and the whole content of the cask, is occasioned by the line of segments being adapted to one particular sort of cask only; which is not material, and near enough the truth in practice.

LXXIV. MALT GAUGING.

PROB. I. *To gauge a maltster's square, or oblong cistern.*

RULE I.

MEASURE the length and breadth of the cistern, in several places, and in case you find any variation, add the lengths or breadths together, and divide their sum by the number of dimensions taken of each, and the quotient will be a mean length or breadth; and at the same time also, the depth of the cistern.

2. To find the area of the cistern, multiply the length by the breadth, and that product multiply or divide by the proper factors for squares malt bushels, in page 346; and the product or quotient will be equal to the area.

EXAMPLE.

EXAMPLE. There is a ciſtern, whoſe length is 114 inches, breadth 58,5 inches, and the depth 36 inches; what is the area at one inch deep?

```
                 114 = Length
                58,5 = Breadth
                ----
                 570
                912
               570
        -----------
2150,42)6669,00(3,1 = Area in buſhels
        645126
        ------
         217740
         215042
         ------
           2698
```

By the Rule.

A B A B
As 2150,42 : 114 :: 58,5 : 3,1 = Area as before.

Note. If any depth in inches be multiplied by the area, the product will be equal to the content of the malt in the ciſtern.

PROB. 2. *To gauge a maltſter's round ciſtern.*

RULE 1. Take mean diameters between every ſix, or ten inches of the depth, and at the ſame time take the depth.

2. Find the area of each mean diameter; then ſquare the diameters, and multiply or divide each ſquare by the circular factors for malt buſhels, in Page 346, and the product or quotient will be equal to the ſeveral areas required.

EXAMPLE. Suppoſe the depth of the ciſtern be 30 inches, and the diameter at 5 inches from the baſe 29,6 inches, at 15 inches from the baſe the diameter is 33 inches, and 25 inches from the baſe the diameter is 36,2 inches; required the reſpective areas of the mean diameters in malt buſhels.

```
Mean diameter = 29,6
                29,6
                ----
                1776
               2664
               592
        ------------
2737,92)876,160(,32 the lower area
        821376
        ------
         547840
         547584
         ------
            256
```

And in this manner the other areas are found to be ,40, and ,48.

By the Rule.

	D		C		D		C	
As	52,32	:	1	::	29,6	:	,32	= 1st area
	52,32	:	1	::	33,0	:	,40	= 2d area
	52,32	:	1	::	36,2	:	,48	= 3d area

Note. All depths that are taken in this ciſtern, muſt be multiplied by the reſpective areas to which they belong.

Prob. 3. *To gauge a couch of malt, in a ſquare or oblong frame, and find the content of the ſame.*

RULE. Multiply the length, breadth, and depth together, and that product divide by the ſquare diviſor for malt buſhels, in Page 346, and the quotient will be the content of malt in the couch.

Example. What is the content of a couch in malt buſhels, whoſe length is 105 inches, breadth 104 inches, and depth 20 inches?

```
   105 = Length
   104 = Breadth
  ----
   420
  1050
  -----
  10920
     20
  ------
2150,42)218400,00(101,5 Content is buſhels.
```

By the Rule.

	A		B		A		B
As	20	:	105	::	104	:	101,5 Content as before.

Prob. 4. *To find the content of a couch, or floor of malt, having the length, breadth, and depth given in inches.*

RULE. Multiply half the length of the floor by the breadth, and that product by the depth; from this laſt product cut off three figures to the right hand, and it will give the content of the floor ſeven buſhels too much in every 100; which exceſs may be deducted, either by ſubtraction or multiplication.

If the malt diviſor had been 2000, this method would have anſwered without any deduction; but ſince it is 2150, the multiplying by half the length or breadth, will give too much; to remedy this, if you multiply the product of the length, breadth, and depth, by 93, you will find the true content.

Or, if you ſubtract ſeven buſhels for every 100, and ,7 for every 10 buſhels, or ,07 for every ſingle buſhel, you will have the content.

Example. Suppoſe the length of a floor be 400 inches, breadth 215, and depth 4 inches; what is the content in malt buſhels?

215 = Breadth	By Subtraction.
200 = Half the length	172 = Product
	12,04
43,000	
4 = Depth	159,96 Content as before.
172	For 100 deduct 7 bushels
,93	For 70 deduct 4,9
	For 2 deduct 0,14
516	
1548	Sum — 12,04
159,96 Content in Bushels.	

LXXV. MONEYING OF CHARGES.

PROBLEM I. *To money Goods at* $1\frac{1}{4}$*d. per pound.*

RULE.

CUT off the right hand figure, which count so many pence and farthings, and those on the left hand will be so many shillings and half-pence.

EXAMPLE. What is the duty of 364 pounds of sheep-skins, at $1\frac{1}{4}$*d.* per pound?

36|4 = 36*s.* 36 half-pence, and four times five farthings.

		£.	s.	d.
36 shillings	=	1	16	0
36 half-pence	=	0	1	6
4 times $1\frac{1}{4}$*d.*	=	0	0	5
Answer		1	17	11 the duty required.

PROB. 2. *To money goods at the rate of 30l. per cent.*

RULE. Divide the value of the goods by 5, and to the quotient add its half, whose sum will be the duty required.

EXAMPLE. Suppose the value be six pounds, ten shillings, and ten pence; what will the duty of the same amount to?

	£.	s.	d.	
5)	6	10	10	
	1	6	2	
+	0	13	1	= One half
Answer	1	19	3	the duty required.

Or the same may be found by multiplying the value of the goods by ,3.

 PROB. 3.

Prob. 3. *To money goods at the rate of 15l. per cent.*

RULE. Divide the given value of the goods by 5, and from the quotient subtract one fourth part; the remainder will be the duty required.

Example. Suppose the value be five pounds, eight shillings, and four-pence; what is the duty?

	£.	s.	d.	
5)	5	8	4	
	1	1	8	
—	0	5	5	$=\frac{1}{4}$
Answer	0	16	3	the duty required.

Or the same may be found by multiplying the valne of the goods by ,15.

Prob. 4. *To money goods at the rate of 18l. per cent.*

RULE. Divide the value of the goods by 5, and from that quotient, subtract half of its one fifth; the remainder will be the duty required.

Example. Let the value of the goods be 23l. to find the duty.

	£.	s.	d.	
5)	23	0	0	
5)	4	12	0	$=\frac{1}{5}$
2)	0	18	$4\frac{1}{4}$	$=\frac{1}{5}$
—	0	9	$4\frac{2}{5}$	$=\frac{1}{2}$ of the $\frac{1}{5}$
Answer £.	4	2	$9\frac{3}{5}$	the duty required.

Or the same may be found by multiplying the value of the goods by ,18.

Prob. 4. *To find the duty of any number of barrels of victuallers strong beer, at 8s. per barrel.*

RULE. Multiply the given number of barrels by 4, and the product will be pounds, except the units figure of the product, which will be so many two shillings; to which add the money for the firkins (if any) and that sum will be equal to the duty of the whole.

Example. What is the duty of $325\frac{1}{2}$ barrels of victuallers strong beer, at eight shillings per barrel?

	325		= number of barrels
	4		
	130,0		
+		4s.	= barrel
Answer £.	130	4	

Prob. 5.

PROB. 5. *To find the duty of any number of barrels of victuallers small beer, at 1s. 4d. per barrel.*

RULE. To the given number of barrels, add one-third of that sum, and if there be any quarters, add proportionably for them. The sum of the whole will be equal to the duty in shillings and pence, which reduce into pounds, and it is done.

EXAMPLE. What is the duty of $295\frac{1}{2}$ barrels of victuallers small beer, at one shilling and four-pence per barrel?

$$\begin{array}{rl} 3)295 & = \text{number of barrels} \\ 98\ \ 4 & = \frac{1}{3} \text{ of the given number} \\ 0\ \ 8 & = \frac{1}{2} \text{ barrel} \\ \hline 2|0)39|4\ \ 0 & \\ \hline £.19\ \ 14\ \ 0 & \text{Answer} \end{array}$$

And in this manner may the amount of the duty of any other sort of goods be found.

PROB. 6. *To find the drawback of any number of barrels of victuallers strong beer, at 1s. 8d. or small beer at 4d.*

RULE. The drawback of one barrel of either strong or small beer, being multiplied by the number of barrels, gives the answer.

E. 1. What is the drawback of 91 barrels of victuallers strong beer, at one shilling and eight-pence per barrel?

$$\begin{array}{rrrl} & s. & d. & \\ & 1 & 8 & \\ & & 9 & \times 10 + 1 = 91 \\ \hline & 15 & 0 & \\ & & 10 & \\ \hline 7 & 10 & 0 & \\ + & 1 & 8 & \\ \hline £.7 & 11 & 8 & \text{Answer.} \end{array}$$

E. 2. Suppose a victualler be charged with 25 barrels of small beer, what must be allowed him for drawback at 4d. per barrel?

$$\begin{array}{rrrl} & & d. & \\ & & 4 & \\ & & 5 & \times 5 = 25 \\ \hline & 1 & 8 & \\ & & 5 & \\ \hline £.0 & 8 & 4 & \text{Answer.} \end{array}$$

PROB. 7.

Prob. 7. *To reduce the groſs buſhels of malt, taken in the ciſtern or couch, and flour, to neat buſhels.*

RULE. It is ſuppoſed that barley, after it is firſt wetted or ſteeped in the ciſtern, and ſtood there its proper time, and from thence emptied into the couch, and lain there about 30 hours, riſes or increaſes to about $\frac{1}{5}$ part more than it was before; therefore, 4 buſhels in every 20 are to be allowed for that increaſe.

But when the malt has been out of the ciſtern above 30 hours, it is deemed to be a floor of malt; and it is ſuppoſed that a buſhel of dry barley, thus wetted and ſteeped, &c. and afterwards thrown out into the floor, and there grown according to the uſual cuſtom, will increaſe or riſe to two buſhels, or double to what it was before; therefore, 10 buſhels in every 20 are to be allowed for that increaſe.

In order to find the proper factors to reduce each of theſe buſhels to their equivalent value in neat buſhels, obſerve the following method.

1. For the ciſtern or couch buſhels.

From 20 = buſhels
Subtract 4 = $\frac{1}{5}$

Remains 16 = $\frac{4}{5}$ = ,8, factor for ciſtern or couch buſhels.

If any number of buſhels, from ciſtern or couch, be multiplied by the above factor, the product will be equal to the neat buſhels.

Example. In 200 buſhels, from ciſtern or couch, how many neat buſhels?

200
,8

Anſwer 160,0 neat buſhels.

2. For the floor buſhels.

There being 10 buſhels in every 20 to be allowed for the increaſe of floor buſhels, therefore the floor buſhel is $= \frac{10}{20} = \frac{1}{2} =$,5, a common factor for floor buſhels.

If any number of floor buſhels be multiplied by the above factor, the product will be equal to the neat buſhels.

Example. In 260 buſhels from the floor, how many neat buſhels?

260
,5

Anſwer 130,0 neat buſhels.

Prob. 8. *To find factors for reducing of couch buſhels into floor buſhels; and on the contrary, for reducing floor buſhels to couch buſhels.*

RULE The factors found in the laſt prob. are to each other as unity to the required factors; therefore, the proportions are as follows:

1ſt. As

1ſt. As ,5 : ,8 :: 1 : 1,6, the factor for couch buſhels.
2d. As ,8 : ,5 :: 1 : ,625, the factor for floor buſhels.

Or the charge may be found, by multiplying the floor buſhels by ,625; and if the product be more than the buſhels from the beſt, the charge will be from the floor, but if leſs, then from the beſt of the ciſtern and couch.

EXAMPLE. Suppoſe the content of a floor gauge of malt be 261 buſhels, and the content of the beſt, ciſtern, or couch, were 200 buſhels; from which will the charge arife?

Unity *Fact.* *Couch*
As 1 : 1,6 :: 200
200
Anſwer 320,0 floor buſhels.
Or thus, 200
1,6
Anſwer 320,0 the ſame as before.

By the Rule.

B A B A
As 1 : 1,6 :: 200 : 320 floor buſhels.

By which I find the amount of the couch is = 320 floor buſhels, that is 59 buſhels more than the number of floor buſhels before found, therefore, the charge will ariſe from the couch.

Or the ſame may be found by this proportion:

B A B A
As 1 : ,625 :: 320 : 200, couch buſhels.

From whence it alſo appears that the couch gauge is the beſt.

PROB. 9. *To find the duty of any number of buſhels from the ciſtern or couch.*

RULE. The duty of 1 buſhel of malt from ciſtern or couch, with the allowance of 4 in 20, or the $\frac{1}{5}$ part is 1*s.* 0$\frac{1}{2}$*d.* ,4 parts, which reduced to the decimal of a pound ſterling, is ,0525, the common factor: now if any number of buſhels from ciſtern or couch be multiplied by the above factor, the product will be equal to the duty in pounds, and decimal parts of a pound.

E. 1 How much will the duty of 120 buſhels of malt, from the ciſtern or couch, amount to, at 1*s.* 0$\frac{1}{2}$*d.* ,4 tenths per buſhel?

,0525 = Factor
120 = No. of buſhels

10500
525

6,3000 Anſw. 6*l.* 6*s.* 0*d.*
20

6,0

E. 2. How much will the duty of 260 buſhels of malt from the beſt of the ciſtern or couch amount to, at 1*s.* 0$\frac{1}{2}$*d.* ,4 per buſhel?

,0525 = Factor
260 = No. of buſhels

31500
1050

Anſ. 13,6500 = 13*l.* 13*s.* duty.

PROB. 10.

Prob. 10. *To find the duty of any number of bushels from the floor.*

RULE. This may be done by a factor, which is found as follows: The duty of 1 bushel of malt from the floor, with the allowance of 10 in every 20, is 7*d.* 3*qrs.* ,5*tenths*, which reduced to the decimal of a shilling, is = ,65625, the factor.—Now if any number of bushels from the floor be multiplied by the above factor, the product will be equal to the duty in shillings, and decimal parts of a shilling.

Example. What is the duty of 400 bushels of malt from the floor, at $7\frac{3}{4}$*d.* ,5*tenths* per bushel?

```
        ,65625 = Factor
           400
   ---------------
   2|0)2|62,50000
   ---------------
        13,125
            20
   ---------------
         2,500
            12
   ---------------
           6,0 Answer 13l. 2s. 6d. the duty required.
```

Or thus: $7\frac{3}{4}$*d.* ,5*tenths*, reduced to the decimal of a pound, = ,0328125, the common factor, by which if you multiply any number of bushels from the floor, the product will be the duty in pounds and decimal parts of a pound.

Example. What is the duty of 80 bushels of malt from the floor, at $7\frac{3}{4}$*d.* ,5 per bushel?

```
        ,0328125 = Factor
              80 = No. of bushels
   ---------------
       2,6250000
              20
   ---------------
          12,500
              12
   ---------------
             6,0 Answer 2l. 12s. 6d. the duty required.
```

And in this manner may the duty of any number of bushels of malt, from the floor, be found:

Prob. 11.

PROB. 11. *To find the duty of any number of barrels of common brewers ſtrong beer, at eight ſhillings per barrel, with the allowance of 2½ in every 23 barrels.*

RULE. This may be done by finding a factor, thus:

		£.	s.
The duty of 23 barrels, at 8*s*. per barrel	=	9	4
Allowance out of 23 barrels is 2½, and duty	=	1	0
Duty of 23 barrels of common brewers X beer	=	8	4

Then the proportion for the factor is

Bar.		*£.*		*Bar.*		*Decimal.*
As 23	:	8,2	::	1	:	,35652, the common factor.

Therefore, if any number of barrels, and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What is the duty of 150½ barrels of common brewers ſtrong beer, at eight ſhillings per barrel, with the allowance of 2½ barrels in 23?

```
  ,35652 = factor for ſtrong beer
   150,5 = given number of barrels
 -------
  178260
 1782600
 35652
 -------
53,656260 = 53l. 13s. 1½d. the duty.
```

PROB. 12. *To find the duty of any number of barrels of common brewers ſmall beer, at one ſhilling and four-pence per barrel, the allowance being 2½ in every 23 barrels.*

There muſt be a factor found for common brewers ſmall beer, as well as ſtrong, in the following manner:

		£.	s.	d.
The duty of 23 barrels at 1*s*. 4*d*. per barrel -	=	1	10	8
The allowance of 2½ barrels - - - - - - - -	=	0	3	4
The duty of 23 barrels of common brewers ſmall beer	=	1	7	4

Then the proportion for the factor is,

Bar.		*£.*		*Bar.*		*Deci. of £.*
As 23	:	1,366	::	1	:	,05942, the factor.

Therefore, if any number of barrels and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What will the duty of $152\frac{1}{2}$ barrels of common brewers small beer amount to, at one shilling and four-pence per barrel?

$$
\begin{array}{r l}
152,5 & = \text{Given number of barrels} \\
,05942 & = \text{Factor for small beer} \\
\hline
3050 & \\
6100 & \\
13725 & \\
7625 & \\
\hline
9,0615{\bf 5}0 & = 9l.\ 1s.\ 2\tfrac{1}{4}d.\ ,3 \text{ The duty.}
\end{array}
$$

PROB. 13. *To find the duty of any number of barrels of common brewers table beer, at three shillings per barrel, with the allowance of $2\frac{1}{2}$ in every 23 barrels.*

By reason of this allowance there will be also a fraction in the price of one barrel; so there must be a factor found for common brewers table beer, as well as for strong and small.

The factor is found in the following manner:

	£.	s.	d.
The duty of 23 barrels at 3s. per barrel is	3	9	0
The allowance of $2\frac{1}{2}$ barrels is - - -	0	7	6
The duty of 23 barrels of common brewers table beer is	3	1	6

Then the proportion for the factor is,

	Bar.		*£.*		*Bar.*		*Decimal of £.*	
As	23	:	3,075	::	1	:	,1336956521739130	+ The factor.

Therefore, if any number of barrels, and quarters of a barrel, of common brewers table beer, reduced to a decimal, be multiplied by the above factor, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What will the duty of 40 barrels of common brewers table beer amount to, at 3s. per barrel?

		£.	s.	d.	qr.	pts.	
,1336956521739130	= Factor for table beer						
40	= Given number of barrels						
5,3478260869565200	=	5	6	11	1	21	The duty required.

The

The common rule by which tables for that purpose are made, is thus:

```
        Bar.   £.  s.  d.     Bar.
     If 23  :  3   1   6  ::  1
               20
               --
          23(61(2s: 8d. qr. 2/3
             46
             --
             15
             12
             --
        23) 186(8
            184
            ---
              2
              4
             --
              8
```

the duty of 1 barrel; therefore, all the fractional parts in common brewers tables for beer, are so many parts of 23 of a farthing.

PROB. 14. *To find factors for reducing the odd gallons of one denomination to their equivalent value in that of another denomination, so that they may produce the same duty.*

RULE 1. If the odd gallons to be reduced only differ in duty, and there be the same number of gallons to the barrel, hogshead, &c. then the factor may be found by dividing the pence that one sort is charged per barrel or hogshead, &c. by the pence that the other denomination is charged at, *vice versa*, and the quotient will be the factor required.

2. When the odd gallons to be reduced not only differ in duty, but also in the number of gallons to a barrel, hogshead, &c. then the factor will be found by multiplying the pence that one sort is charged with duty, by the number of gallons in a barrel, hogshead, &c. of the other for a dividend; which divided by the product of the number of gallons to the barrel, hogshead, hogshead, &c. and the duty of the other, *vice versa*, the quotient will be the factor required.

E. 1. Required to find a factor to reduce strong beer, at eight shillings per barrel to small, at one shilling and four-pence per barrel?

By Rule 1.—First 8*s*. = 96*d*. ÷ 16 = 6, the factor required.

E. 2. It is required to find a factor for reducing small beer to strong.—See the first example.

By Rule 1.—96) 16,000 (,166, the factor required.

E. 3. It is required to find a factor to reduce odd gallons of cyder, at 18*s*. 9*d*. 0 *qrs*. ,8 per hogshead, containing 63 gallons, to its equivalent value in gallons of small beer, at 1*s*. per barrel, whose barrel contain 34 gallons?

By Rule 2.

The duty of a hogshead of cyder, 18*s*. 9*d*. 0 *qr*. ,8 = 900,8 *qrs*.
And 900,8 *qrs*. × 34 gallons = 30627,2, the dividend.
Also 48 *qrs*. × 63 gallons = 3024, the divisor.
∴ 30627,2 ÷ 3024 = 10,128, the factor required.

 And

And in this manner you may find factors to reduce any number of odd gallons of one denomination, to their equivalent value in those of another.

Note. The drawback must be taken out of both strong and small beer before it be reduced, as in the above example.

PROB. 15. *To find factors for reducing ale measure to wine and corn measure; and e'contra, corn to ale and wine measure.*

To perform which, oblerve the following proportions, which are wrought by the Rule of Three Inverse.

282	:	1	::	231	:	1,220779	factor for ale to wine.
231	:	1	::	282	:	,819148	———wine to ale.
282	:	1	::	2150,42	:	,131137	———ale to corn.
2150,42	:	1	::	282	:	7,625602	———corn to ale.
231	:	1	::	2150,42	:	,107422	———wine to corn.
2150,42	:	1	::	231	:	9,309177	———corn to wine.

The USE *of the foregoing* FACTORS

Multiply any number of gallons of ale, wine, or corn measure, by its proper factor; the product will be the number of gallons, reduced to the measure required.

EXAMPLE. Required to reduce 63 gallons of wine to ale measure?

$$\begin{array}{r} ,819148 = \text{Factor for wine to ale.} \\ 63 \\ \hline 2457444 \\ 4914888 \\ \hline \end{array}$$

Answer 51,606324 gallons, ale measure.

And thus may any number of gallons of ale measure be reduced to corn or wine measure by the help of the foregoing factors.

The way to find any FACTOR *is as follows:*

Let a = number of gallons, a hogshead, &c. b = number of gallons in a barrel, &c. c = duty of a hogshead, &c. d = duty of a barrel, &c. and x = the factor required.

Then $a : b :: c : d \times x$, and $a\,d\,x = b\,c$, whence $x = \frac{bc}{ad}$; the factor required.

When the number of gallons in each are the same, and the difference is only in the price, then $a = b$ and $x = \frac{c}{d}$.

PROB. 16. *To find factors for salaries, both for common and leap years, at any rate per annum.*

RULE. As the number of days in a year is to the salary per annum, so is one day to its salary; which decimal of the salary, for one day, will be a proper factor to find the salary of any number of days at that rate.

EXAMPLE.

EXAMPLE. If the salary be 5*l.* per annum, what will the factors be at that rate, both for a common and leap year?

	Days.		*£.*		*Day.*		*Decimal.*	
As	365	:	5	::	1	:	,013699,	the factor for a common year.

	Days.		*£.*		*Day.*		*Decimals.*	
As	366	:	5	::	1	:	,013661,	the factor for a leap year.

And in this manner was the following Table of Factors computed.

A TABLE OF FACTORS FOR SALARIES.

Salaries per annum. £.	*s.*	*d.*	*Factors for a common year.*	*Factors for a leap year.*	*Salaries per annum.* £.	*s.*	*d.*	*Factors for a common year*	*Factors for a leap year.*
5	0	0	,013699	,013661	90	0	0	,246575	,245901
10	0	0	,027397	,027322	100	0	0	,273072	,273224
15	0	0	,041051	,040983	115	0	0	,316439	,315574
20	0	0	,054794	,054645	120	0	0	,328766	,327869
25	0	0	,068493	,068106	200	0	0	,547974	,546448
30	0	0	,082191	,081967	300	0	0	,821917	,819672
40	0	0	,109589	,109289	400	0	0	1,095948	1,022896
48	2	6	,131849	,131489	500	0	0	1,369863	1,366119
50	0	0	,136986	,136610	600	0	0	1,643834	1,639344
52	0	0	,142465	,142076	700	0	0	1,917808	1,912568
60	0	0	,164383	,163934	800	0	0	2,191896	2,185792
70	0	0	,191780	,191256	900	0	0	2,465811	2,459616
80	0	0	,219178	,218579	1000	0	0	2,739726	2,732240
86	12	6	,237329	,233948					

The Use of the Table of Factors.

By the above table the salary due for any number of days at any rate therein mentioned may be found, both for a common or leap year; for if you take the factor of the rate, and multiply it by the number of days, the product will be equal to the salary due in pounds, and decimal parts of a pound.

E. 1. Suppose the salary to be forty pounds per annum; how much would be due to a person for sixty days?

The factor for a common year, at 40*l.* per ann. is ,109589
Multiplied by the number of days .... 60 £. s. d.
Answer .... 6,575340 = 6 11 6

E. 2. Suppose the salary was fifty pounds per annum; how much would be due to an officer for 80 days?

The factor for 50*l.* = ,136986
Number of days = 80
Answer 10,958880 = 10*l.* 19*s.* 2*d.*

Note. If any of the factors in the table are reduced, they will shew the amount of a day's salary, at any rate therein mentioned.

In

In the foregoing examples, only the groſs ſalary is found; but the Officers of Exciſe are deducted 9*d*. in the pound for tax and charity; therefore, to find the neat ſalary, you muſt multiply the groſs ſalary by ,9625, and the product will be the neat ſalary.

EXAMPLE. Suppoſe the groſs ſalary to be fifty pounds per annum; what is the neat money?

```
 ,9625
    50
------
48,1250
    20
------
 2,5000
    12    Anſwer 48l. 2s. 6d. the neat ſalary.
  ----
  6,0
```

The conſtruction of the ſalary tables for a common year is thus;

```
 Days.  £.  Day.
As 365 : 50 :: 1
          20
365)1000(2s.
     730
     ---
     270
      12
365)3240(8d.                  s.  d.  qr.  pts.
    2920            Anſwer    2   8   3    37
    ----
     320
       4
365)1280(3qrs.
    1095
    ----
     185   37
     --- = --  in its loweſt terms; ſo that all the fractional
     360   73     parts in thoſe tables are ſo many 73 parts
                  of a farthing.
```

After the ſame manner are ſalary tables for a leap year calculated; only, inſtead of 365, take 366 days, and work as before, and the anſwer will be 2*s*. 8*d*. 3*qr*. $\frac{54}{366} = \frac{9}{61}$ in its loweſt terms; ſo that all the fractional parts in thoſe tables are ſo many 61 parts of a farthing.

LXXVI.

LXXVI. SURVEYING.

IN the following inſtructions for ſurveying, I ſhall make uſe of no other inſtrument but the chain, for that uſed properly, is the compleateſt for the purpoſe both in accuracy and expedition.

The chain now moſt in uſe (commonly called Gunter's) contains 4 poles, or 22 yards in length, and is divided into 100 decimal parts, or links, each link containing 7,92 inches; and an acre contains 10 ſquare chains, viz. 10 in length and 1 in breadth; the are likewiſe 100,000 ſquare links in an acre.

And for the learner's better information, I have inſerted the following new Table of Square Meaſure.

A TABLE OF SQUARE MEASURE.

Inches 1								
62,7264	Links 1							
144	2,295	Feet 1						
1296	20,75	9	Yards 1					
3600	57,38	25	2,7	Paces 1				
39204	625	272,25	30,25	10,89	Perches 1			
627264	10000	4356	484	174,24	16	Chains 1		
6272640	100000	43560	4840	1742,4	160	10	Acres 1	
4014489600	6400000	2787840	309760	1115136	102400	6400	640	Mile 1

The above table may be read thus, viz. in one chain there are 16 ſquare perches, 174,24 ſquare paces; 484 ſquare yards; 4356 ſquare feet; 10000 ſquare links; and 627264 ſquare inches; and ſo of the reſt.

The

The laying down of plain figures being already taught in practical geometry, Part 4. Sec. 66, it is quite unneceſſary to enlarge upon that ſubject in this place: I ſhall therefore obſerve to the learner, that in meaſuring with the chain, he muſt be careful to get the ſhorteſt diſtance between any two objects, otherwiſe he will make more of the land than it really is.

In taking dimenſions of a field, it is beſt to begin at ſome remarkable place, and from thence proceed according to the ſituation of the field.

It is proper for the learner, at his entrance into the field, to draw a figure of the ſame at random, which will enable him to plan the ſame with more certainty, but when he is ready in the practical part of ſurveying, ſuch draughts will be unneceſſary.

When the field is bounded by irregular hedges, care muſt be taken to ſtraighten them by taking up off-ſets, which muſt be as near the fence as poſſible; and every perpendicular muſt be taken, ſo, that if a right line was drawn from the end of any one to the next, that line would neither include your neighbour's ground, nor exclude any part of that you are meaſuring.

Prob. I. *To meaſure and find the content of a ſquare piece of land.*

RULE. Multiply the ſide by itſelf, the product is the content.

Example. How many acres are contained in a ſquare field, each of whoſe ſides are 12 chains and 20 links, or 12,20?

```
      12,20 = ſide.
      12,20
      -----
      24400
     2440
    1220
    --------
    14,88400
           4
    --------
     3,53600
          40
    --------
    21,44000
    A. R. P.
Anſwer 14 3 21
```

Note. Having multiplied the ſide by itſelf the product is 1488400, which to reduce into acres, roods, and perches, point off five places of figures to the right hand, the remainder to the left hand are acres, then multiply the figures pointed off to the right hand by 4, and point off five figures to the right hand as before, and the left hand figure is roods; laſtly multiply the figures pointed off by 40, and point off five figures to the right as before, the remaining figures to the left are perches.

If you work the links as the decimal of a chain, and divide the product by 10, the ſquare chains in an acre, the anſwer in acres, &c. will be the ſame.——See the laſt example worked by this method.

```
       12,20 = ſide
       12,20
       -----
       24400
      2440
     1220
     --------
 10) 148,8400
     --------  A. R. P.
Anſwer 14,88400 = 14 3 21 the ſame as before.
```

Prob. 2.

PROB. 2. *To find the content of a rectangular piece of land.*

RULE. Multiply the length by the breadth, the product is the content.

EXAMPLE. How many acres are contained in a rectangular field, whose length is 12,15, and breadth 5,40 chains?

$$
\begin{array}{r l}
12,15 & = \text{length} \\
5,40 & = \text{breadth} \\
\hline
48600 & \\
6075 & \\
\hline
6,56100 & \text{Answer} \quad \begin{array}{ccc} A & R & P \\ 6 & 2 & 9 \end{array} \\
4 & \\
\hline
2,24400 & \\
40 & \\
\hline
9,76000 &
\end{array}
$$

PROB. 3. *To find the content of a triangular piece of land.*

RULE. Multiply the base by half the perpendicular, the product is the content.

EXAMPLE. There is a triangular piece of land whose base measures 10,42, and perpendicular 5,22, what is the content in acres, &c.?

$$
\begin{array}{r l}
10,42 & = \text{base} \\
2,61 & = \text{half perpendicular} \\
\hline
1042 & \\
6252 & \\
2084 & \\
\hline
2,71962 & \\
4 & \text{Answer} \quad \begin{array}{ccc} A & R & P \\ 2 & 2 & 35 \end{array} \\
\hline
2,87848 & \\
40 & \\
\hline
35,13920 &
\end{array}
$$

PROB. 4. *To find the content of a field that is comprehended under four unequal sides, called a trapezium.*

RULE. Multiply the diagonal by half the sum of the perpendiculars, the product is the content.

EXAMPLE. Let the following figure represent a four sided field, whose diagonal measures 40 chains, one perpendicular 7 chains, and the other 14 chains; what is the content in acres?

First $14 + 7 = 21$ sum of the perpendiculars; and $21 \div 2 = 10,5$ half sum of perpendiculars; then $10,5 \times 40 = 420$, and $420 \div 10 = 42$ acres the answer.

 The

The above dimenſions being taken by a four pole chain, the product as above is 420 ſquare chains, which, divided by 10, the quotient is 42 acres, as appears by the work; therefore when one or any of your numbers conſiſts of chains only, there is no neceſſity to prefix cyphers in the place of links.

The common way of meaſuring a field of four unequal ſides, is by meaſuring from one corner to the oppoſite one, which divides it into two triangles, and in meaſuring this line perpendiculars muſt be erected to the other corners of the field, the places from whence theſe perpendiculars riſe are found by the croſs ſtaff or triangle, fixed on your ſtaff for that purpoſe, ſo this diagonal is a baſe line common to both perpendiculars, which may be meaſured by the preceding rule.

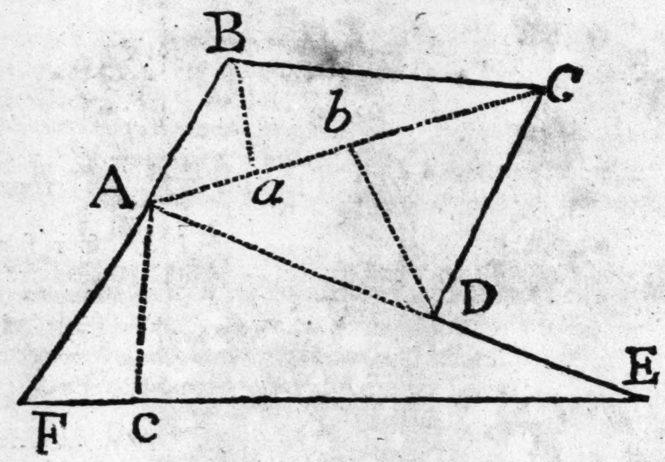
But you may plot the former field very expeditiouſly, if you meaſure round it, putting down each ſide ſeparately, and likewiſe meaſure one diagonal, which divides it into two triangles, each of which may be truly plotted by PROB. 14, ſec. 66.

PROB. 5. *To find the content of any irregular field, conſiſting of any number of ſides.*

RULE, Meaſure round it, and put down each ſide ſeparately, then reduce it into trapeziums and triangles, and meaſure each ſeparately, which ſeveral dimenſions collected into one ſum will be the ſuperficial content of the field.

EXAMPLE. Admit a field, A B C D E F, conſiſting of ſix unequal ſides, whoſe dimenſions in chains and links are as follow, viz. from A to B 3,15; B to C 5,90; C to D 4,40; D to E 4,20; E to F 11,40; and from F to A 4,00; what is the content?

Note. This and the following figures are laid down by a ſcale of ſixteen ſtatue poles, or four chains to an inch.



To plot this field, proceed as in the laſt example; thus:

1. Let fall a perpendicular from B to *a*, and D to *b*, to the diagonal A C; and meaſure them on your ſcale, which products add together, and multiply that ſum by half A C, and you have the content of the trapezium A B C D.

Let

2. Let fall a perpendicular from A to c, to the baſe F E, and meaſure it on your ſcale, which multiply by half F E, and the product is the content of the triangle F A E; then add the contents together, the ſum will be the content of the whole field. See the work.

2,70 = Ba	3,55 = Ac
3,30 = Db	5,70 = $\frac{1}{2}$F E
6,00 = Sum	24850
3,55 = $\frac{1}{2}$ A C	1775
3000	2,02350 = Area of the triangle F A E.
3000	
1800	

2,13000 = Area of the trapezium A B C D.
2,02350

	A	R	P
4,15350 = Area of the whole field =	4	0	24.

4

,61400
40

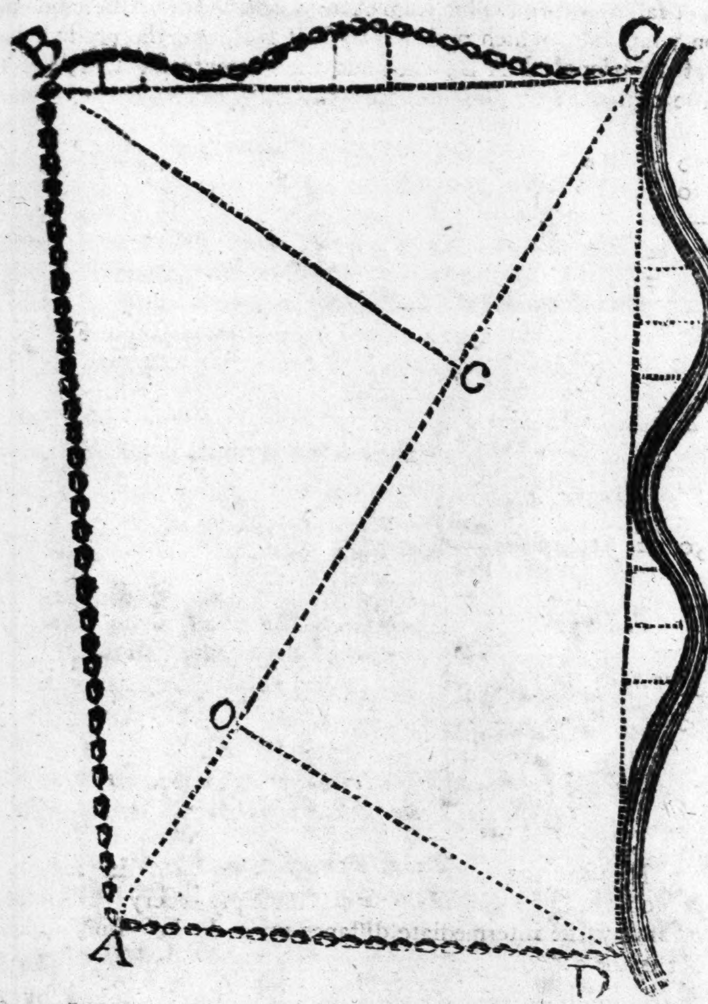
24,56000

Prob. 6. *To meaſure and find the content of any irregular field, whoſe boundaries are curved or circular.*

RULE. Meaſure the triangles and trapeziums as directed in the foregoing Problems; and for the off-ſets multiply every two adjacent perpendiculars by the intermediate diſtance upon the baſe line, and half the product is the content.

Note. The intermediate diſtance upon a baſe line is found by ſubtrcting the foregoing length or diſtance from the following.

Example. Let the following figure repreſent a field, bounded on one ſide by a ſmall river; I deſire to know the content in acres.



First, suppose you enter the field at A; measure the side A B, which you will find is 16,00.

Secondly, measure the side B C, which is 10,60, and as you measure take up the off-sets, and enter them in your field-book, thus; when you have measured 1 chain from B towards C, take up the first off-set with your staff, which is 0,50; at 2 chains another off-set 0,35; and at 6,25 you will perceive it necessary to take up another off-set, which you will find 0,65; and at 7,45 another, which is 0,60.

Thirdly, measure the side C D, which you will find 16,30, and as you advance take up the off-sets; thus as you measure from C towards D at 3,65, you take up the first off-set = 0,80; at 4,60 — 1,45; at 6,00 — 1,00; at 9,40 — 0,60; at 10,20 — 1,10; and at 11,10 — 1,00.

Fourthly,

Fourthly, meaſure the ſide DA, which is 9,00; and laſtly meaſure the diagonal AC 19,25, and your dimenſions are finiſhed in the field, which may be caſt up as follows:

See the work of each off-ſet and trapezium.

Top or north off-ſets.

,50	,50	,35	,65	,65	1,20
1,	,35	1,30	1,70	,60	60
,50	,85	1050	4550	1,25	,7200
	1	35	65	1,20	
	,85	,4550	1,1050	2500	
				125	
				1,5000	

,50
,85
,4550
1,1050
1,5000
,7200 } The ſeveral products collected

2) 5,1300

A. R. P.

10) 2,565 = Square chains = 0 0 4 content of the north off-ſets.

,2565
4
,10260
40
4,10400

Brook ſide off-ſets.

,20	,55	1,55	,80	,5250	The ſeveral products collected.
,55	1,55	,80	2,55	2,1000	
				3,4075	
,75	2,10	2,35	400	2,0400	
,70	1,00	1,45	400	,5000	
			160	1,6000	
,5250	2,1000	1175		1,7100	
		940	2,0400	2,3400	
		235			
				2)14,2225	
		3,4075			
				10)7,1112	A. R. P.
,50	,50	1,00	,90	,71112	= 0 2 33
1,00	1,00	,90	2,60	4	Content of the brook ſide off-ſets.
,5000	1,50	1,90	5400	2,84448	
	1,00	,90	18	40	
	1,6000	1,7100	2,3400	33,77920	

The

The trapezium.

8,70 = B *c*
8,00 = Ditto

16,70 = Sum of the perpendicular B *c* Ditto
9,62 = ½ AC the diagonal

3340
10020
15030

A. R. P.
16,06540 = 16 0 10 content of trapezium
4

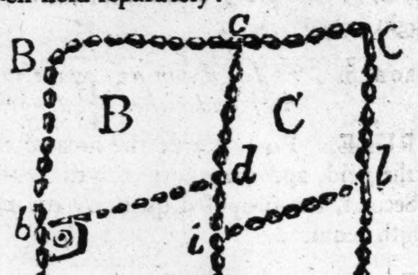
,26160
40

10,46400

A	R.	P.	
16	0	10	= area of trapezium
0	2	33	= area of brookside off-sets
0	0	4	= area of north off-sets
16	3	7	= area of the whole field.

Note. Some practitioners cast up off-sets by dividing the sum of the perpendiculars by the numbers thereof taken for a mean breadth, contained between the straight line and the hedge, which part they cast up as a parallelogram, this erroneous practice is, I fear, too much used, on account of ease and expedition.

EXAMPLE. Let the following figure represent the ground plot of a small estate, left by Mr. Geo. Fentham for charitable uses to the town of Birmingham, and it is required to measure the same with the chain only, so that the dimensions thereof may be cast up and planned; what is the content of the whole, and each field separately?



Suppose you enter the estate at A, 1st measure from A to *a* 1 chain, from *a* to *b* 4 chains, and from *b* to B 3 chains, 15 links. Having chained from A to B, proceed to measure the other sides; but as you advance take care to lay down the off-sets. as before directed.

2d. Measure from B to *c* 3 chains 20 links, from *c* to *d* 2,35, from *d* to *i* 1,15, from *d* to *b* 3,26, the dimensions of the field B being finished, walk to *c*, and measure from *c* to C 2,65, from C to *b* 3,05, and from *l* to *i* 2,90, and as *c i* was measured before, there will be no need of mea-

ſuring or ſtraightening that ſide again, ſo the dimenſions of the field C being finiſhed, proceed from *i* to *e* 4,45 from *e* to *n* 1 chain, from *n* to D 2,90, and from D to *l* 5,30; the dimenſions in the field D being finiſhed, I walk to *e* and meaſure *e a* 3.10, and as *d i* and *i e* was meaſured before, there is no need to meaſure that over again; therefore the dimenſions in the field A are finiſhed, but if the firſt field A is not a true ſquare you muſt meaſure the diagonal from *d* to *a* and caſt up the dimenſions as two triangles, as before directed.

Laſtly, meaſure from *n* to A 3,25, and your dimenſions in the field are finiſhed, which may be caſt up and planned at pleaſure.

Note. As the off ſets, &c. are meaſured and caſt up as before directed, I think it will be unneceſſary to inſert the operations at length, as it will give the learner an opportunity of caſting up the ſame, and if the reſult of his work ſhould agree with the following contents of each field, his work is undoubtedly true.

		A	R	P
Houſe, garden, &c.	F	0	1	13
Well piece	A	1	1	14
Hill piece	B	0	3	18
Calf's croft	C	1	2	23
Pit piece	D	1	1	37
Total of the whole eſtate		5	2	25

PROB. 7. *To meaſure woods or large pools of water.*

RULE. Meaſure round it, and at every bending or turn take the angle, and meaſure the diſtance from one turn to another, and enter all down in your field-book, and when you come round to the place where you began, if your plot cloſe, your work is right; when you have plotted your work, take off the baſes and perpendiculars from the ſame ſcale you laid down the plot, and from thence caſt up the content in acres, as taught before.

PROB. 8. *To lay down any quantity of land in a field, when the quantity and either the length or breadth are given.*

RULE. Firſt, divide the area of the given quantity, by the length of the field, and the quotient will be the breadth required.

Second, the propoſed quantity divided by the breadth will give the length required.

EXAMPLE 1. Suppoſe a farmer lets an acre of meadowing, to be laid out on one ſide of a field that is 22 chains long, how broad muſt the land be to make an acre?

$$\begin{array}{l} 22 \\ \underline{4} \qquad\qquad\quad (4 \\ 88)160,00(\underline{1,818} \\ \qquad\qquad\quad ,454 \text{ Links anſwer.} \end{array}$$

But

But for the assistance of those who do not understand decimal arithmetic, I have inserted the following table.

Breadth	Length		
C.	C.	L.	P.
1	10	00	,0
2	5	00	,0
3	3	33	,33
4	2	02	,50
5	2	00	,0
6	1	66	,6
7	1	42	,28
8	1	25	,0
9	1	11	,11
Length	Breadth		

The Use of the Table.

If the length or breadth be given in chains, links, &c. the other may be found by inspection, viz. if the length be even chains, look on the contrary dimensions and you will see how many chains, links, &c. must be measured for an acre.

EXAMPLE. Suppose the length of a field was 5 chains, how much in breadth will make an acre? against 5 chains in the table you will find 2 chains; the breadth required to make an acre; and so of the rest.

PROB. 9. *To Reduce a large plot of land or map into a lesser compass, or, on the contrary, to enlarge it.*

RULE. If it be a field or two it is the best way to plot it over again by a greater or lesser scale; but if it be large, as the map of a county or manor, &c. the readiest way is to circumscribe it with a geometric square, and divide that square into several other lesser squares, and by this means every field, house, &c. in one will fall in the same square in the other.

PROB. 10. *To find the exact distance to any visible object without any instrument, or actually measuring the same.*

RULE. First, get four straight sticks, of any length you please, then let it be required to find the distance A B upon level ground, at B put down one of your sticks, there stand and order an assistant to put down another at F; so that standing at B you may see the staff F and the object at A, both in a straight line (now it matters not at what distance the staff B is from the staff F, but if your distance required be far, then the further F is from B the better,) then take a third staff and go from F any number of yards, chains, or any other measure to D, so that the line F D may be at right angles with B A, and at D put down the third staff.

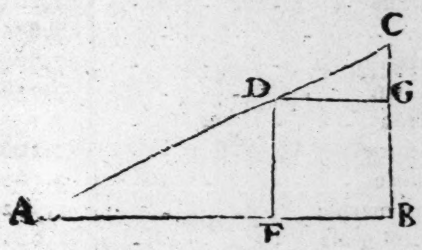
Lastly, take the other staff, and go from B (square-wise, as before) so far till you can see the staff D and the object A in a right line; which suppose at C; here make a mark, and measure the distance C G, 25 feet, &c. and G B 39,1; then C G 25 + G B 39,1 = B C, 64,1 feet; by measuring F D = B G = 39,1; the truth of this problem is grounded upon similar triangles, for the triangle C G D is similar to the triangle C B A. Therefore it will always hold.

As

As C G 25 is to G D 39, so is C B 64,1 to B A 100 the answer.

The work at Length.

As 25 : 39 :: 64,1
39
5769
1923
25)2499,9(99,9 = 100 nearly, the distance required.
225
249
225
249
225
24



N. B. As the narrow limits of this treatise would not admit of room to give the whole art of surveying, I have given such examples as I thought would be the most useful to the young *Tyro*, as well as the farmer and grazier, which examples carefully considered and duly regarded, will enable the learner to find the content, and plan a single field, or estate with certainty and expedition.

LXXVII. SPECIFIC GRAVITY.

THE specific gravity of a body, is the relation that the weight of a body of one kind hath to the weight of an equal magnitude of a body of another kind; the knowledge of which is of great use, not only in natural philosophy, but also in common life, in computing the weights of such bodies as are too unweildy to have their weights discovered by other means.

The following table shews the specific gravity to rain water, of metals, and other bodies; and the weight of a cubic inch of each, in parts of a pound avoirdupoise and of ounces troy.

N. B. If the specific gravity of any solid in the table be less than 1000, it will swim in water; but if greater than 1000 it will sink.

Bodies.	Spe. Gra.	Wt. lb. Avo d.	Wt. oz. Troy.
Fine gold	19,639	0,7103587	10,359273
Standard gold	18,887	0,7060185	9,962625
Guinea gold	17,793	0,6828703	9,911707
Quick-silver	13,762	0,4976574	7,384411
Lead	11,313	0,4091696	5,984010
Fine silver	11,091	0,4011501	5,850035
Standard silver	10,629	0,3844400	5,556769
Copper	8,769	0,3171658	4,747121
Plate brass	8,350	0,2942593	4,404273
Cast brass	8,104	0,2929832	4,272409
Steel	7,850	0,2839265	4,142127
Bar iron	7,764	0,2808159	4,031361
Block tin	7,238	0,2417901	3,861519
Cast iron	7,135	0,2380647	3,806568
Load stone	5,106	0,1846788	2,724083
Blue slate	3,500	0,1264914	1,867272
Veined marble	2,702	0,0977286	1,429411
Common glass	2,600	0,0940393	1,360841
Flint stone	2,582	0,0933883	1,351419
Portland stone	2,570	0,0929543	1,345139
Free stone	2,352	0,0915788	1,231038
Brick	2,000	0,0723379	1,046801
Alabaster	1,888	0,0683061	0,988456
Ivory, Horn	1,832	0,0662606	0,958489
Brimstone	1,800	0,0651042	0,949424
Clay	1,712	0,0619213	0,902498
Lignum-vitæ	1,327	0,0479862	0,699936
Coal	1,255	0,0553921	0,661956
Pitch	1,150	0,0415943	0,606759
Mahogany wood	1,063	0,0384475	0,560691
Dry box wood	1,030	0,0372530	0,543282
Milk, Sea water	1,033	0,0372530	0,543742
Rain water	1,000	0,0361690	0,527458
Red wine	0,993	0,0359158	0,523766
Bees' wax	0,996	0,0359881	0,524820
Linseed oil	0,932	0,0337095	0,491591
Proof spirits or brandy	0,927	0,0335503	0,489268
Dry oak	0,915	0,0330946	0,489008
Olive oil	0,913	0,0330222	0,481569
Beech	0,854	0,0308883	0,450449
Dry elm, Dry ash	0,800	0,0289352	0,421966
Dry wainscot	0,747	0,0270182	0,394011
Dry yellow fir	0,657	0,0237630	0,346539
Man's body	1,111	0,0413100	0,600354
Cedar	0,613	0,0221715	0,323332
Dry white deal	0,569	0,0205801	0,300123
Cork	0,240	0,0186805	0,126590
Air	0,0012	0,0000434	0,000633

Note. If you take away the points from the numbers, in the ſecond column, and reckon them to be whole numbers, they will ſhew how many avoirdupoiſe ounces are contained in a cubic ſoot of each of the above bodies in the table.

CASE I. The dimenſions, or ſolidity of any body being given to find its weight.

RULE. Multiply the cubic inches contained in that body, by the tabular weight correſponding, and it will give the weight in pounds avoirdupoiſe, or ounces troy.

EXAMPLE I. What is the weight of a piece of oak, of a rectangular form, whoſe ſolidity is 11880 cubic inches?

```
      ,0330946
         11880
      --------
      26475680
      330946
     330946
     ---------
112)393,1638480(3,5104 cwt. the anſwer.
```

E. 2. There is a bar of iron, in length 156 inches, and 1 inch ſquare; I deſire to know how many pounds avoirdupoiſe it doth weigh?

```
     ,2808159
          156
    ---------
     16848954
   14040795
   2808159
   ---------
   43,8072804 lb. the anſwer.
```

E. 3. What is the weight of a piece of fir, whoſe girt is 20 inches and length 40 feet?

Firſt, $20 \div 5 = 4$, alſo $4 \times 4 = 16$ ſquare of $\frac{1}{4}$ girt. And 40 feet $=480$ inches. Then $480 \times 2 \times 16 = 960 \times 16 = 15360$ cubic inches (per rule 2. Sect. 73.)

```
Therefore  ,0237630
              15360
           --------
           14257800
           712890
          1188150
          237630
          ---------               c. qr. lb.
Anſwer 364,9997800 lbs. =          3  1   1
```

 E. 4.

E. 4. What is the weight of an iron ſhot, of 8 inches diameter?
Firſt, 8×8×8×,5236=268,0832 Solid inches, (per Prob. 10, Sect. 68.)

```
Therefore 268,0832
           ,2580647
           18765824
          10723328
         16084992
        21446656O
       13404160
       5361664
       69,18281058304 lb. the anſwer.
```

E. 5. What is the diameter of an iron ſhot, weighing 69,18281058304*lb.* avoirdupoiſe?

First, ,2580647)69,18281058304(268,0832 Solid inches.

Then ,5236)268,0832(=512 Cube of the diameter.

Therefore $\sqrt[3]{512}=8$ The diameter ſought.

E. 6. What is the weight of an iron bomb ſhell, of 3 inches thick, the greateſt diameter being 14 inches?

Firſt, 14—6=8, Diameter of the concavity.

Then 14×14×14×,5236=1436,7584 Content of the whole.
And 8× 8× 8×,5256= 268,0832 Ditto of the concavity.
Solidity of the ſhell = 1168,6752 Inches.

Therefore 1168,6752×,2580647=301,59381488544 *lb.* the weight required.

E. 7. In the walls of Balbeck, in Turkey, there are three ſtones laid end to end, now in ſight, that meaſure in length 61 yards; one of which, in particular, is 63 feet long, 12 feet thick, and 4 yards over: now, if this ſtone was marble, what power would ballance it, ſo as to prepare it for moving?

Firſt, 63×12×12=9072 Solid feet.

Then 9072×1728=15676416 Cubic inches.

```
Therefore 15676416
           ,0977286
           94058496
          125411328
         31352832
        109734912
       141087744
2240)1532034,1886976(683,9438=638 tons, 18 cwt. 97 lb. the anſ.
```

Case 2,

CASE. 2. The weight of any body being given to find the solidity, and the specific gravity thereof.

RULE. Divide the given weight by the tabular weight, corresponding to the name of the same kind, and the quotient will be the solidity in cubic inches.

EXAMPLE 1. How many solid feet are there in a block of marble that weighs 8 tons; and what will it come to at 5 shillings per foot solid?

First, 8 tons = 17920 *lb.*

Then ,0977286) 17920,00000000 (183328,45 inches.

Now 1728)183328,45(106,09 cubic feet (nearly) at 5*s.* or ,25*l.*

```
      106,09
         ,25
      ------
       53045
      21218
      ------
      26,5225 = 26l. 10s. 5¼d. answer.
```

E. 2. In the Spectators club of fat people, it is said that each person weighed no less than 4 cwt. how many solid inches was there in one of their bodies?

First, 4 cwt. = 448 *lb.*

Then ,041310(448,000000(10844,8 solid inches the answer.

E. 3. Suppose that a man of war, with all its ordnance, rigging, and appointment, draws so much water as to displace 1300 tons of sea water, London beer measure: the weight of the vessel is required?

First, 1300 × 4 = 5200 hhds. and a hhd = 282 × 54 = 15228 cubic inches.

Therefore 15228 × 5200 = 79185600 cubic inches displaced.

```
Then     79185600
          ,037253
         --------
          2375568
         3959280
        1583712
       5542992
      2375568
      -------                                  T.  cwt. lb.
2240)2949901,156800(1316,92015 tons = 1316  18  17 the weight
                                                    required.
```

E. 4. Hiero, king of Scilly, ordered his jeweller to make him a crown, containing 63 ounces of gold; the workmen thought by substituting part silver therein, to have a proper perquisite, which taking air, Archimedes was appointed to examine it, who, on putting it into a vessel of water, found it raised the fluid, or that itself contained 8,2245 cubic inches of metal, and having discovered that the cubic inch of gold

gold more critically weighed 10,36 ounces, and that of ſilver but 5,85 ounces, he, by calculation, found what part of his Majeſty's gold had been changed, and you are deſired to repeat the proceſs?

Firſt, 10,36) 63,00 (6,08108, had it been all gold.

Alſo, 5,85) 63,00 (10,76923, if all ſilver.

Then by Sect. 28.

$$\text{Mean rate } 8,2245 - \begin{cases} 6,08108 = 2,54473 \\ 10,76903 = 2,14342 \end{cases}$$

Sum 4,68815

4,68815) 2,54473 (,5428, oz. part gold
4,68815) 2,14342 (,4572, oz. part ſilver

$$\text{Then} \begin{Bmatrix} ,5428 \\ ,4572 \end{Bmatrix} \times 63 = \begin{cases} 34,1964 = 34 \quad 3 \quad 22,272, \text{ G} \\ 28,8036 = 28 \quad 16 \quad 1,728, \text{ S} \end{cases}$$

	oz.	dwts.	grs.
Proof	63	00	00,000

Case 3. The weight and magnitude being given, to find the ſpecific gravity.

RULE. Divide the weight in ounces, by the ſolidity in cubic feet, the quotient will be the ſpecific gravity.

Example 1. I have a piece of marble that contains 4 ſolid feet; and weighs 675 *lb.* what is its ſpecific gravity?

Firſt, $675 \times 16 = 10800$ ounces.

Then $10800 \div 4 = 2700$ the ſpecific gravity.

E. 2. I have a piece of timber that contains 6 feet, and weighs 300 lb. what wood it it?

Firſt, $400 \times 16 = 4800$ ounces.

Then $48000 \div 6 = 800$ the ſpecific gravity.

Now, in the table of ſpecific gravity, againſt 800 you will find dry aſh or elm, the wood required.

N. B. All bodies of what nature or kind ſoever, being weighed in open air, and balanced by thoſe whoſe ſpecific gravity is greateſt; thoſe bodies whoſe ſpecific gravity is leaſt, will weigh the heavieſt in vacuo.

Thus, if a piece of lead, at the end of a nice balance, and a piece of cork at the other end, are in equilibrio in the air, and thus placed under the receiver of an air-pump, as ſoon as the air begins to be exhauſted, the equilibrium will begin to be deſtroyed, till at laſt when all the air is taken away, the cork will deſcend and ſhew itſelf really heavier than the lead.—(And for the ſame reaſon, a pound of feathers is heavier than a pound of lead, which may ſeem a paradox to ſome;) but the reaſon is very evident from the laws of Hydroſtatics; for both bodies being weighed

weighed in air, each would lose the weight of an equal bulk of air, consequently the feathers will lose a greater weight than the lead, because it is of greater bulk, therefore when the air is taken away from both, the weight that is restored to the feathers, being the greatest, will cause it to preponderate or weigh down the lead in vacuo.

CASE 4. The solidity of any piece of timber being given to find how far it will sink.

RULE. Divide the specific gravity of the timber by the specific gravity of the water: multiply this sum by the depth of the timber, and it will give the inches under water.

EXAMPLE 1. How many inches will a cubic foot of elm sink in common water?

First, 1,000) ,8000 (,800
12

9,600 inches the answer.

E. 2. How many inches will a cubit foot of deal sink in common water?

First, 1,000) ,6570 (,657
12

7,884 inches under water.

CASE 5. The solidity of any timber being given, to find how much it will carry.

RULE. Subtract the specific gravity of the timber from that of the water, the remainder is the number of ounces that one solid foot will carry.

EXAMPLE 1. How much weight is just necessary to sink a cubic foot of deal in common water?

1,000
,657

16) ,343 (21,437 *lb.* the answer.

E. 2. How much will a raft, made of 12 pieces of yellow deal, carry in sea water, if each piece be a foot square, and 20 feet long?

First, $1033 - 657 = 376$, and $12 \times 20 = 240$
Then $240 \times 376 = ,90240$
$\therefore 9024 \div 16 = 5640$ *lb.* the answer.

To make a deceitful balance, or pair of scales, whose beam will hang in equilibrio without the scales, or with the empty scales; and yet shall also be in equilibrio when unequal weights are placed in

in the scales; so as to cheat in any proportion intended, in making the balance at first. See Plate 1. Fig. 32.

To the beam A B = 23 inches long, whose arm C B, of 11 inches in length, keeps in equilibrio about the point C, the arm A C; of 12 inches in length, by being made so much thicker, or having so much more matter, as may make amends for its being shorter; hang the scales D, E, in such a manner, that D, which weighs one part in twelve less than E, shall hang at the longest end of the beam, and they will keep each other in equilibrio; then placing 12 pounds weight at G, in the scale E, it will keep in equilibrio no more than 11 pounds of F, the commodity to be sold, if placed in the scale D; because then, F will be to G in a reciprocal proportion of B C to A C.

Now, though such a balance may be so nicely made as to deceive the eye, the cheat is immediately discovered by changing the weights, and the commodity F, from one scale to another; for then, the owner of the balance, must either confess the fraud, or add to the commodity he sells, &c. not only what was wanting, but also as much as he intended to cheat him of; and a fraction of the added weight, proportionable to the inequality of the arms of the balance, that is, in this case, the buyer, instead of the 11 pounds offered him for 12, his due, will have, by changing the scales, $13\frac{1}{11}$ pounds. For whereas, in the first position of the balance $F = 11 \times 12 = A C$ was equal to $G = 12 \times 11 = B C$ when G or 12 pounds is placed in the scale D, then 12×12 will be equal to no less than $C B = 11 \times 13\frac{1}{11} = G$. Or,

As the arm C B, 11 inches long, is to the arm C A, 12 inches long, so is F, or the weight 12, placed in the scale D: to G $13\frac{1}{11}$, or the weight of the commodity keeping the weight in equilibrio.

And therefore, as this analogy gives a reciprocal proportion between the weights and their velocities, the momenta will be equal; which, with contrary directions, destroy one another.

N. B. In all these cases, we suppose the weight to hang freely from those ends of the balance to which they are fastened.

A TABLE,

A TABLE by which the quantity and weight of water in a cylindrical pipe of any given diameter of bore, and perpendicular height may be found: and conseqently, the power may be known that will be sufficient to raise the water to the top of the pipe, in any pump, or any or any other hydraulic machine.

Diameter of the Cylindric Bore, 1 Inch.

Feet high.	Quantity of water in cubic inches.	Weight of water in troy ounces.	In avoirdupoise ounces.
1	9,4247781	4,971234	5,4541539
2	18,8495562	9,942468	10,9083078
3	28,2743343	14,913702	16,3624617
4	37,6991124	19,884936	21,8166156
5	47,1238905	24,856170	27,2707695
6	56,5486686	29,827404	32,7249234
7	65,9734467	34,798638	38,1790773
8	75,3982248	39,769872	43,6332312
9	84,8230029	44,741106	49,0873851

For tens of feet high, remove the decimal points one place forward; for hundreds of feet, two places, for thousands, three places, and so on. Then multiply the sums by the square of the diameter of the given bore, and the products will be the answer.

EXAMPLE. The quantity and weight of water in a cylindric pipe, 85 feet high, and 10 inches diameter.

The Square of 10 = 100.

Feet high.	Cubic inches.	Troy ounces.	Avoirdupoise ounces.
80	753,982248	397,698720	436,332312
5	47,123890	24,856170	27,270769
85	801,106138	422,554890	463,603081
	×100	×100	×100
Answer	80110,613800	42255,489000	46360,308100

Which number 80110,6 of cubic inches being divided by 231, the cubic inches in a wine gallon, gives 342,6 for the number of gallons: and the respective weights 42255,489 and 46360,3, being divided, the former by 12, and the latter by 16, give 3521,29 for the number of troy pounds, and 2897,5 for the number of avoirdupoise pounds, that the water in the pipe weighs. So much power would be required to balance or support the water in the pipe, and as much more to work the engine, as the friction thereof amounts to.

In all pumps, the pressure of the column of water, or its weight felt by the working power, when raised to any given height above the surface of the well, is in proportion to the height of the column, considered throughout, as if it were equal in diameter to that part of the bore, in which the piston or bucket works.

 The

The advantage or power gained by the handle of the pump, is the ſame as in the common leaver; that is, as great as the length from the axes of the handle to its end, where the power is applied, exceeds the length of the other part of the handle, from the axes on which it turns, to the pump rod, wherein it is fixed, for lifting the piſton and water.

In the making of pumps, the diameter of the bore, where the bucket works, ſhould be proportioned to the height which the pump raiſes water above the ſurface of the well, as that a man of ordinary ſtrength might work all pumps equally eaſy, let their heights be what they will. The annexed table ſhews how this may be done, and what quantities of water may be raiſed in a minute by one man, ſuppoſing the handle of the pump to be a leaver, increaſing the power five times.

The Uſe of the Table.

Find the given height of the pump, in the firſt column of the table; and againſt it in the ſecond column, you have the diameter, which the bore muſt be of, in inches, and hundredth parts of an inch; and in the third column, you have the quantity of water in gallons and pints, that a man of common ſtrength can raiſe to that height in a minute.

With reſpect to the power required to work the pump, or the quantity of water diſcharged thereby, it matters not what the diameter of the bore be in any other part, than that wherein the piſton or bucket works.

A TABLE FOR PUMP-MAKERS.

Height of the pump above the ſurface of the well	Diameter of the bore, where the piſton works.	Water diſcharged in a minute in gallons and pints.	
Feet.	*Inches.*	*Gallons.*	*Pints.*
10	6,93	81	6
15	5,65	54	4
20	4,90	40	7
25	4,38	32	6
30	4,00	27	2
35	3,70	23	3
40	3,47	20	4
45	3,26	18	1
50	3,10	16	3
55	2,95	14	7
60	2,83	13	5
65	2,71	12	4
70	2,62	11	5
75	2,53	10	7
80	2,44	10	2

Before I quit this ſubject, I ſhall obſerve one thing more to my reader, concerning the preſſure of fluids, which is this:

Let a body be ever ſo heavy, it may be made to ſwim in liquids, by knowing the ſpecific gravity.

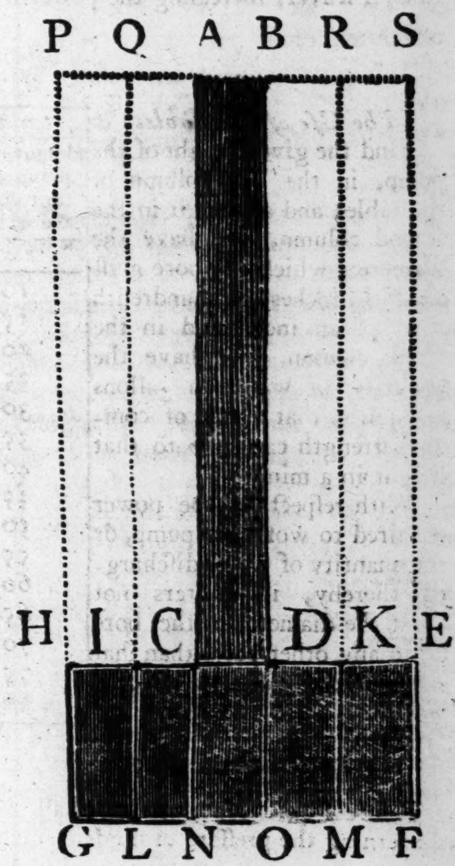
Thus,

Thus, becaufe the fpecific gravity of gold is to that of water, as 19 to 1; therefore if you hold a guinea to the bottom of a tube of equal diameter (fo as no water can get in) by means of a ftring; then put the tube down in the water, above 19 times the thicknefs of the guinea in depth, and letting the ftring go, the guinea will not fink, but ride fuftained by the preffure of the fub-adjacent water, which now is ftronger than the force of gravity in the guinea; and thus you make any body fwim, let it be ever fo large and weighty.

SCHOLIUM.

The writers on Hydroftatics demonftrate, that the preffure of liquids on the bottom and fides of veffels, is always proportional to the height thereof, and every way equal at the fame depth.

To illuftrate this, let G E. in the annexed figure, be a veffel, from whofe upper part H E proceeds a tall tube, A B C D, communicating therewith. Let this tube and veffel be filled with water, then will the preffure of the water on the bottom G F be as great, and every way the fame, as it would be, were the veffel itfelf as high as the tube, and filledwith water to the level of P S; that is the column of water, A N O B in the prefent cafe has the fame effect on the bottom of the veffel G F, as the column of water P G F S would have.



This is no fmall paradox, but is, notwithftanding that very eafy to to conceive; for fince fluids act in every direction, or prefs every way equally; and action and re-action is equal and contrary; it muft follow, that the parts of the bottom L N and G L (being equal to N O) will fuftain the fame preffure as N O, or as they would do, were the columns

of water continued to the height of P Q A. For in the line C N, the force of the column of water A O, is exerted on each side equally, and has the same effect at I L as at D O, and therefore the lateral pressure being equal, the perpendicular pressures also on L N and N O will be equal.

Or thus: If the pressure of the part of I L were less than on the part D O, the fluid in the column C O would, by reason of its greater gravity, have a motion towards the part I L, and the surface A B would descend: but since there is a perfect quiescence of all the parts of the fluid, aud that in the column C O is as much at rest as that in the column C L, it is evident their pressures and effects are every way the same, and consequently that the column C L presses as much on the part L N, as the column C O does on the part N O. What is thus proved of the column I N, is to be proved of all the rest, H L, D M, and K F, which makes the proposition manifest.

This property of fluids is not only in itself very curious, but of great importance in many affairs in life. They who would see more of this wonderful property, may consult Dr. Gravesande's Elements.

The weight of the whole Atmosphere.

On a square inch, it is 15 pounds; on a square foot, 2160; on a square yard, 19,440; on a square mile, 60,217,344,000; and on the whole surface of the earth, and sea together,——

12,014,118,565,447,680,000 pounds.

The surface of the body of a middle sized man, is about 14 square feet; and as the weight or pressure of the air is equal to 2160 pounds on every square foot, on or near the earth's surface, and as the pressure of the air is equal in all manner of directions, its pressure on the whole body of a middle sized man is equal to 30,240 pounds, or $13\frac{1}{2}$ tons. But because the spring of the internal air is of equal force with the pressure of the external, the pressure is not felt.

The cause of the ebbing and flowing of the sea, at the same time, on opposite sides of the globe.

The reason why the tides rise on the side of the earth, which is at any time turned towards the moon, is plain to every one; because her attraction must occasion a swelling of the waters towards her on that side: but the cause of as great swell, at the same time, on the opposite side of the earth, which is then turned away from the moon, has been very hard to account for; because the rising of the tide there is in a direction quite contrary to the attraction of the moon. But this difficulty is immediately removed, when we consider, that all bodies moving in circles, have a centrifugal force, or constant tendency to fly off from the centers of the circles they describe; and this centrifugal force is always in proportion to the distance of the body from the center of its orbit, and the velocity with which it moves therein.

When

When the body is large, the side of it which is farthest from the center of its orbit will have a greater degree of centrifugal force than the center of the body has; and the side of it which is nearest the center of its orbit, will have a less degree of centrifugal force than its center has.

As the moon goes round the earth every month in her orbit, the earth also goes round an orbit every month, which is as much less than the moon's orbit, as the quantity of matter in the moon is less than the quantity of matter in the earth, which is 40 times. For, by the laws of nature, when a small body moves round a great one, in free and open space, both these bodies must move round the common center of gravity between them.

The moon's mean distance from the earth's center is 240,000 miles; divide therefore this distance by 40, the difference between the quantity of matter in the earth and moon, and the quotient will be 6000 miles, which is the distance of the common center of gravity between the earth and moon, from the center of the earth.

Now, as the earth and moon move round the common center of gravity between them, once every month; it is plain, that whilst the moon moves round her orbit, at 240,000 miles from the earth's center, the center of the earth describes a circle of 6000 miles radius, round the center of gravity between the earth and the moon; the moon's attraction balancing the centrifugal force of the earth at its center.

The diameter of the earth is 8000 miles (nearly) and consequently its semi-diameter is 4000: so that the side of the earth which is at any time turned towards the moon, is 4000 miles nearer the common center of gravity between the earth and moon, than the earth's center is; and the side of the earth, which is then farthest from the moon, is 4000 miles farther from the center of gravity between the earth and moon, than the earth's center is at that time.

Therefore, the radius of the circle described by the parts of the earth which come about towards the moon, by the earth's diurnal motion, is 2000 miles; the radius of the circle described by the earth's center is 6000; and the radius of the circle described by those parts of the earth which, in revolving on its axis, are farthest from the moon, is 10,000 miles.

The centrifugal forces of the different parts of the earth being directly as their distances from the above-mentioned common center of gravity, round which both the earth and moon move, these forces may be expressed by 2000 for the side of the earth nearest the moon, by 6000 for the earth's center, and by 10,000 for the side of the earth which is farthest from the moon.

But the moon's attraction is greatest on the side of the earth next her, where the centrifugal force or tendency to fly off from the common center of gravity (and consequently, from the moon) is least; and therefore, the tides must rise on the side of the earth which is nearest the moon, by the excess of the moon's attraction.

As her attraction balances the centrifugal force at the earth's center, it is plain that the centrifugal force of the side of the earth which is farthest from the moon is greater than her attraction; and therefore, the tides will rise as high upon that side from the moon, by the excess of the centrifugal force, as they rise on the side next her by the excess of her attraction. And as the earth is in constant motion on its axis, so as that any given meridian revolves from the moon to the moon again in 24 hours $50\frac{1}{2}$ minutes, each place will come to the two eminences of water, under and opposite to the moon, in 24 hours $50\frac{1}{2}$ minutes, or have two tides of flood and two of ebb in that time. For, as much as the waters rise above the common level of the surface of the sea, under and opposite to the moon, so much they must fall below that level half way between the highest places, or at 90 degrees from them.

On these principles, it is equally easy to account for the rising of the tides, at the same time, on both sides of the earth; and this rising is made evident to sight in Mr. Ferguson's Lecture on the Central Forces.

CHRONOLOGY.

PART V.

SECTION LXXVIII.

CHRONOLOGY is the art of estimating and comparing together the times when any memorable transaction hath happened.

It also takes a view of the various tracts, calendars, and methods of computing time, practised by different nations; compares them together, and settles such order among them, that the exact time in which any remarkable event happened may be certainly known.

RULES for finding the corresponding years of the Julian Period, with the years of the world, and years before and since the birth of Christ; supposing that the creation of the world was in the 706th year of the Julian Period; and that the birth of Christ was (according to the vulgar æra thereof) in the 4713th year of the Julian Period.

From any given year of the Julian Period subtract 706, and the remainder will be the year of the world's age.

If the number of the given year of the Julian Period be less than 4713, subtract it from 4713; and the remainder will be the number of years before the year of Christ's birth.

If the given year of the Julian Period is greater than 3967, subtract 3967 from it; and the remainder will be the number of years after the famous æra of Nabonassar.

Sub-

Subtract 1 from any given year of the Julian period, and divide the remainder by 4; if nothing remains, the given year is a leap-year; but if 1, 2, or 3 remains, it is the first, second, or third year after leap-year, in the old stile.

If any year before the year of Christ's birth be given, subtract its number from 4713, and the remainder will be the year of the Julian period: and if you subtract the said year from 4007, the remainder will be the year of the world's age.

If any year after the year of Christ's birth be given, add 4713 to it, and the sum will be the year of the Julian period; or if you add 4007 to it, the sum will be the years of the world's age.

If any year of the world's age is given, add 706 to it, and the sum will be the year of the Julian period. If the given year of the world be less than 4007, subtract it from 4007, and the remainder will be the number of years before the year of Christ's birth. But, if the given year of the world be more than 4007, subtract 4007 from it; and the remainder will be the number of years after the year of Christ's birth.

A Table of remarkable Æras and Events.

		Julian Period.	World's Age.	Before Christ.
1	The creation of the world - - -	706		4007
2	The flood - - - - - -	2362	1656	2351
3	The Assyrian monarchy founded by Nimrod	2537	1831	2176
4	The birth of Abraham - - -	2714	2008	1999
5	The destruction of Sodom and Gomorrah -	2816	2110	1897
6	The kingdom of Athens founded by Cecrops	3157	2451	1556
7	Moses receives the ten commandments from God	3222	2516	1491
8	The Israelites enter Canaan - - -	3262	2556	1451
9	The destruction of Troy - - -	3529	2823	1184
10	The beginning of King David's reign -	3650	2944	1063
11	The founding of Solomon's temple - -	3701	2995	1012
12	The Argonautic expedition - - -	3776	3070	937
13	Lycurgus formed his excellent laws -	3829	3103	884
14	Arbaces, first king of the Medes - -	3838	3132	875
15	Manducus, the second - - -	3865	3159	848
16	Sosarmus, the third - - - -	3915	3029	798
17	The beginning of the Greek Olimpiades -	3938	3232	775
18	Artica, the fourth king of the Medes -	3945	3239	768
19	The Catonian epocha of the building of Rome	3961	3255	752
20	The æra of Nabonasiar - - -	3967	3261	746
21	The destruction of Samaria by Salmanesser -	3992	3286	721
22	The first eclipse of the moon on record -	3993	3287	720
23	Cardicea, the fifth king of the Medes -	3996	3290	717
24	Phraortes, the sixth - - - -	4058	3352	655
25	Cyaxares, the seventh - - -	4080	3374	633
26	The first Babylonish captivity by Nebuchadnezzar	4107	3401	606
27	The long war ended between the Medes and Lydians - - - - -	4111	3405	602

		Julian Period.	Word's Age.	Before Christ.
28	The 2d Babylonish captivity and birth of Cyrus	41 4	3408	599
29	The destruction of Solomon's temple –	4124	3419	588
30	Nebuchadnezzar struck with madness –	4144	3438	569
31	Daniels vision of the four monarchies –	4158	3452	555
32	Cyrus begins to reign – – –	4177	3471	536
33	The Battle of Marathon – – –	4223	3517	490
34	Artaxerxes Longimanus begins to reign –	4249	3543	464
35	The beginning of Daniels seventy weeks of years	4256	3350	457
36	The beginning of the Peloponnesian war –	4282	3576	431
37	Alexander's victory at Arbela – –	4383	3677	330
38	His death – – – – –	4390	3684	323
39	The captivity of 100,000 Jews by King Ptolomy	4393	3687	320
40	Colossus of Rhodes thrown down by an earthquake	4491	3875	222
41	Antiochus defeated by Ptolomy Philopater –	4496	3700	217
42	The famous Archimides murdered at Syracuse	4506	3800	207
43	Jason butchered the inhabitants of Jerusalem	4543	3837	170
44	Corinth taken and plundered by the Consul Mummius – – – – –	4567	3861	146
45	Julius Cæsar invaded Britain – –	4659	3953	54
46	He corrects the calendar – – –	4667	3961	46
47	Is killed in the senate-house – – –	4671	3965	42
48	Herod made king of Judea – – –	4671	3967	40
49	The battle at Actium – – –	4683	3977	30
50	Agrippa builds the Pantheon at Rome –	4668	3982	25
51	The true æra of Christ's birth – –	4709	4003	4
52	The death of Herod – – – –	4710	4004	3
				Since Christ.
53	The Dionysian, or vulgar æra of Christ's birth	4713	4007	
54	The true year of his crucifixion – –	4746	4040	33
55	The destruction of Jerusalem – –	4783	4077	70
56	Adrian built the long wall in Britain –	4833	4127	120
57	Constantius defeated the Picts in Britain –	5019	4313	306
58	The Council of Nice – – – –	5038	4332	725
59	The death of Constantine the Great –	5050	4344	337
60	The Saxons invited to Britain – –	5158	4452	445
61	The Arabian Hegira, or flight of Mahomed	5335	4629	662
62	The death of Mahomed – – –	5343	4637	630
63	The Persian Yesdegird – – –	5344	4638	631
64	The art of printing discovered – –	6153	5447	1440
65	The reformation begun by Martin Luther –	6230	5524	1517
66	Oliver Cromwel died – – – –	6371	5665	1658
67	Sir Isaac Newton born at Woolstrope, in Lincolnshire, December 25 – – –	6355	5649	1642
—	Made President of the Royal Society –	6416	5700	1703
—	Died, March 20 – – – –	6440	5734	1727

		Years of Christ
68	The Scotch rebels defeated at Cullodon, April 16 - -	1746
69	Westminster bridge finished, cost £389,500 - -	1750
70	The Style and Calendar altered, Sept. 2 - -	1752
71	A large Comet appeared, foretold by Dr. Halley -	1758
72	Otaheite in the South Seas discovered - - -	1765
73	Longitude found by Harrison's time piece, £18,750 given him	1765
74	Blackfriars bridge finished, cost £152,840 - -	1770
75	Georgium Sidus discovered by Herschel, - -	1781
76	King of Sweden shot at a masquerade by Ankerstrom -	1792
77	King and Queen of France beheaded - - -	1793
78	French fleet defeated in the channel by Earl Howe -	1794
79	Spanish fleet defeated by Lord St. Vincent - -	1797
80	Dutch fleet defeated by Viscount Duncan - -	1797
81	French fleet defeated near Egypt by Lord Nelson -	1798

In the foregoing table, the years before and since Christ are reckoned exclusive from the year of his birth.

By the following tables (pages 418, 419, and 420,) the day of the month answering to any given day of the week, and the day of the week answering to any given day of the month, may be found, in the old stile, within the limits of 5500 years, before the year of Christ's birth, and 5500 years after it; and in the new stile, from A. D. 1752, to 1837, inclusive.

1. For any given year before Christ, look for the complete hundreds of that year (when its number amounts to hundreds) at the head of the table on page 418, and for the years below or less than an hundred, to make up the number of the given year, at the left hand; and where the columns meet, you have the dominical letter for the given year.

EXAMPLE. What was the dominical letter for the 584th year before Christ's birth?

Under 500, at the head of the table, and against 84, at the left hand, I find FE, the dominical letter required, and shews the said year to have been leap-year, as every leap-year has two dominical letters; the first of which serves for January and February, and the last for the rest of the year.

EXAMPLE 2. For the year 1741, I demand the dominical letter?

Under 1700 at the head of the table, and downwards thence in that column against 41, at the left hand, I find D; the dominical letter required.

Note. These two tables shew the dominical letter for the old stile, and the table on page 420 shews it for the new stile.

A TABLE, shewing the Dominical Letters for Years before the Year of Christ's Birth.

Old Stile.				*Hundreds of Years.*						
Years less than an hundred.				0	100	200	300	400	500	600
				700	800	900	1000	1100	1200	1300
				1400	1500	1600	1700	1800	1900	2000
				2100	2200	2300	2400	2500	2600	2700
				2800	2900	3000	3100	3200	3300	3400
				3500	3600	3700	3800	3900	4000	4100
				4200	4300	4400	4500	4600	4700	4800
				4900	5000	5100	5200	5300	5400	5500
0	28	56	84	D C	C B	B A	A G	G F	F E	E D
1	29	57	85	E	D	C	B	A	G	F
2	30	58	86	F	E	D	C	B	A	G
3	31	59	87	G	F	E	D	C	B	A
4	32	60	88	B A	A G	G F	F E	E D	D C	C B
5	33	61	89	C	B	A	G	F	E	D
6	34	62	90	D	C	B	A	G	F	E
7	35	63	91	E	D	C	B	A	G	F
8	36	64	92	G F	F E	E D	D C	C B	B A	A G
9	37	65	93	A	G	F	E	D	C	B
10	38	66	94	B	A	G	F	E	D	C
11	39	67	95	C	B	A	G	F	E	D
12	40	68	96	E D	D C	C B	B A	A G	G F	F E
13	41	69	97	F	E	D	C	B	A	G
14	42	70	98	G	F	E	D	C	B	A
15	43	71	99	A	G	F	E	D	C	B
16	44	72		C B	B A	A G	G F	F E	E D	D C
17	45	73		D	C	B	A	G	F	E
18	46	74		E	D	C	B	A	G	F
19	47	75		F	E	D	C	B	A	G
20	48	76		A G	G F	F E	E D	D C	C B	B A
21	49	77		B	A	G	F	E	D	C
22	50	78		C	B	A	G	F	E	D
23	51	79		D	C	B	A	G	F	E
24	52	80		F E	E D	D C	C B	B A	A G	G F
25	53	81		G	F	E	D	C	B	A
26	54	82		A	G	F	E	D	C	B
27	55	83		B	A	G	F	E	D	C

A TABLE

A TABLE, shewing the Dominical Letters for Years after the Year of Christ's Birth.

Old Stile.				*Hundreds of Years.*						
Years less than an hundred.				0 700 1400 2100 2800 3500 4200 4900	100 800 1500 2200 2900 3600 4300 5000	200 900 1600 2300 3000 3700 4400 5100	300 1000 1700 2400 3100 3800 4500 5200	400 1100 1800 2500 3200 3900 4600 5300	500 1200 1900 2600 3300 4000 4700 5400	600 1300 2000 2700 3400 4100 4800 5500
0	28	56	84	D C	E D	F E	G F	A G	B A	C B
1	29	57	85	B	C	D	E	F	G	A
2	30	58	86	A	B	C	D	E	F	G
3	31	59	87	G	A	B	C	D	E	F
4	32	60	88	F E	G F	A G	B A	C B	D C	E D
5	33	61	89	D	E	F	G	A	B	C
6	34	62	90	C	D	E	F	G	A	B
7	35	63	91	B	C	D	E	F	G	A
8	36	64	92	A G	B A	C B	D C	E D	F E	G F
9	37	65	93	F	G	A	B	C	D	E
10	38	66	94	E	F	G	A	B	C	D
11	39	67	95	D	E	F	G	A	B	C
12	40	68	96	C B	D C	E D	F E	G F	A G	B A
13	41	69	97	A	B	C	D	E	F	G
14	42	70	98	G	A	B	C	D	E	F
15	43	71	99	F	G	A	B	C	D	E
16	44	72		E D	F E	G F	A G	B A	C B	D C
17	45	73		C	D	E	F	G	A	B
18	46	74		B	C	D	E	F	G	A
19	47	75		A	B	C	D	E	F	G
20	48	76		G F	A G	B A	C B	D C	E D	F E
21	49	77		E	F	G	A	B	C	D
22	50	78		D	E	F	G	A	B	C
23	51	79		C	D	E	F	G	A	B
24	52	80		B A	C B	D C	E D	F E	G F	A G
25	53	81		G	A	B	C	D	E	F
26	54	82		F	G	A	B	C	D	E
27	55	83		E	F	G	A	B	C	D

Dominical Letters for the New Stile.

1752	B A	1795	D
1753	G	1796	C B
1754	F	1797	A
1755	E	1798	G
1756	D C	1799	F
1757	B	1800	E
1758	A	1801	D
1759	G	1802	C
1760	F E	1803	B
1761	D	1804	A G
1762	C	1805	F
1763	B	1806	E
1764	A G	1807	D
1765	F	1808	C B
1766	E	1809	A
1767	D	1810	G
1768	C B	1811	F
1769	A	1812	E D
1770	G	1813	C
1771	F	1814	B
1772	E D	1815	A
1773	C	1816	G F
1774	B	1817	E
1775	A	1818	D
1776	G F	1819	C
1777	E	1820	B A
1778	D	1821	G
1779	C	1822	F
1780	B A	1823	E
1781	G	1824	D C
1782	F	1825	B
1783	E	1826	A
1784	D C	1827	G
1785	B	1828	F E
1786	A	1829	D
1787	G	1830	C
1788	F E	1831	B
1789	D	1832	A G
1790	C	1833	F
1791	B	1834	E
1792	A G	1835	D
1793	F	1836	C B
1794	E	1837	A

A TABLE, shewing the days of the months for ever, both in the old and new stile, by the dominical letters.

Months.	A	B	C	D	E	F	G
January 31 *October* 31	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31	—	—	—	—
February 28 *March* 31 *November* 30	—	—	—	1	2	3	4
	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30	31	—
April 30 *July* 31	—	—	—	—	—	—	1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31	—	—	—	—	—
August 31	—	—	1	2	3	4	5
	6	7	8	9	10	11	12
	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31	—	—
September 30 *December* 31	—	—	—	—	—	1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
	31	—	—	—	—	—	—
May 31	—	1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31	—	—	—
June 30	—	—	—	—	1	2	3
	4	5	6	7	8	9	10
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	

The

The dominical letter for the New Stile may be found by the following:

RULE. To the given year add its fourth part, omitting fractions; divide that sum by 7; the remainder, taken from 8, leaves the index of the letter in the common years reckoning

1	2	3	4	5	6	7
A	B	C	D	E	F	G

But in leap-years, this letter and its succeeding one are the dominical letters.

E. 1. For the year 1801, I demand the dominical letter?

$$\begin{array}{l} 4)1801 \\ \quad 450 = \tfrac{1}{4} \\ \hline 7)2251 \\ \hline \quad 321 - 4 \text{ remains} = \text{D the dom. letter.} \end{array}$$

Note. The time in which the sun performes his annual revolution, is not exactly 365 days, 6 hours, but 365 days, 5 hours, 48 minutes, and 49 seconds, the civil year must, therefore, have exceeded the solar year by 11 minutes, and 11 seconds; which in the space of about 130 years, amounted to a whole day, since the correction of the calender; which was the reason of leap-year being omitted this present year 1800.

And the above rule will hold good, for finding the dominical letter untill the year 1900, from this time, but till the year 1804, you must work as in the above example, viz. omit subtracting the remainder as the above rule directs as the remainder itself is the index of the letter.

Note. The year 1900 will not be a leap year, on the above account, but there will then be 13 days to be added, according to the following table.

E. 2. What will be the dominical letter for the year 1814?

$$\begin{array}{lcl} 4)1814 & & \\ \quad 453 & = & \tfrac{1}{4} \\ \hline 7)2267 & & 8 \\ \hline \quad 323 & - & 6 \\ \hline & & 2 = \text{B, the dominical letter.} \end{array}$$

To find whether any given year be leap-year.

RULE. Divide the given year by 4; if nothing remains, it is leap-year; but if 1, 2, or 3 remains, it is so many years after.

E. 1. I desire to know whether 1801 will be a leap-year or not?

$$\begin{array}{l} 4)1801 \\ \hline \quad 450 - 1 \text{ remains} \therefore 1 + 4 = 5\text{th year after.} \end{array}$$

Note. Until the year of our Lord 1804, you must add 4 to the remainder, as above, and after that period, proceed as the rule directs.

E. 2. Will the year 1805 be leap-year or not?

First, $1805 \div 4 = 451$, and 1 remains $\therefore$ it is the first year after leap-year.

Pope

Pope Gregory XIII. finding the Julian account erroneous, reformed it after the manner of this table. The Julian year consists of 365 days, six hours; which is 10 minutes, 34 seconds more than the real tropical year, consisting of 365 days, 5 hours, 48 minutes, 49 seconds. Now these 11 minutes 11 seconds, from the first year of the Julian account (which took date before the birth of Christ) to the year 1600, amounted to 12 days; and so many days the Julians were too soon in reckoning the vernal equinox, and consequently too late in reckoning their month. The Pope looking back no farther than the council of Nice, added 10 days to his own birth-day, Oct. 5, 1582, and called it the 15th; and by that means the vernal equinox fell on March 21. This reformation of the calendar is called the Gregorian account, or new style; which if it had not been corrected, the vernal equinox would have fallen on Christmas-day in the year 10200.

A TABLE *to reduce the* Julian *to the* Gregorian *Year.*

	Days added.	10
1600		10
1700		11
1800		12
1900		13
2000	Leap-Year,	13
2100		14
2200		15
2300		16
2400	Leap-Year.	16
2500		17
2600		18
2700		19
2800	Leap-Year.	19
2900		20
3000		21
3100		22
3200	Leap-Year.	22
3300		23
3400		24
3500		25
3600	Leap-Year.	25
3700		26
3800		27
3900		28
4000	Leap-Year,	28
4100		29
4200		30
4300		31
4400	Leap-Year,	31
4500		32
4600		33
4700		34

To know on what day of the week any proposed day of the month will fall.

Having found the dominical letter for the given year, look for that letter at the top of the table shewing the days of the month (page 420) and under the said letter you have all the days of the months which are Sundays in that year, in the divisions of the months. Under the next, towards the right hand, all the days in the column are Mondays; those under

under the next are Tuesdays; and so on. When you are out at the right hand of the table, go back to the left, and so reckon on according to the order of the days of the week.

Thus, suppose for the year 584, before Christ, for which the dominical letter was F E; the first serving for January and February, and the last for all the rest of the year; in the table (page 420) I find, under F, the 6th, 13th, 20th, and 27th of January; and the 3d, 10th, 17th, and 24th of February; and then under E, I find the 2d, 9th, 16th, 23d, and 30th of March and November; the 5th, 12th, 19th, and 26th of October; the 6th, 13th, 20th, and 27th of April and July; the 3d, 10th, 17th, 24th, and 31st of August; the 7th, 14th, 21st, and 28th of September and December; the 4th, 11th, 18th, and 25th of May; and the 1st, 8th, 15th, 22d, and 29th of June; which being all Sundays in that year, the rest of the days of the months, answering to given days of the week, are easily found.

E. 1. In the 584th year before the birth of Christ, I desire to know on what day of the week the 18th of May fell on?

Look for the 28th of May in the table, and you will find A stands at the top of the column in which that day is found: And as the 25th of May fell on Sunday, it is plain, that the 28th of May must have been on Wednesday.

E. 2. On what day of the week does Christmas-day fall on for the year 1801?

The dominical letter for this year is D. Then under D, in the division for December, in the table (420) I find that the 6th, 13th, 20th, and 27th are Sundays; and consequently, as the 20th of December falls on Sunday, the 25th (or Christmas-day) must be on Friday.

To find the solar cycles.

RULE. To the given year add 9, divide the sum by 28, the remainder is the cycle of the sun.

Note. The solar cycle, or cycle of the sun, is a period of 28 years; in which time all the varieties of the dominical letters will have happened, and the 29th year the cycle begins again, when the same order of the letters will return as they were 28 years before. At the birth of Christ, 9 years had passed in this cycle.

EXAMPLE. Required the year of the solar cycle, for the year 1801.

$1801 + 9 = 1810 \div 28 = 64$, remains 18 cycle of the sun.

To find the lunar cycle.

RULE. To the given year add 1, and divide this sum by 19; the remainder shews the cycle of the moon, or lunar cycle.

Ex. What

Ex. What is the golden number, or lunar cycle for the year 1783?

$$\begin{array}{r l} 1783 & \\ +1 & \\ \hline 19)1784(93 & \text{revolution since the birth of Christ.} \\ 171 & \\ \hline 74 & \\ 57 & \\ \hline \end{array}$$

Answ. 17 lunar cycle, or golden number.

Note. The lunar cycle, or golden number, is a period of 19 years, containing all the variations of the days on which the new and full moons happen; after which time they fall on the same days they did 19 years before; and then she begins again with the sun.

But when a centissimal or hundredth year falls in the cycle, the new and full moon, according to the new stile, will fall a day later than otherwise. The birth of Christ happened in the second year of this cycle.

To find the Roman indiction.

RULE. To the given year add 3, and divide the sum by 15; the remainder is the number of indiction.

Ex. What is the Roman indiction for the year 1783?

$$\begin{array}{r} 1783 \\ +3 \\ \hline 15)1786(119 \\ 15 \\ \hline 28 \\ 15 \\ \hline 136 \\ 135 \\ \hline \end{array}$$

1 Roman indiction.

N. B. The Roman indiction is a cycle of 15 years, which first began the third year before Christ.

To find the epact till the year 1900.

RULE. Multiply the golden number for the given year by 11, divide that product by 30, and from the remainder take 11, leaves the epact. If the remainder is less than 11, add 19 to it, and the sum will be the epact.

Ex. Required the epact for the year 1782?

First, the golden number for this year is 16.

Then per rule 17

$$\begin{array}{r} 11 \\ \hline 3|0)18|7 \\ \hline \end{array}$$

6—7+19=26 the epact.

N. B.

N. B. The epact of any year is the moon's age at the beginning of that year; that is, the days past since the last new moon.

To find the moon's age.

RULE. To the epact add the month of the year, and the day of the month; their sum, if under 30, is the moon's age; but if that sum is greater, then 30 taken from it leaves the moon's age.

The moon's age taken from 30, leaves the day of the next change.

When the solar and lunar cycles begin together, the moon's age on the first of each month, or the monthly epacts, are called the numbers of the month, and are as follows, viz.

These,	0.	2.	1.	2.	3.	4.
For	Jan.	Feb.	Mar.	April	May	June
These,	5.	6.	8.	8.	10.	10.
For	July	Aug.	Sept.	Oct.	Nov.	Dec.

E. 1. Required the moon's age on January 1st, 1783?

First, the epact is 26
The month 0
1

Answ. 27 days, the moon's age.

E. 2. The moon's age is 27; how many days is there to the day of her change, which age never exceeds 30 days?

30
27

Answ. 3 days to the change.

To find the time of the moon's southing.

RULE. Multiply the moon's age by 4; divide the product by 5, and the quotient gives the hours; the remainder, multiplied by 12, gives the additional minutes.

If this time is less than 12 hours, it is the time of southing after mid-day; but if greater, 12 hours taken from it, leaves the southing after mid-night.

E. 1. Required the time of the moon's southing at London, on the 21st of May, 1776?

3 moon's age
4

5)12

2—2
12

24

Answ. 2 h, 24 m, afternoon.

E. 2. Required the time of the moon's fouthing at Birmingham, on the 1ft day of January, 1783?

$$
\begin{array}{l}
27 \text{ moon's age} \\
\underline{\quad 4} \\
5)108 \\
\overline{21 - \tfrac{3}{5} = 36} \\
\underline{12} \\
9h.\ 36.\ \text{after midnight.}
\end{array}
$$

Anfwer 9*h.* 36*m.* after midnight.

To find the time of high water at any place.

RULE. To the time of the moon's fouthing, add the time the moon has paffed the meridian, to make high water at that place, and the fum will fhew the time of high water.

The diftance of the moon from the meridian, when high water at the following places, is; at London, ☽ bears N. E. or S. E. 3*h.* 0*m.* Briftol key. ☽ bears E. by S .and W. by N. 6*h.* 45*m.*

E. 1. On the 21ft of May, 1776, at what time was it high water at London?

	H.	*M.*	
The moon fouths at –	2	24	*P. M. N.*
At London ☽ bears N.E. or S. E.	3	00	
Sum	5	24	

Anfwer, 24 minutes paft 5 in the afternoon.

High water is the ftate of the tide when higheft, or the time it ceafes to flow up.

ASTRONOMY.

PART VI.

SECTION LXXIX.

ASTRONOMY is derived from two Greek words, viz. After, a ftar, and Nomos, a law, or rule. It is a fcience, which by infallible demonftration, teaches us the motions diftances, and magnitudes of the heavenly bodies; their revolutions, anomalies, aplelions, eccentricities, enlongations, and parallaxes of the planets, eclipfes of the luminaries, occultations of the primary planets and fixed ftars, by the moon;

moon; as alſo, the riſing, culminating, ſetting, and amplitudes; and, in ſhort, whatever belongs to the right underſtanding of the true ſyſtem of the world.

Of eclipſes there are four ſorts, viz. 1ſt, partial; 2d, total without continuance; 3d, total with continuance; and laſtly, annular.

1ſt. Partial, is when part of the ſun's diameter is obſcured from ſome particular tract of the earth, and theſe happen more frequently than any other.

2d. Total without continuance, is when, at the time of the viſible conjunction, the true latitude of the moon is equal to her parallax in latitude from the ſun; that ſo the center of one is exactly ſeen in the center of the other; and then alſo their viſible diameters are equal; that ſo the ſun is no ſooner hid from our ſight by the dark body of the moon, but very ſpeedily he is ſeen to recover his light on the other ſide.

3d. Total with continuance, is when, at the viſible conjunction, the eclipſe is central (which is always when the north latitude and parallax are equal) the moon is in perigon, and the ſun in apogeon; then the apparent diameter of the moon exceeds that of the ſun, and this exceſs can never amount to one minute, ſo that the total darkneſs of any ſolar eclipſe, can never exceed 4′ or 5′ in time.

4th. Laſtly, annular, is when the viſible central conjunction happeneth, the ſun being in perigeon, and the moon in apogeon; here the diameter of the ſun exceeds that of the moon, and conſequently there will then be a ring of light round the moon.

That the ſun's eclipſe always begins on the weſt ſide, and goes off, or ends, on the eaſt ſide of his body, is a moſt manifeſt truth; becauſe the moon (who is always the cauſe of this obſcurity) moving in her annual motion always in conſequence, or according to the order of the ſigns, muſt, of neceſſity, firſt touch the ſun's weſtern limb, and laſt leave his eaſtern.

Aſtronomers have divided the ſun's diameter into 12 equal parts, which they call digits; ſo that if we ſpeak of a digit, or finger's breadth, it is no more than $\frac{1}{12}$ part of the ſun's diameter.

Of the cauſes and times of eclipſes.

An eclipſe of the ſun is cauſed by the moon's opaque body paſſing between the ſun, and thoſe parts of the earth from which ſhe hides the whole or part of the ſun; and this can never happen, but at time of new moon.

An eclipſe of the moon is cauſed by the whole, or part, of her body paſſing through the earth's ſhadow; which can never happen, but when the moon is full.

If the moon's orbit lay in the plane of the ecliptic (in which the earth always moves, and the ſun appears to move) the ſun would be eclipſed at the time of every new moon; and the moon would be eclipſed at the time of every full.

But one half of the moon's orbit lies on the north side of the ecliptic, and the other half on the south side of it; therefore, the moon's orbit intersects the ecliptic only in two opposite points, which are called the moon's nodes; and the angle which the moon's orbit makes with the ecliptic is 5° 18′. The intersection from which the moon ascends northward from the ecliptic, is called the moon's ascending node; and the opposite intersection, from which the moon descends southward from the ecliptic, is called the moon's descending node. These nodes move backward in the ecliptic $19\frac{1}{5}$ degrees every year, from the consequent toward the antecedent signs; and therefore they go quite round the ecliptic, in 18 years, 223 days and 5 hours.

From the time of the sun's being in conjunction with either of the moon's nodes, to the time of his being in conjunction with the other, is about $173\frac{1}{2}$ days, at a mean rate; within which number of days the eclipses must always happen, in different times of the year.

When the moon changes within 18 days before or after the day of the sun's being in conjunction with either of her nodes, the sun will be eclipsed. And when the moon is full within 12 days before or after the day of the sun's conjunction with either of the nodes, the moon will be eclipsed. At greater distances of the sun from the nodes, there can be no eclipses of these luminaries.

The period of eclipses, according to the learned Dr. Halley, is 18 years, 10 days, 7 hours, 43 minutes; in leap year, 11 days, 7 hours, 43 minutes; which is near two hundred and twenty-three mean lunations. And, therefore, in that time, there will be a regular period of eclipses or returns of the same eclipses for many ages. But the falling back of the line of conjunction of the sun and moon, with respect to the line of the nodes, in every period, will at length exhaust it, and not return again in less than twelve thousand four hundred and ninety-two years.

Another period for comparing and examining eclipses, which happen after long intervals of time, is that which consists of six thousand eight hundred and ninety mean lunations, or about five hundred and fifty-seven years and twenty-one days; in which time the sun and node will meet again so nearly, as to be little more than eleven seconds distant: but it will not be the same eclipse that returns, as in the shorter period above-mentioned.

These periods are said to have been discovered by the Chaldeans six or seven hundred years before the birth of Christ; but M. Bailly, in his Histoire de l'Astronomie Ancienne, has shewn that the invention is of a much earlier date, and were known to the Arabs, Indians, Chinese, and Tartars, long before the sciences were cultivated in Greece. And as a knowledge of this kind could have only been obtained from a long series of observations, or a general and perfect acquaintance with the celestial motions, it is probable, that these, as well as many other discoveries of equal importance, are due to the most ancient inhabitants of the earth.

In

In China, where astronomy is made subservient to the interest of the state, under the reign of the emperor Choukang, the two principal astronomers, Ho and Hi; were condemned to death, on account of their ommiting, to announce the precise time of an eclipse of the sun. This eclipse which happened two thousand one hundred and sixty-nine years before Christ, and a remarkable conjunction of four of the planets, which their annals affirm to have taken place at a still earlier period, are strong proofs of the authenticity of the Chinese chronology.

There is a general tradition, which is countenanced by sacred and profane history, that great changes and revolutions have taken place in our globe since its first formation: and the bare inspection of the earth gives great weight to this opinion. We can perceive, in many instances, that the waters of the ocean have not always been confined within their present bounds. The vegetables and fishes of India, which are found in the petrefactions of Europe; and the number of shells, and other marine productions, discovered in ranges of mountains very remote from the sea, can be accounted for upon no other principle. This was a doctrine taught by Pythagoras and his followers; and Ovid expresses himself thus,

"The face of places, and their forms, decay;
And what was solid earth converts to sea;
Seas, in their turn, retreating from the shore,
Make solid lands what ocean was before;
And far from strands are shells of fishes found;
And rusty anchors fixed on mountain ground:
And what were fields before, now mark'd and worn
By falling floods, from hills to vallies turn:
And crumbling still descend to level lands;
And lakes, and trembling bogs, are barren sands:
And the parch'd desert floats in streams unknown,
Wondering to drink of waters not her own."

To these testimonies, may be added another still more singular; which is that of the ancient Egyptians, who maintained that the sun, in former ages, had risen in the west and set in the east; and Herodotus, Plato, Diogenes Laertius, and Plutarch, all mention this revolution, and some are induced to believe, that the idea, was not without foundation.

The best modern astronomers are now generally agreed, that the angle which the ecliptic makes with the equator is continually decreasing, at the rate of about one minute in a hundred years; and therefore, if this diminution should proceed, the two circles, in about one hundred and forty thousand years, would coincide, and the sun, moving in or near the equator, would make equal days and nights all over the globe for many ages. A revolution of this kind, sufficient to reverse the four cardinal points of the compass, could not have been accomplished in less than two million of years: a length of duration to the world that few will admit.

We

We are told by Diodorus Siculus, that the philosophers of Babylon, at the time of Alexander's entry into that city, reckoned four hundred and three thousand years from the beginning of their astronomical observations. And upon a supposition that the ecliptic was first perpendicular to the equator, and afterwards began to approach towards it, according to the rate above-mentioned, this period nearly agrees with the diminution of the angle, which in that time, had taken place, and reduced the obliquity to twenty-three degrees and a half.

Some, however, are of a contrary opinion; and are disposed to believe, that the obliquity of the ecliptic has been always the same. But in this they are certainly mistaken; for besides the apparent decrease of this angle, which has been observed by most astronomers since the time of Hipparchus, the variation of latitude in the fixed stars is such as could arise from no other cause.

Ptolemy tells us expressly that he determined the obliquity, for several years together, to be twenty-three degrees fifty-one minutes, and it is now known to be twenty-three degrees twenty-eight minutes; the attention which has been bestowed upon this subject for nearly a century past, has enabled us to decide with certainty that the diminution is real, and according to a certain law.

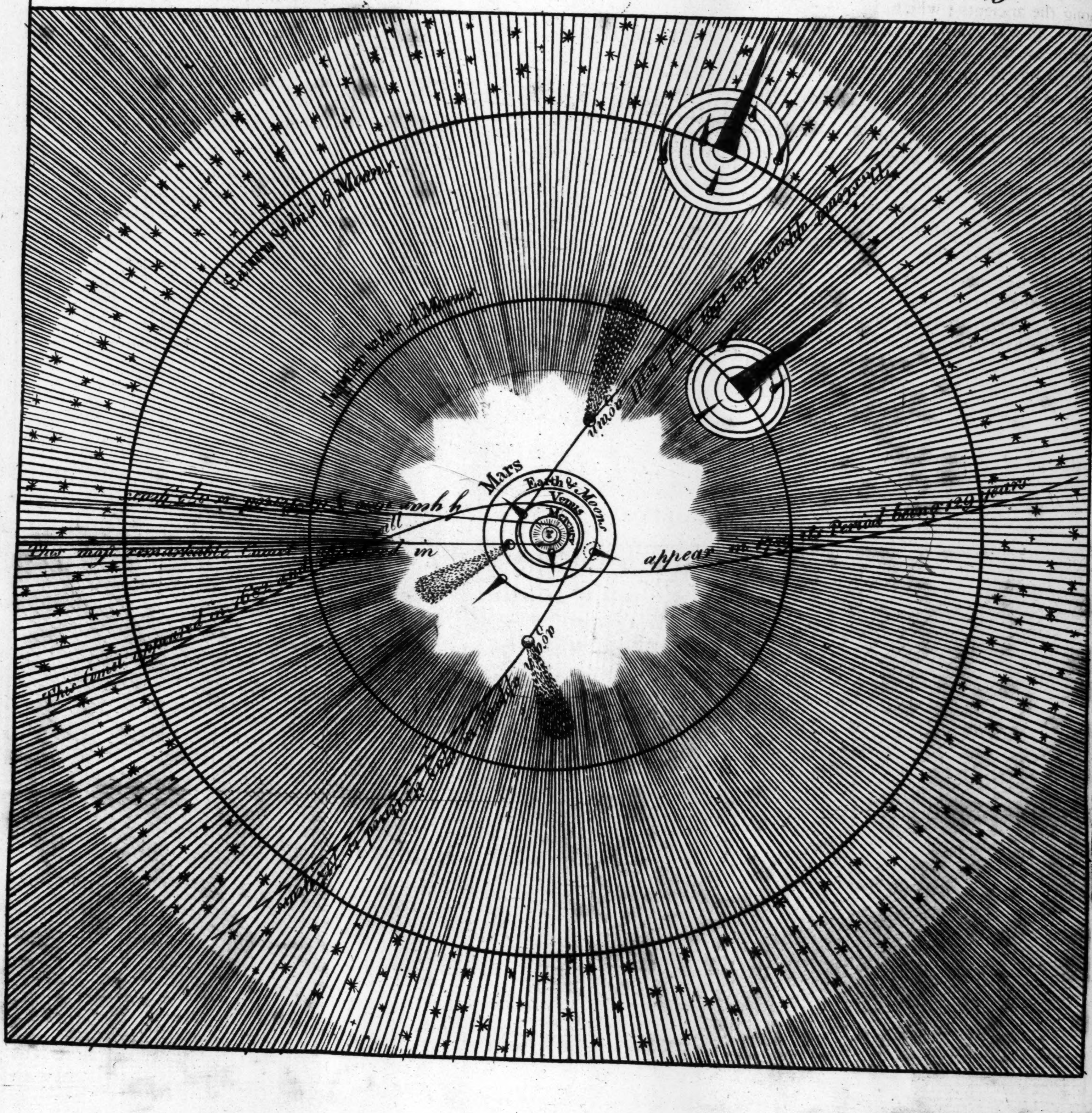
Hipparchus, in comparing his observations with those of Timocharis, which had been made at Alexandria about a century before, first perceived that the stars changed their positions, and appeared to have a slow motion from west to east with regard to the equinoctial points.

This change of the stars in longitude, which has now become sufficiently apparent, is owing to a small retrograde motion of the equinoctial points, of about fifty seconds in a year, which is occasioned by the attraction of the sun and moon upon the protuberant matter about the equator. The same cause also occasions a small deviation in the parallelism of the earth's axes, by which it is continually directed towards different points in the heavens, and makes a complete revolution round the axes of the ecliptic in about twenty-five thousand nine hundred and twenty years.

In consequence of this shifting of the equinoctial points, an alteration has taken place in the signs of the ecliptic; those stars which in the infancy of astronomy were in Aries, being now got into Taurus; those of Taurus into Gemini; &c.

The

Pl. 2. *The* Copernican *or* Solar System. *Page 431*



The Copernican, or solar system.

This system of the world, as described on Plate 2, is not a late invention, but was known and taught by the wise Samian Pythagoras, and others among the ancients; which, in after-times, was lost; till, in the 15th century, it was again revived by the famous Polish philosopher, Nicholas Copernicus, who was born at Thorn, in the year 1473. In this he was followed by the greatest mathematicians and philosophers that have since lived, as KEPLER, GALILEO, DESCARTES, GESSENDUS, and Sir ISAAC NEWTON, who have established this system on such an everlasting foundation of mathematical and physical demonstration, that the gates of ignorance shall never prevail against it.

The most famous of the antiquated systems are two, viz. one taught by PTOLEMY, the EGYPTIAN, astronomer, said to have lived 138 years before Christ; the other by the noble Dane, TYCHO BRABE, born in Schonen, A. D. 1546.

The PTOLEMEAN SYSTEM supposed the earth immoveably fixed in the center of the world, about which moved seven planets, viz. the *Moon*, *Mercury*, *Venus*, the *Sun*, *Mars*, *Jupiter* and *Saturn*; above these is placed the firmament of the fixed stars, then the two crystalline spheres; all which were included in, and received motion from, the primum mobile; which constantly revolved about the earth in 24 hours from east to west.

The TYCHONIAN SYSTEM succeeded the PTOLEMEAN, but was never so universal. This supposed the earth in the center of the world, or firmament of fixed stars; as also of the two luminaries, the Moon and Sun: but then he supposes the Sun the center of the planetary motions, viz. of Mercury, Venus, Mars, Jupiter, and Saturn; these, with the Sun, all revolved about the earth in the space of a year, to account for the annual motion; and the earth he made to revolve about the axes every 24 hours, from west to east. This hypothesis being part false, was embraced by few, and soon gave way to the only true and rational solar system, restored by COPERNICUS as aforesaid.

A BRIEF DESCRIPTION OF THE SOLAR SYSTEM.

The sun, a stupendous body of fire, is placed in the centre of the system, round whose orb, the planets, satellites, and comets, perform their revolutions, with an order and regularity which must fill our minds with the most exalted conceptions of their divine Original.

Mercury, the nearest planet to the sun, goes round him in about 87 days and 23 hours, or a little less than three months; which is the length of his year. But being seldom seen, on account of his nearness to the sun, and no spots appearing on his surface, or disk, the time of his rotation upon his axis, or the length of his days and nights, is not yet determined. His distance from the sun is computed to be about thirty-seven millions of miles, and his diameter three thousand two hundred; and

and in his courſe round the ſun, he moves at the rate of a hundred and five thouſand miles an hour.

Venus,* the next planet above Mercury, is computed to be ſixty-eight millions of miles from the ſun, and by moving at the rate of ſeventy-ſix thouſand miles an hour, ſhe completes her annual revolution in two hundred and twenty-four days and ſeventeen hours, or about ſeven months and a half. Her diameter is ſeven thouſand ſeven hundred miles, and her diurnal rotation on her axis, is performed in twenty-three hours and twenty-two minutes. When this planet appears to the weſt of the ſun, ſhe riſes before him in the morning, and is called the Morning Star; and when ſhe appears to the eaſt of the ſun, ſhe ſhines in the evening, after he ſets, and is then called the Evening Star; being in each ſituation, alternately, for about two hundred and ninety days.

The next planet above Venus, in our ſyſtem, is the *Earth*. Its diſtance from the ſun is ninety-five millions of miles, and by travelling at the rate of fifty-eight thouſand miles an hour, its annual revolution is performed in three hundred and ſixty-five days, five hours, and forty-nine minutes, or the ſpace of a year; which motion, though one hundred and twenty times ſwifter than that of a cannon ball, is but little leſs than half the velocity of Mercury in his orbit. The earth's diameter is ſeven thouſand nine hundred miles; and as it turns round its axis every twenty-four hours, from weſt to eaſt, it occaſions an apparent motion of all the heavenly bodies, from eaſt to weſt, in the ſame time.

Next above the earth's orbit, is *Mars*, whoſe diſtance from the Sun is computed to be about one hundred and forty-four millions of miles. He moves at the rate of fifty-five thouſand miles an hour, and completes his revolution round the Sun in a period little leſs than two of our years. His diameter is four thouſand two hundred miles; and his diurnal rotation upon his axis is performed in about twenty-four hours and thirty-nine minutes.

Jupiter, the largeſt of all the planets, is ſtill higher in the ſyſtem than Mars. He is reckoned to be about four hundred and ninety millions of miles from the Sun, and by going at the rate of twenty-nine thouſand miles an hour, completes his annual revolution in ſomething leſs than twelve of our years. His diameter is computed to be eighty-nine thouſand miles, and, by a prodigiouſly rapid motion upon his axis, he performs his diurnal rotation in nine hours and fifty-ſix minutes.

Saturn, the next planet in the ſyſtem above Jupiter, is about nine hundred millions of miles from the Sun; and by travelling at the rate of twenty-two thouſand miles an hour, he performs his annual circuit round that luminary in about twenty-nine and an half of our years. His diameter is computed to be ſeventy-nine thouſand miles; but, on account of his immenſe diſtance, and the deficiency of light occaſioned by ſuch a remote ſituation, the time of his diurnal rotation upon his axis has not yet been aſcertained.

* There will be a Tranſit of this beautiful Planet over the Sun's diſk, December 6, 1882.—For an Explanation, and Orthographical Projection of this Phenomenon, ſee Frontiſpiece to this work.

This

This was thought the most remotest planet in our system; but Dr. HERSCHEL, on the 13th of March, 1781, discovered another at a still greater distance, called the GEORGIUM SIDUS, his distance from the sun is computed to be one thousand eight hundred millions of miles, its magnitude is about eighty-nine times greater than the earth's; and that it revolves round the sun in an orbit, which is nearly circular, in about eighty-two years.

SATELLITES OR MOONS.

Besides the primary planets here mentioned, there are ten others, called secondary planets, or satellites, which regard their primaries as the centres, of their motions, and revolve round them in the same manner as those primaries rovolve round the Sun.

The most conspicuous of these satellites is the Moon, who is a constant attendant on our Earth; and, whilst she accompanies it in its annual progress through the heavens, keeps revolving round it continually, by a different motion in the space of a month.

The moon's diameter is two thousand one hundred and eighty miles; her distance from the Earth two hundred and forty thousand miles; and in bulk she is about sixty times less than the Earth. Jupiter has four such moons, and Saturn five; and from the continual change of their phases, or appearances, it is evident that these also are opaque bodies, like the planets, and shine only by means of the borrowed light which they receive from the Sun.

It may also be observed, that our earth is a moon to the moon, waxing and waneing in the same manner, but appearing about thirteen times larger, and, of course, affording a proportional quantity of light. When she changes to us, the Earth will appear full to her, and when she is in her first quarter to us, the Earth will be in her third quarter to her. And, as her axis is almost perpendicular to the plane of the ecliptic, one half of her orb will be constantly illuminated by the reflected light of the Earth in the Sun's absence, whilst the other half will have a fortnight's darkness, and a fortnight's light, alternately.

The rotation of the moon upon her axis, is also performed in the same time that she goes once round the earth, as is evident from her always presenting the same face to us during the whole of her monthly revolution; on which account, it is plain that the inhabitants of one half of the lunar world, are totally deprived of a sight of the earth, whilst the inhabitants of the other hemisphere have a full view of our globe, moving through the heavens at the rate of fifty-eight thousand miles an hour, and appearing to them near thirteen times larger than that of the sun.

The reason of the moon's rising and setting an hour later every night than it did the night before, is owing to its diurnal motion of about 13° 10′ from west to east round her orbit; which, together with the motion of the earth round her orbit, makes very near an hour's difference of rising and setting, one night with another; but it is not so every night, being sometimes as much again as at others, according to the different

times of the year, and the different parts of the ecliptic, the earth and moon may then happen to be in; as is evident from the different aspects of the moon towards the autumn, vulgarly called the harvest moon; which does not set the next night after full, but rises about the same time for several nights together.

In an oblique sphere, all great circles intersecting the equinoctial, will, in the revolution of the sphere, intersect the horizon with different angles, at every different part thereof. Thus, with respect to the ecliptic, when the beginning of Libra is orient, or rising in the east, it then makes the greatest angle with the horrizon; when Capricorn is orient, the angle is mean; and when Aries is orient, the angle is least of all; therefore, when the moon is full in the beginning of Libra, one day's motion depresses her farthest below the horizon, and least when in the beginning of Aries; consequently, the difference of her rising each day at the vernal equinox will be greatest, and least of all in the autumnal equinox.

The revolution of the moon through the zodiac is called a lunation, and 12 of these lunations or revolutions is a lunar year; which takes up the space of 354 days, 8 hours, 48′ 38″. The difference between this and the solar year, which contains 365 days, 5 hours, 48′ 57″ is almost 11 days, which chronologers call the epact.

And because the moon's motion about her axis is performed in the same time as about the earth, the lunarians have their natural days equal to their months.

Besides the moons, these planets are accompained with, Saturn is known to be encompassed with a ring; which surprizing phænomenon was discovered about 132 years since. It is said, the inner border of the ring, from the body of Saturn, is equal to the breadth of the ring itself; each is computed to be, at least, 21,000 miles; though others make the interval between the ring and Saturn's body to be 210,265, and the breadth of the ring to be 29,200 miles; its thickness is unknown, being too small for observation: it hath a variety of aspects, sometimes appearing a large ellipsis, then a smaller; sometimes only a straight line, and sometimes not visible at all. These are the most remarkable particulars of this prodigy of nature, known by astronomers; as to the matter, of which it doth consist, it is not known by any.

OF COMETS.

Comets, or blazing stars, according to Sir Isaac Newton, are solid, compact, fixed, and durable substances; and are a kind of planets, which move about the sun in stated periods of time, and shine by the light of the sun-beams reflected from them; and that the orbits are very eccentric and elliptical, but some more and some less; so consequently their periods are longer or shorter. The forms of three remarkable cometary orbs are described in the solar system before-going.

Dr.

Dr. Halley has determined the longest axis of the orbit of that comet, which appeared in 1680, and whose period is 575 years, to be 1382975 parts, of which the mean distance of the earth from the sun is 10000; therefore, supposing this mean distance to be 81000000 English miles, then the length of that comet's orb will be above eleven thousand and two hundred millions of English miles.

And Sir Isaac Newton has computed the heat of the aforesaid comet, when nearest the sun, to be 2000 times hotter than red-hot iron; and it is computed, that a ball of iron, as big as the globe of our earth, would, if red-hot, require 50,000 years to grow cold in; and the bodies of comets being so much greater than our earth, can never be cold at their greatest distance: and the learned Dr. Halley has compiled a set of tables, whereby the places in the zodiac, of above 20 comets, may be determined for any given time.

OF THE FIXED STARS.

Fixed Stars, are so called, in opposition to the planets, or moving stars, because they always keep the same place in the heavens, and do not seem to move for several ages together; yet they have an apparent motion, occasioned by a certain contrary motion of the earth, arising from the spheroidical figure thereof.

This motion of the fixed stars does not exceed 50″ of a degree in a year, or one degree in 70 years; therefore, to complete one revolution of a circle, is required 25,920 years; after which time the stars all return again to their former places.

The distance of the fixed stars is but imperfectly known; however, the famous Hugens has computed the brightest, and of course the nearest of all the fixed stars, viz. *Syrius*, to be, in appearance, 27,664 times less than the sun; and since their distances are greater, as their magnitudes are lesser, therefore this star must be at the rate of above two millions of millions of miles; which is so great, that a cannon ball would spend almost 700,000 years in passing through it; and it is probable, that all fixed stars are equally distant from each other, in proportion to the distance of the nearest of them from our sun.

The number of visible fixed stars, whose places have been rectified by astronomers, are these:

	Hypparchus, 140 years before Christ, had a catalogue of stars, containing - - - - - - - - -	1022
	Pliny - - - - - - - - - - - -	1600
	Ptolemy, 135 years after Christ - - - - - -	1026
1437	Uligh Beighi - - - - - - - - - -	1017
1572	Tycho Brabe - - - - - - - - - -	777
1620	John Kepler - - - - - - - - -	1163
1630	Dr. John Bayerus - - - - - - - - -	1725
	Prince of Hesse - - - - - - - - -	400

1635	Brachius - - - - - - - -	1672
1651	John Ricciolus - - - - - - - - -	1468
1670	John Hevetius - - - - - - - - -	1888
1676	Dr. Edmund Halley - - - - - - - -	373
1690	John Flamstead - - - - - - - -	3000

Aſtronomers have divided the ſtars into ſix ſeveral ſizes or magnitudes, of which the greateſt or brighteſt of them are called ſtars of the firſt magnitude, as *Acturus*, *Regulus*, *Sirius*, &c. and the next to them in brightneſs, are called the ſtars of the ſecond magnitude; next to them in brightneſs, are ſtars of the third magnitude, &c. till we come to ſtars of the ſixth magnitude, which comprehend the ſmalleſt ſtars that can be diſcerned with the naked eye; and in the above catalogues, the moſt complete contains only 3000 ſtars, though aſſiſted by the beſt glaſſes; but the moſt that can be diſcovered by the naked eye, in the moſt ſerene night, are not above three or four hundred; and Dr. KEILL, in his Aſtronom. Lect. vi. p 51, 52, 53, 54, ſays of the 3000 ſtars in Mr. FLAMSTEAD's catalogue, it is ſeldom that a very good eye can reckon more than one hundred together; and the famous Mr. FLAMSTEAD himſelf aſſerts, that the naked eye cannot diſcover above 384 ſtars in the ſereneſt night, in both the hemiſpheres.

The Galaxy, via lectea, or milky way, is a broad white tract, encompaſſing the whole heavens, and extending itſelf in the ſign of *Capricorn*, from the equinoctial to the tropic of *Cancer*, with a double path, and the reſt of it is a ſingle one. Some of the ancients, as ARISTOTLE imagined that this path conſiſted only of a certain exhalation hanging in the air; but by the obſervations with the teleſcope, made in this age, it has been diſcovered to conſiſt of an innumerable quantity of fixed ſtars, different in ſituation and magnitude; from the confuſed mixture of whoſe light, its white colour is ſuppoſed to be occaſioned.

The fixed ſtars are known from the planets by their ſcintillation, or ſparkling; for the planets have no ſuch vibration, twinkling, or glimmering of light; but, generally, all the fixed ſtars, more or leſs, and at ſome times more than others. The cauſe of their ſcintillation is variouſly diſcourſed of, both by philoſophers and aſtronomers. ARISTOTLE among the ancients, aſſigns the cauſe thereof to their remoteneſs from our ſight, by which they are weakly, and as it were by a trembling wearineſs reached. Some, again, will have the cauſe of this twinkling to proceed from refraction; others aſſign the cauſe to ariſe from the unequal ſuperfices of the fluctuating air or medium; as ſtones in the bottom of a river, by the rapid motion of the water, ſeem to have a kind of tremulous motion: but GASSENEUS, more probably, conceives this ſcintillation of the fixed ſtars to proceed from that native and primogenial light they are indued with, like that of the ſun's ſparkling, caſting forth ſuch quick darted rays, as our weaker ſight cannot behold without that trembling paſſion.

GEOGRAPHY.

PART VII.

SECTION LXXX.

THE word Geography comes from the Greek; and, in a proper ſenſe, ſignifies nothing more than a deſcription of ſuch parts of the ſurface of the earth as are really land; the other parts, which deſcribes the water, being called Hydrology. For the globe of our earth, having its external ſurface partly land and partly water, has been from thence always denominated the terraqueous globe, which is the foundation of the two above-mentioned ſciences, Geography and Hydrology.

The figure of the earth has been long well known to be globular, or ſpherical: it was originally ſuppoſed flat, or a plane: but this was too great an error for any perſon to continue in long; becauſe if a perſon walks directly north or ſouth, it will cauſe the ſtars to have a greater or leſſer elevation above the horizon; but no alteration, in that reſpect, would happen to them, in walking on a plane, though the diſtance be ever ſo great. This, therefore, afforded an evident proof, that the ſurface of the earth was of a curvilineal form; and becauſe walking over equal ſpaces occaſioned an equal difference in the meridian altitude of the ſtars, it was a proof that the curve ſurface was of the ſpherical kind; and therefore, the body of the globe was in form of a globe or ſphere.

And this was the general opinion, till the beginning of the laſt century; when experiments on pendulums, the nature of gravity, a centrifugal force in revolving bodies, and ſome other phyſical principles, came to be underſtood, there was great reaſon to ſuſpect, that the figure of the earth could not poſſibly be that of a globe, but that of a ſpheriod as above-mentioned.

Theſe diſcoveries excited a great deſire among the learned, to be ſatisfied (experimentally) of the true figure of the earth; which they eaſily knew could not be done, without actually meaſuring a degree on the ſurface of the earth, in ſeveral different parts of it; and the more remote from each other, the better. At length, by the munificence of Kings, and great propenſity of philoſophers and mathematicians, the arduous undertaking was attempted, proſecuted, and finiſhed, with ſucceſs.

NORWOOD, and others, make the circumference of the globe 25020 miles; the diameter in the equator is 7964 miles, at the poles 7930 miles; but to form a general idea of theſe things, we may, without much error, look upon the earth as a globe or ſphere; and ſo the dimenſions of its ſurface, computed in ſquare miles, 60 to a degree, will be expreſſed in the following table; whereby you may ſee, at one view, the ſuperficial content of the whole, and its ſeveral parts.

The

The superficial Content of the whole Globe, in square Miles, 60 to a Degree.

	Square Miles.	*Islands.*	*Square Miles.*
The globe	148510627	Flores	6000
Seas & unknown parts	117843821	Ceram	5400
The habitable world	30666806	Briton	4000
Europe	2749349	Socatora	3600
Asia	10275487	Candia	3220
Africa	8506208	Porto Rico	3200
North America	3699087	Corsica	2520
South America	5454675	Zealand	1935
Persian empire under Darius	1650000	Majorca	1400
		St. Jago	1400
Roman empire in its utmost height	1610000	Negropont	1300
		Teneriff	1272
Russian	3303485	Gotland	1000
Chinese	1749000	Madeira	950
Great Mogul	1116000	St. Michael	920
Turkish	960057	Skye	900
Present Persia	800000	Lewis	880
		Funen	768
Islands.		Yvica	625
Borneo	228000	Minorca	520
Madagascar	168000	Rhodes	480
Sumatra	129000	Cephalonia	420
Japan	118000	Amboyna	400
Great Britain	72926	Orkney Pomona	324
Celebes	68400	Scio	300
Manilla	58500	Martinico	260
Iceland	46000	Lemnos	220
Terra del Fuego	42075	Corfu	194
Mindinao	39200	Providence	168
Cuba	38400	Man	160
Javan	38250	Bornholms	160
Hispaniola	36000	Wight	150
Newfoundland	35500	Malta	150
Ceylon	27730	Barbadoes	140
Ireland	27457	Zant	120
Formosa	17000	Antigua	100
Arion	11900	St. Christopher	80
Gilolo	10400	St. Helena	80
Sicily	9400	Guernsey	50
Timor	7800	Jersey	43
Sardinia	6600	Bermudas	40
Cyprus	6300	Rhode	36
Jamaica	6000		

Geographical

Geographical Definitions.

The ſituation of places upon the earth is determined by their latitude and longitude.

1. The latitude of any place upon the earth is its neareſt diſtance, either north or ſouth, from the equator; and if the place be in the northern hemiſphere, it is called north latitude; if in the ſouthern hemiſphere, it is called ſouth latitude, and is meaſured by an arch of the meridian paſſing through the zenith of the ſaid place, and intercepted betwixt it and the equator: and all places that lie on the ſame ſide, and at the ſame diſtance from the equator, are ſaid to be in the ſame parallel of latitude; the parallels of latitude in geography, are the ſame with the parallels of declination in aſtronomy.

Corollaries. 1. No place can have above 90 degrees of latitude, either north or ſouth.

2. Thoſe places that lie under the equinoctial have no latitude, it being from thence that the calculation of latitude is counted; and thoſe places that lie under the poles have the greateſt latitude, thoſe points being at the greateſt diſtance from the equator, or equinoctial line.

3. The latitude of any place is always equal to the elevation of the pole in the ſame place, above the horizon; and is therefore often expreſſed by the pole's height, or elevation of the pole: the reaſon of which is, becauſe from the equator to the pole there is always the diſtance of 90 degrees, and from the zenith to the horizon the ſame number of degrees, each of theſe including the diſtance from the zenith to the pole. That diſtance, therefore, being taken away from both, will leave the diſtance from the zenith to the equator (which is the latitude) equal to the diſtance from the pole to the horizon.

4. The elevation of the equator in any place, is always equal to the complement of the latitude of the ſame place.

5. A ſhip ſailing directly towards the equator, leſſens her latitude, or depreſſes the pole juſt ſo much as her diſtance ſailed; and ſailing directly from the equator augments her latitude, or raiſes the pole juſt ſo much as her diſtance ſailed.

2. *Difference of Latitude*, is the neareſt diſtance betwixt any two parallels of latitude, ſhewing how far the one is to the northward or ſouthward of the other, which can never exceed 180 degrees; and when the two places are in the ſame hemiſphere, or on the ſame ſide of the equator, the leſſer latitude ſubtracted from the greater, and when they are on different ſides of the equator, the two latitudes added gives the difference of latitude.

3. *The Longitude* of any place upon the earth is an arch of the equator, contained betwixt the meridian of the given place, and ſome fixed or known meridian; or it is equal to the angle formed by the two meridians; which properly can never exceed 180 degrees, though ſometimes the longitude is counted eaſterly quite round the globe.

Since the meridians are all moveable, and not one that can be fixed in the heavens, the longitude of places cannot ſo well be fixed from any one meridian; but every geographer is at his liberty to make which he pleaſes

pleafes his firft meridian, from whence to calculate the longitudes of other places: hence it is, that the geographers of different nations, reckon their longitudes from different meridians, commonly chufing the meridian paffing through the metropolis of their own country for their firft; thus, the Englifh geographers generally make the meridian of London to be their firft; the French, that of Paris, &c. And mariners generally reckon their longitude from the laft know land they faw. This arbitary way of reckoning the longitude from different places, makes it neceffary, whenever we exprefs the longitude of any place, that the place from whence it is counted be alfo expreffed.

4. *Zones* are large tracts of the furface of the earth, diftinguifhed by the tropics and polar circles, being five in number, viz. one torrid, two temperate, and two frigid.

The torrid, or burning zone, is all the fpace comprehended between the two tropics: the ancients imagined this tract of the earth to be uninhabitable. All the inhabitants of the torrid zone have the fun in their zenith, or exactly over their heads, twice in every year; excepting thofe who live exactly under the two tropics, where the fun comes to their zenith only once in every year.

The two temperate zones lie on either fide of the globe between the tropics and the polar circles.

The two frigid zones are thofe fpaces upon the globe that are included within the two polar circles.

The inhabitants of the earth are alfo diftinguifhed by the diverfity of their fhadows; thofe who live in the torrid zone are called Amphifcians, becaufe their noon-fhadow is caft different ways, according as the fun is to the northward or fouthward of their zenith; but when the fun is in their zenith, they are called Afcians.

The inhabitants of the temperate zones are called Heterofcians, becaufe their noon-fhadow is always caft the fame way: but thofe who live under the tropics are called Afcians-Heterofcians. Thofe who live in the frigid zones are called Perifcians, becaufe fometimes their fhadow is caft round about them.

The inhabitants of the earth are alfo diftinguifhed into three forts, in refpect to their fituation one to another; and thefe are called, the Periœci, Antœci, and Antipodes.

5. The Periœci are thofe who live under oppofite points of the fame parallel of latitude; they have the feafons of the year at the fame time, and their days and nights always of the fame length with one another; but the one's noon is the other's midnight; and when the fun is in the equinoctial, he rifes with the one, when he fets with the other; thofe who live under the poles have no Periœci.

6. The Antœci live under the fame meridan, and in the fame latitude, but on the different fides of the equator; their feafons of the year are contrary, and the days of the one are equal to the nights of the other; but the hour of the day and night is the fame with both; and when the fun is in the equinoctial, he rifes and fets to both exactly at the fame time. Thofe who live under the fame equator have no Antœci.

7. The

7. The Antipodes are thoſe who live diametrically oppoſite to one another, ſtanding, as it were, exactly feet to feet; their days and nights, ſummer and winter, are at direct contrary times.

8. A climate is a tract of the ſurface of the earth, included between two ſuch parallels of latitude, that the length of the longeſt day in the one, exceeds that in the other, by half an hour.

The whole ſurface of the earth is conſidered as being divided into 60 climates, viz. from the equator to each of the polar circles, 24, ariſing from the difference of half an hour in the length of their longeſt days; and from the polar circles to the poles themſelves are ſix, ariſing from the difference of an entire month; the ſun being ſeen, in the firſt of theſe, a whole month without ſetting; in the ſecond, two, and in the third, three months, &c. Theſe climates continually decreaſe in breadth, the farther they are from the equator. How they are framed, viz. the parallel of latitude in which they end (that being likewiſe the breadth of the next) with the reſpective breadth of each of them, is ſhewed in the following table.

A TABLE OF THE CLIMATES.

Climates between the Equator and the Polar Circles.

Climates	Longeſt day.	Latitude. D.	Latitude. M.	Breadth. D.	Breadth. M.	Climates.	Longeſt day.	Latitude. D.	Latitude. M.	Breadth. D.	Breadth. M.
1	12½	8	25	8	25	13	18½	59	58	1	29
2	13	16	25	8	00	14	19	61	18	1	20
3	13½	23	50	7	25	15	19½	62	25	1	07
4	14	30	20	6	30	16	20	63	22	0	57
5	14½	36	28	6	08	17	20½	64	06	0	44
6	15	41	22	4	54	18	21	64	49	0	43
7	15½	45	29	4	07	19	21½	65	21	0	32
8	16	49	01	3	32	20	22	65	47	0	26
9	16½	51	58	2	57	21	22½	66	06	0	19
10	17	54	27	2	29	22	23	66	20	0	14
11	17½	56	37	2	10	23	23½	66	28	0	08
12	18	58	29	1	51	24	24	66	31	0	03

Climates between the Polar Circles and the Poles.

Length of days. Months.	Latitude. D.	Latitude. M.	Length af days. Months.	Latitude. D.	Latitude. M.
1	67	21	4	78	30
2	69	48	5	84	05
3	37	37	6	90	00

Of the Coſmical, Achronical, and Heliacal, Riſing and Setting of the Stars.

A ſtar is ſaid to riſe or ſet coſmically, when it riſes or ſets at ſun-riſing; and when it riſes or ſets at ſun-ſetting, it is ſaid to riſe or ſet achronically. A ſtar riſes heliacally, when firſt it becomes viſible, after it has been ſo near the ſun, as to be hid by the ſplendor of his rays; and a ſtar is ſaid to ſet heliacally, when it is firſt immerſed, or hid by the ſun's rays.

The fixed ſtars, and the three ſuperior planets, Mars, Jupiter, and Saturn, riſe heliacally in the morning; but the moon riſes heliacally in the evening; becauſe the ſun is ſwifter than the ſuperior planets, and ſlower than the moon.

Of the Surface of the Earth, conſidered as it is compoſed of Land, and Water,

The earth conſiſts naturally of two parts, land and water; and therefore it is called the terraqueous globe: each of theſe elements are ſubdivided into various forms and parts, which accordingly are diſtinguiſhed by different names.

I. OF THE LAND.

The Land is diſtinguiſhed into Continents, Iſlands, Peninſulas, Iſtmuſſes, Promontories, Mountains, or Coaſts.

9. A Continent is a large tract of land, comprehending ſeveral countries, not ſeparated by the ſea; ſuch as Europe, Aſia, Africa, and America; which four are the principal diviſions of the earth.

10. An Iſland is a portion of the earth, entirely ſurrounded with water, ſuch as Great Britain and Ireland; alſo a ſmall part of dry land, in the midſt of a river, is called an iſland.

11. A Peninſula is a part of land almoſt environed with water, ſave one narrow neck of land adjoining it to the Continent; ſuch as Denmark joining to Germany; alſo Africa is properly a large peninſula, joining to Aſia.

12. An Iſthmus is a narrow neck of land, joining a peninſula to the Continent; as the Iſthmus of Suez, which joins Africa to Aſia; that of Panama, joining North and South America, &c.

13. A Promontory is a high part of land, ſtretching out into the ſea; and is often called a Cape or Headland; ſuch is the Cape of Good Hope, in the ſouth of Africa; the Lizard Point, &c. A Mountain is a high part of land, in the midſt of a country, overtoping the adjacent parts.

14. A Coaſt, or Shore, is that part of land which borders upon the ſea, whether it be an Iſland, or a Continent; and that part of the land, which is far diſtant from the ſea, is called the inland country. Theſe are the uſual diſtinctions of the land.

II. OF WATER.

The Water is diſtinguiſhed into Oceans, Seas, Lakes, Gulfs, Straits, and Rivers.

15. The Ocean, or Main Sea, is a vaſt ſpreading collection of water, not divided by lands running between; ſuch as the Atlantic, or Weſtern Ocean, between Europe and America; the Pacific Ocean, or South Sea, &c.

Note.

Note. Those parts of the Ocean, which border upon the land, are called by various names, according to those of the adjacent countries; as the British Sea, the Irish Sea, the French and Spanish Sea.

16. A Lake is a collection of standing water, surrounded by land, and having no communication by sea; as the Lake of Geneva, the Lake of Aral, and the Caspian Sea, which is properly a Lake.

17. A Gulf is a part of the sea, almost encompassed with land, or that which runs up a great way into the land; as the Gulf of Venice, &c. but if it be very large, it is rather called an Inland Sea; as the Baltic Sea, the Mediterranean Sea, the Red Sea, or the Arabian Gulf, &c. And a small part of the sea, thus environed with land, is usually called a Bay. If it be but a very small part, or as it were, a small arm of the sea, that runs but a few miles between the land, it is called a Creek or Haven.

18. A Strait is a narrow passage lying between two shores, whereby two seas are joined together; as the Straits of Dover, between the British Channel and the German Sea; the Straits of Gibraltar, between the Atlantic and the Mediterranean Sea. These are all the necessary terms used in Geography.

The names of the several countries, seas, and all the principal divisions of the earth, the reader will find expressed upon the terrestrial globe.

To give a proper account of the produce of each country, the genius of the people, their political institutions, &c. is, properly, a subject of itself.

A DESCRIPTION OF THE GLOBES.

A globe is a spherical, or round body, whereon those circles, that are fixed, are drawn; those that are moveable, are supplied by the brass meridian, the wooden horizon, and the quadrant of altitude.

The appurtenances of a globe are, 1st. the axes, represented by a wire run through both poles. 2. A brass circle, representing the first meridian, wherein the globe turns on its axis. 3. A wooden frame, representing the horizon, on which the course of the sun is inscribed; and within which, the brazen meridian turns, by means of the notches. 4. The horary circle; it is fixed in the brazen meridian, in such a manner, as to make the pole its center. 5. The quadrant of altitude; which is a thin brass plate, screwed to the brazen meridian, and graduated with 90 degrees, answering to one-fourth part of the equator. 6. The semi-circle of position; this is a thin narrow plate of brass, exactly answering to one-half of the equator, containing 180 degrees. 7. The compass; it is a round circle, like a wheel, with 32 points issuing from its center; one of which is a flower-de-luce, and points due north: it usually stands on the pedestal of the horizon.

The things above described are common to both globes; but there are some others, which are peculiar, or proper to one sort of globe. The two colours, and the circles of latitude, from the ecliptic, belong only to the celestial globe; also the ecliptic itself does properly belong only to this globe, though it is drawn on the terrestrial, for the sake of

thofe that might not have the other by them. The equinoctial, on the celeftial globe, is always numbered into 360 degrees, beginning at the equinoctial point ♈; but on the terreftrial, it is arbitrary, where thefe numbers commence, according to the meridian of what place you intend for your firft; and the degrees may be counted, either quite round to 360, or both ways, till they meet in the oppofite part of the meridian, at 180.

The globe is of great ufe to explain geography: is very eafy and pleafant to learners, and will explain a great number of problems; fome of which are the following.

THE USE OF THE TERRESTRIAL GLOBE.

PROBLEM 1. *To find the latitude and longitude of any given place upon the globe.*

Turn the globe round its axis, till the given place lie exactly under the brazen meridian; then that degree upon the meridian which is directly over it is the latitude; likewife that degree upon the equator which is cut by the brazen meridian, is the longitude required from the firft meridian upon the globe.

EXAMPLE. What is the latitude and longitude of Mexico, Pekin in China, and Cape Horn?

	Latitude.	*Longitude.*
Mexico - -	20° N.	102° W.
Pekin - -	39° 45 N.	111° E.
Cape Horn -	57 S.	80 W.

PROB. 2. *The latitude and longitude being known, to rectify the globes fit for ufe.*

Raife the pole to the given latitude, as fuppofe London; then fix the quadrant of altitude in the zenith, and by the compafs on the pedeftal fet the globe fo that the brazen meridian may ftand due north and fouth, according to the needle, and then it is done.

EXAMPLE. By the preceding problem I find the latitude of London to be $51\frac{1}{2}$ degrees north latitude; therefore I count $51\frac{1}{2}$ degrees from the pole downwards, and turn the meridian through the notches of the horizon till thofe $51\frac{1}{2}$ degrees come exactly to the uppermoft edge of the north point in the horizon; and then is the meridian rectified to the latitude of London.

2. Next rectify the quadrant of altitude, by fcrewing the edge of the nut that is even with the graduated edge of the thin plate, to $51\frac{1}{2}$ degrees of the brazen meridian counted from the equinoctial, which is the zenith of London; and thus is your globe rectified for the folution of fuch queftions as are to be wrought thereby in that latitude.

PROB. 3.

PROB. 3. *The latitude and longitude being given, to find the place.*

Seek for the given longitude in the equator, and bring that point to the meridian; then count from the equator on the meridian, the degree of latitude given towards the arctic or antarctick pole, according as the latitude is northerly or ſoutherly; and under that degree of latitude lies the place required.

EXAMPLE. What is the name of that place, whoſe latitude is 18° N. and longitude 76½ W.? Anſwer, *Jamaica.*

PROB. 4. *To find the difference of latitude between any two given places.*

Bring each of the places propoſed ſucceſſively to the meridian, and obſerve where they interſect it; then the number of degrees upon the meridian, contained between the two interſections, will be the difference of latitude required; or, if the places propoſed are on the ſame ſide of the equator, having firſt found their latitudes, ſubtract the leſſer from the greater; but if they are on the contrary ſides of the equator, add them together; and the difference in the firſt caſe, and the ſum in the latter, will be the difference of latitude required.

EXAMPLE. What is the difference of latitude between London and Rome; alſo between Paris and Cape Bona?

The difference of latitude between London and Rome, is 90° 45″; and between Paris and Cape Bona, is 83°.

PROB. 5. *To find the difference of longitude between any two given places.*

Bring each of the places ſucceſſively to the meridian, and ſee where the meridian cuts the equator each time; the number of degrees contained between thoſe two points, if it be leſs than 180 degrees, otherwiſe the remainder to 360 degrees, will be the difference of longitude required.

EXAMPLE. What is the difference of longitude between Rome and Conſtantinople?

By working as above, you will find the difference of longitude to be 19° (which are reduced into miles, by multiplying the degrees by 60, and allowing for every minute, one mile) makes 1140 miles for their diſtance.

PROB. 6. *The day of the month being given, to find the ſun's place in the ecliptic, and his declination.*

Firſt, to find the ſun's place, look for the day of the month, given in the kalendar of months, upon the horizon; and againſt it, you will find that ſign and degree of the ecliptic, which the ſun is in. The ſun's place being thus found, look for the ſame in the ecliptic line, which is drawn upon the globe, and bring that point to the meridian; then that degree of the meridian, which is over the ſun's place, is the declination required; which is either north or ſouth, according as the ſun is in the northern or ſouthern ſigns: thus,

Sun's

	Sun's place.			Declination.	
		Deg.	*Min.*	*Deg.*	*Min.*
April 12, 1800	♈	22	19	8	42 N.
July 20, ——	♋	28	23	20	42 N.

Prob. 7. *To find the angle of position of places; or, the angle formed by the meridian of one place, and a great circle passing through both the places.*

First, rectify the globe, for the latitude and zenith of one of the given places; then bring the said place to the meridian, and turn the quadrant of altitude about, until the fiducial edge thereof cuts the other place; and the number of degrees upon the horizon, contained between the said edge and the meridian, will be the angle of position sought.

Thus, the angle of position at the Lizard, between the meridian of the Lizard, and the great circle passing from thence to Barbadoes, is 69 degrees south-westerly; but the angle of position between the same places, at Barbadoes, is but 38 degrees, north-easterly.

Scholium.—The angle of position between two places, is a different thing from what is meant by the bearings of places; the bearings of two places is determined by a sort of spiral line, called a rhumb line, passing between them in such a manner, as to make the same or equal angles, with all the meridians through which it passeth. But the angle of position is the very same thing with what we call the azimuth in astronomy; both being formed by the meridian, and a great circle, passing through the zenith of a given place, and a given point, either in the heavens, then called the azimuth, or upon the earth, then called the angle of position.

From hence may be discovered the error of that geographical paradox, viz. if a place, A, bears from another, B, due west, B shall not bear from A due east; for if it be admitted, that the east and west lines make the same angles with all the meridians through which they pass, it will follow, that these lines are the parallels of latitude: for the path described in travelling from east to west, is the continuation of the surface of a cone, whose sides are the radii of the sphere, and base the parallel of latitude of the traveller; and it is evident, that all the meridians cut the said surface at right (and therefore at equal) angles; whence it follows, that the rhumbs of east and west are the parallels of latitude; though the case may seem different, when we draw inclining lines (like meridians) upon paper, without carrying our idea any further.

Prob. 8. *To find the Antœci, Periœci, and Antipodes, to any given place.*

Bring the given place to the meridian, and having found its latitude, count the same number of degrees on the meridian, from the equator towards the contrary pole, and that will give the place of the Antœci. The globe being still in the same position, set the hour index to 12 at noon; then turn the globe about, till the index points to the lower 12; the

the place which then lies under the meridian, having the ſame latitude with the given place, is the Perioeci required. As the globe now ſtands, the Antipodes of the given place are under the ſame point of the meridian that its Antœci ſtood before.

Example. Required the Antœci, Periœci, and Antipodes, to London, whoſe latitude is 51° 30′ North.

That place which lies under the ſame meridian, and in the latitude of 51° 30′ ſouth, is the Antœci. That which lies in the ſame parallel with London, and 180° of longitude from it, is the Periœci, and the Antipodes is that place whoſe longitude from London is 180°, and latitude 51° 30′ ſouth.

Prob. 9. *The hour of the day at one place, being given, to find the correſponding hour, or what o'clock it is at that time in any other place.*

The difference of time between two places, is the ſame with their difference of longitude; wherefore, having found their difference of longitude, reduce it into time, by allowing one hour for every 15 degrees, &c. and if the place, where the hour is required, lies eaſterly from the place where the hour is given, add the difference of longitude, reduced into time, to the hour given, and the ſum will be the hour required; and if the place lies weſterly, ſubtract the difference of longitude, reduced into time, the remainder will be the hour required. Or,

Bring the place, where the hour is given, to the meridian, and ſet the hour index to the given hour, then turn the globe about, until the place, where the hour is required comes to the meridian, and the hour index will point out the hour at the ſaid place.

Thus, when it is noon at London, it is

	H.	*M.*	
At Rome - - - - -	0	52	P. M.
Conſtantinople - -	2	07	P. M.
Vera-Cruz - - -	5	30	A. M.
Pekin, in China -	7	50	P. M.

Prob. 10 *The day of the month being given, to find thoſe parts on the globe where the ſun will be vertical, or in the zenith that day.*

Having found the ſun's place in the ecliptic, bring the ſame to the meridian, and note the degree over it: then turning the globe round, all places that paſs under that degree, will have the ſun vertical that day.

Prob. 11.

Prob. 11. *A place being given in the torrid zone, to find those two days in which the sun shall be vertical to the same.*

Place the town in question this present year 1800 (suppose Goa, which is in the 16th degree of north latitude) under the brazen meridian, and observe what degree of the eliptic will pass under this latitude, when the globe is turned about, and you will have two; the 9th degree of Gemini, and the 17th degree of Leo; when the sun, therefore, shall be in the 9th degree of Gemini, on the 30th of May, or in the 17th degree of Leo, on the 10th of August, it will be perpendicular to Goa.

Prob. 12. *To find where the sun is vertical at any given time assigned, or the day of the month, and the hour, at any place, being given, to find in what place the sun is vertical at that very time.*

Having found the sun's declination, and brought the place (suppose London) to the meridian, set the index to the given hour, and turn the globe about, until the index points to 12 at noon; which being done, that place upon the globe, which stands under the point of the sun's declination upon the meridian, has the sun that moment in the zenith.

Prob. 13. *The day, and hour of the day, at any one place being given, to find all those places upon the earth where the sun is then rising, setting, culminating (or on the meridian); also when it is day, night, twilight, dark night, and midnight; where the twilight then begins, and where it ends; the height of the sun, in any part of the illuminated hemisphere, also his depression in the obscure hemisphere.*

Having found the place where the sun is vertical at the given hour, rectify the globe for the latitude, and bring the said place to the meridian.

Then all those places, that are in the western semi-circle of the horizon, have the sun rising at that time.

Those in the eastern semi-circle, have it setting.

To those who live under the upper semi-circle of the meridian, it is 12 o'clock at noon. And,

Those who live under the lower semi-circle of the meridian, have it at midnight.

all those places that are above the horizon, have the sun above them, just so much as the places themselves are distant from the horizon; which height may be known by fixing the quadrant of altitude in the zenith, and laying it over any particular place.

In all those places, that are 18 degrees below the western side of the horizon, the twilight begins in the morning; and in all those places, that are 18 degrees below the eastern side of the horizon, the twilight ends, and the total darkness begins.

The twilight is in all those places, whose depression below the horizon does not exceed 18 degrees. And,

All those places, that are lower than 18 degrees, have dark night.

Prob. 14.

PROB. 14. *The day of the month being given; to ſhew, at one view, the length of day and night in all places upon the earth at that time; and to explain, how the viciſſitudes of day and night are really made by the motion of the earth round her axes in 24 hours; the ſun ſtanding ſtill.*

The ſun always illuminates one half of the globe, or that hemiſphere which is next towards him, while the other remains in darkneſs; and if we elevate the globe, according to the ſun's place in the ecliptic (by prob. 13.) it is evident, that the ſun (he being at an immenſe diſtance from the earth) illuminates all that hemiſphere, which is above the horizon; the wooden horizon itſelf will be the circle terminating light and darkneſs; and all thoſe places, that are below it, are wholly deprived of the ſolar light.

The globe ſtanding in this poſition, thoſe arches of the parallels of latitude, which ſtand above the horizon, are the diurnal arches, or the length of the day in all thoſe latitudes at that time of the year, and the remaining part of thoſe parallels, which are below the horizon, are the nocturnal arches, or the length of the night in thoſe places. The length of the diurnal arches may be found, by counting how many hours are contained between the two meridians, cutting any parallel of latitude in the eaſtern and weſtern parts of the horizon.

In thoſe places that are in the weſtern ſemi-circle of the horizon, the ſun appears riſing; for, the ſun ſtanding ſtill in the vertex (or above the braſs meridian) appears eaſterly, and 90 degrees diſtant from all thoſe places that are in the weſtern ſemi-circle of the horizon; and therefore, in thoſe places he is then riſing.—Now, if we pitch upon any particular place upon the globe, and bring it to the meridian, and then bring the hour index to 12 at noon, and afterwards turn the globe about, until the aforeſaid place be brought to the weſtern ſide of the horizon; the index will then ſhew the time of ſun-riſing at that place. Suppoſing the hour circle numbered the contrary way, then turning the globe gradually about from weſt to eaſt, and minding the hour index, we ſhall ſee the progreſs made in the day every hour, in all latitudes upon the globe, by the real motion of the earth round its axes; until, by their continual approach to the braſs meridian (over which the ſun ſtands ſtill all the while) they at laſt have noon-day, and the ſun appears at the higheſt; and then by degrees, as they move eaſterly, the ſun ſeems to decline weſtward, until, as the places ſucceſſively arrive in the eaſtern part of the horizon, the ſun appears to ſet in the weſtern; for the places that are in the horizon, are 90 degrees diſtant from the ſun.

We may obſerve, that all places upon the earth, that differ in latitude, have their days of different length (except when the ſun is in the equinoctial) being longer or ſhorter, in proportion to what part of the

parallels ſtand above the horizon: thoſe that are in the ſame latitude, have their days of the ſame length: but have them commence ſooner or later, according as the places differ in longitude.

PROB. 15. *The latitude of any place, not exceeding 66½ degrees, and the day of the month being given, to find the time of ſun-riſing and ſetting, and the length of the day and night.*

Having rectified the globe according to the latitude, bring the ſun's place to the meridian, and put the hour index to 12 at noon; then bring the ſun's place to the eaſtern part of the horizon, and the index will ſhew the time when the ſun riſes; again, turn the globe until the ſun's place be brought to the weſtern ſide of the horizon, and the index will ſhew the time of ſun-ſetting.

The hour of ſun-ſetting, doubled, gives the length of the day; and the hour of ſun-riſing, doubled, gives the length of the night.

EXAMPLE. Let it be required to find when the ſun riſes and ſets at London on the 20 of April, 1800.

Rectify the globe for the latitude of London, and having found the ſun's place, correſponding to April the 20th, viz. ♉ 0¼ degrees, bring ♉ 0¼ degrees to the meridian, and ſet the index to 12 at noon; then turn the globe about, till ♉ 0¼ degrees be brought to the eaſtern part of the horizon, and you will find the index point to 4¾ hours, the time of ſun-riſing: again, bring the ſun's place to the weſtern part of the horizon, and the index will point to 7¼ hours, which is the time of ſun-ſetting.

PROB. 16. *The latitude, the ſun's place, and his altitude, being given, to find the hour of the day, and the ſun's azimuth from the meridian.*

Having rectified the globe for the latitude, the zenith, and the ſun's place, turn the globe and the quadrant of altitude, ſo that the ſun's place may cut the given degree of altitude; then the index will ſhew the hour, and the quadrant will cut the azimuth in the horizon.

Thus, if at London, on the 10th of Auguſt, the ſun's altitude be 36 degrees in the forenoon, the hour of the day will be 9, and the ſun's azimuth about 58 degrees from the ſouth part of the meridian.

PROB. 17. *The latitude, the ſun's place, and his azimuth, being given, to find his altitude, and the hour.*

Rectify the globe for the latitude, the zenith, and the ſun's place; then put the quadrant of altitude to the ſun's azimuth in the horizon, and turn the globe till the ſun's place meets the edge of the quadrant; then the ſaid edge will ſhew the altitude, and the index point to the hour. Thus,

On

On May the 10th, at London, when the ſun is due eaſt, his altitude will be about 24 degrees, and the hour 7 in the morning; and when his azimuth is 60 degrees ſouth-weſterly, the altitude will be about $44\frac{1}{2}$ degrees, and the hour about $2\frac{3}{4}$ in the afternoon.

PROB. 18. *The latitude, the ſun's altitude, and his azimuth being given; to find his place in the ecliptic, and the hour.*

Rectify the globe for the latitude and zenith, and ſet the edge of the quadrant to the given azimuth, then turning the globe about, that point of the ecliptic, which cuts the altitude, will be the ſun's place. Keep the quadrant of altitude in the ſame poſition, and having brought the ſun's place to the meridian, and the hour index to 12 at noon, turn the globe about till the ſun's place cuts the quadrant of altitude, and then the index will point the hour of the day.

PROB. 19. *The declination and meridian altitude of the ſun, or of any ſtar, being given, to find the latitude of the place.*

Mark the point of declination upon the meridian, according as it is either north or ſouth from the equator, then ſlide the meridian up or down in the notches, till the point of declination be ſo far diſtant from the horizon, as is the given meridian altitude; that elevation of the pole will be the latitude. Thus,

If the ſun's, or any ſtar's meridian altitude, be 50 degrees ſouth, and its declination $11\frac{1}{2}$ degrees north, the latitude will be $51\frac{1}{2}$ degrees north.

PROB. 20. *The day and hour of a lunar eclipſe being known, to find all thoſe places upon the globe, in which the ſame will be viſible.*

Find, by Prob. 12. where the ſun is vertical at the given hour, and bring that point to the zenith, then the eclipſe will be viſible in all thoſe places that are under the horizon, or if you bring the Antipodes to the place where the ſun is vertical into the zenith, you will have the places where the eclipſe will be viſible above the horizon.

Note. Becauſe lunar eclipſes continue ſometimes for a long while together, they may be ſeen in more places than one hemiſphere of the earth; for, by the earth's motion round its axes, during the time of the eclipſe, the moon will riſe in ſeveral places, after the eclipſe began.

When an eclipſe of the ſun is central, if you bring the place where the ſun is vertical at that time into the zenith, ſome part of the eclipſe will be viſible in moſt places within the upper hemiſphere; but, by reaſon of the ſhort duration of ſolar eclipſes, and the latitude which the moon has at that time (though but ſmall) there in no certainty in determining the places where thoſe eclipſes will be viſible by the globe, but recourſe muſt be had to calculations.

THE USE OF THE CELESTIAL GLOBE.

PROB. 1. *To find the right aſcenſion and declination of the ſun, or any fixed ſtar.*

Bring the ſun's place, in the ecliptic, to the meridian, then that degree of the equator, which is cut by the meridian, will be the ſun's right

 aſcenſion;

ascension; and that degree of the meridian, which is exactly over the sun's place, is the sun's declination.

After the same manner, bring the place of any fixed star to the meridian, and you will find its right ascension in the equinoctial, and declination on the meridian. Thus, the right ascension and declination of the sun, 1800,

		Right Ascension.		Declination.	
		Deg.	Min.	Deg.	Min.
Is on	Jan. 20 - - - -	21	29	20	7
	Mar. 25 - - - -	1	32	1	50
	July 10 - - - -	6	43	22	16
	Nov. 15 - - - -	14	50	18	32

	Right Ascension.	Declination.
	Degrees.	Degrees.
Aldebaran - - - - - -	$4\frac{1}{2}$	16 N.
Syrius, or the dog-star - -	$6\frac{1}{2}$	$16\frac{1}{2}$ S.

Note. The insensible change in the longitude, right ascension, and declination of the fixed stars, made by their slow motion, parallel to the ecliptic, is not worth observation in this place. The right ascension and declination of the sun varies every day.

PROB. 2. *The right ascension and declination of any star being given, to find the star upon the globe.*

Bring the given degree of right ascension on the equator to the meridian, then look under the degree of declination on the meridian, and you will find the star at the meridian, under the given degree of declination.

Thus, suppose I wanted to find Aldebaran, whose right ascension is $4\frac{1}{2}°$, and his declination 16° N. I first bring $4\frac{1}{2}°$ of the equinoctial to the meridian; and looking under 16° N. declination on the meridian, I find Aldebaran: proceed in the same manner for any other star.

PROB. 3. *To find the longitude and latitude of a given star.*

Having brought the solstitial colour to the meridian, fix the quadrant of altitude over the proper pole of the ecliptic, whether it be north or south; then turn the quadrant over the given star, and the arch contained between the star and the ecliptic, will be the latitude, and the degree cut on the ecliptic will be the star's longitude.

Thus you will find the latitude of Arcturus to be 31° N. and the longitude 200° from ♈, or 20° from ♎. Also the latitude of Fomalhaunt, in the southern Fish, 21° S. and longitude $299\frac{1}{2}$ degrees, or ♒ 29° 30′.

The distance between two stars, or the number of degrees contained between them, may be found, by laying the quadrant of altitude over each of them, and counting the number of degrees intercepted, after the same manner, as we found the distance between two places on the terrestrial globe,

PROB. 4.

PROB. 4. *To find the rising and setting of the stars; and the point of the compass any star rises or sets upon in any latitude, and on any day of the year.*

Rectify the globe, and bring the sun's place to the meridian, then turn the globe till the given star comes to the eastern verge of the horizon, and the index will point to the time of rising, and the horizon will shew the point it rises upon. Turn it to the west, and the index will point to the time of setting, and the horizon will shew you the point it sets upon.

Proceed thus, and you will find that Syrius in latitude $51\frac{1}{2}°$ rises on the 10th of August, and sets on the 15th of November.

Note. The stars rise and set every day on the same point of the compass, though at contrary hours.

PROB. 5. *To find the time, viz. how many hours any star continues above the horizon, from its rising to its setting, in any latitude.*

Rectify the globe, then bring the star to the eastern verge, and note the time of rising; then turn the globe to the western side, and the number of hours that passed through the dial-plate, tells you the continuance of that star above the horizon. Thus,

I find Aldebaran, at London, on the 1st of January 1800, continues up from the time of his rising about 7 hours and 28 minutes.

PROB. 6. *The latitude of the place, and the day of the month being given, to know where to find any star, or tell the name of any star, at pleasure.*

Rectify the globe for the day, and turn it till the index points to the given hour; then, by a quadrant, take the height of the required star, or, for want of this (in a common way of guessing) observe well what part of the heavens it is in, viz. whether E. N. E. S. W. or the like; as also its height, as near as you can guess. This being done, set the globe in due order, for the day and hour, and you will find the same star on the globe; and, by applying the quadrant, you will find the exact point of the compass, and the real height the star then was; which, though not, perhaps, near to what you guessed it at, yet, if it be any noted star, you may assure yourself it was right; as there is no other star of note near it about that height, and upon the same point.

SCHOLIUM.—The globe being rectified to the latitude of any place, if you turn it round its axes, all those stars that do not go below the horizon, during a whole revolution of the globe, never set in that place; and those that do not come above the horizon, never rise.

PROB. 7.

Prob. 7. *To find the time of the achronical rising and setting of any star.*

Bring the sun's place, for the given day, to the western side of the horizon, and all those stars that are on, or near the eastern side of the horizon, rise achronically; and those on the western verge of the horizon, set achronically.

Thus I find, that Aldebaran rises achronically in latitude $51\frac{1}{2}°$ on the 8th of December, and sets on the 21st of May.

Prob. 8. *To find the cosmical rising and setting of the stars, in any latitude.*

Rectify the globe for the latitude, and bring the sun's place to the eastern side of the horizon, for the given day; then all those stars, cut by the eastern verge of the horizon, rise cosmically. The globe still remaining in the same position, look at the western verge, or edge of the horizon; and all those stars cut by it, or that are very near it, set on that day cosmically.

Thus I find, that Syrius, rises cosmically on the 10th of August. Also two stars in Eridanus, Essengue in Lyra, &c. set cosmically.

Again, for the cosmical setting. Turn the globe, till the star comes to the western side of the horizon, and observe the degree of the ecliptic; then cut by the eastern side of the horizon, for that will answer to the day of the cosmical setting.

Prob. 9. *To tell the heliacal rising or setting of the stars.*

Rectify the globe, and bring the given star to the eastern verge of the horizon; then fix the globe, and turn the quadrant to the western side, till 12 degrees of the quadrant touches the ecliptic; this done, note the degree of the ecliptic that is cut by 12 degrees of the quadrant, on the western side (for then will the real place of the sun be depressed 12 degrees on the eastern side) and that degree, sought in the calendar, gives the heliacal rising. The same is to be observed with the quadrant on the eastern side for the heliacal setting. Thus you will find, Arcturus rises heliacally on the 8th of October, and sets heliacally on the 2nd of December.

Prob. 10. *The sun's declination and hour, when he is due east, given; to find the latitude, viz. the elevation of the pole.*

Rectify the globe to the same latitude as the given number of degrees of declination, and fix the quadrant in the zenith; then convert the hours, that the sun is due east before or after six o'clock, into degrees; and count the same number of degrees on the horizon, from the east point, southward,

ſouthward, and bring the quadrant to that degree of the horizon; ſo ſhall the degree on the quadrant, that is cut by the equator, be the complement of latitude; which taken from 90°, gives the latitude itſelf, or height of the pole.

EXAMPLE. Sailing May 31, I made an obſervation, that the ſun was due eaſt about ſeven minutes paſt ſeven in the morning, and his declination 20° N. I demand what latitude I was in?

Proceed as above directed, and you will find the latitude to be $51\frac{1}{2}$° nearly.

PROB. 11. *Having the ſun's azimuth at ſix o'clock, and declination; to find the latitude.*

As many degrees as are contained in the azimuth given, ſo much elevate the pole, and fix the quadrant in the zenith, and bring ♈ to the meridian; this done, count on the quadrant upwards, the complement of the ſun's declination, to ninety, and bring that degree to the equator; then the degree of the horizon, cut by the quadrant, ſhall be the complement of latitude, counted from the ſouth point, or elſe from the north, as it may happen; and the remainder to 90, is the latitude required; or otherwiſe, the degrees counted from the other two cardinal points, either E. or W. as it may happen, will give the latitude.

Thus, I find the ſun's azimuth, at 6 o'clock, to be 12° 15′, and his declination 20° 10′; What is the latitude? Work as taught above, and you will have the anſwer $38\frac{1}{2}$° complement; that is, $51\frac{1}{2}$° latitude required.

PROB. 12. *The ſun's altitude E. and his declination given; to prove the elevation of the pole.*

Elevate the pole to the complement of the ſun's altitude at E. and fix the quadrant in the zenith, and bring ♈ to the meridian; then number, on the quadrant of altitude, the degree of declination; and bringing the ſame to the equator, obſerve what degree the quadrant cuts the equator in; for its complement to 90° is the height of the pole.

EXAMPLE. The ſun's declination is 20° 10′ N. his altitude at eaſt (at London) is nearly 26°; I would know, whether the ſuppoſed latitude ($51\frac{1}{2}$°) agrees with the operation?

Firſt, I ſubtract 26° from 90°, and there remains 64° complement of altitude, and I elevate the pole accordingly; then I bring ♈ to the meridian, and cauſe 20° 10′ on the quadrant to cut the equator, and find it nearly $38\frac{1}{2}$° the complement of latitude required; which ſubtracted from 90°, gives $51\frac{1}{2}$°, the real latitude of the place.

PROB.

Prob. 13. *To find the place of any planet upon the globe; and so, by that means, to find its place in the heavens. Also to find at what hour any planet will rise or set, or be on the meridian, at any day in the year.*

You must first seek in an ephemerides, for the place of the planet proposed on that day; then mark that point of the ecliptic, either with chalk, or by sticking on a little black patch; and then, for that night, you may perform any problem, as before by a fixed star.

Prob. 14. *To find all that space upon the earth, where an eclipse of one of the satellites of Jupiter will be visible.*

Having found that place upon the earth, in which the sun is vertical at the time of the eclipse, by problem 12, on the terrestrial globe, elevate the globe according to the latitude of the said place, then bring the place to the meridian, and set the hour index to 12 at noon. If Jupiter be in consequence of the sun, draw a line with black lead, or the like, along the eastern side of the horizon; which line will pass over all those places where the sun is setting at the time; then count the difference between the right ascension of the sun and that of Jupiter, and turn the globe westward until the hour index points to this difference, keeping the globe from turning round its axes, and elevate the meridian according to the declination of Jupiter. The globe being in this position, draw a line along the eastern side of the horizon, the space between this line and the line before drawn, will comprehend all those places of the earth where Jupiter will be visible, from the setting of the sun to the setting of Jupiter.

But if Jupiter be in antecedence of the sun (i. e. rises before him, having brought the place where the sun is vertical to the zenith, and put the hour index to 12 at noon, draw a line on the western side of the horizon, elevating the globe according to the declination of Jupiter, and turn it about eastwards until the index points to so many hours distant from noon, as is the difference of right ascension of the sun and Jupiter. The globe being in this position, draw a line along the western side of the horizon; then the space contained between this line and the other line last drawn, will comprehend all those places upon the earth where the eclipse is visible, between the rising of the sun and Jupiter.

ALGEBRA.

ALGEBRA.

PART VIII.

SECTION LXXXI.

ALGEBRA is a ſcience which teaches, in a general manner, the relations and compariſons of abſtract quantities; by means wherof, ſuch queſtions are reſolved whoſe ſolutions would be ſought in vain from common arithmetic.

There are two kinds of Algebra, Numeral, and Specious or Literal.

Numeral Algebra is that wherein all the given quantities are repreſented by numbers, and the unknown quantity by a letter or other ſymbol, as was uſed by the ancients.

Specious or Literal Algebra, is that wherein all the quantities as well known as unknown, are expreſſed by letters of the alphabet; the given ones, for diſtinction ſake, being uſually denoted by the initial letters, a, b, c, d, &c. and the unknown or required ones by the final letters, u, w, x, y, &c. There are, likewiſe, certain ſigns and characters made uſe of to ſhew the relation and dependence of quantities one upon another, which are the foundation of this celebrated ſcience. See the mathematical abbreviations facing page 1.

When the reader has a clear underſtanding of what the ſigns and characters uſed in algebra are intended to expreſs, it will be neceſſary to inform him, that when any quantity is to be taken more than once, the number is to be prefixed, which ſhews how many times it is to be taken; thus, $5a$ denotes that the quantity a is to be taken five times, and $3ab$ ſtands for three times ab, or the quantity which ariſes by multiplying ab by 3; alſo $8\sqrt{a^2+b^2}$ ſignifies that $\sqrt{a^2+b^2}$ is to be taken eight times, and ſo of others.

The numbers thus prefixed are called coefficients, and that quantity which ſtands without a coefficient is always underſtood to have an unit prefixed, or to be taken once, and no more; thus, a is the ſame as $1a$.

Thoſe quantities are ſaid to be alike, that are expreſſed by the ſame letters under the ſame powers, or which differ only in their coefficient; thus $3bc$, $8bc$, $10bc$, are like quantities; and the ſame is to be underſtood of the radicals, $2\sqrt{\frac{b+c}{a}}$ and $8\sqrt{\frac{b+c}{a}}$. But unlike quantities, are thoſe which are expreſſed by different letters, or by the ſame letters under different powers; thus $2ab$, $3abc$, $8ab^2$, and $5ba^2$, are all unlike.

When

When a quantity is expressed by a single letter, or by several single letters joined together in multiplication, without any sign between them, as a, $3a$ or $4ab$, it is called a simple quantity.

But when these are connected by the signs $+$ or $-$, as $b+dc$, $ab-d$, $dy+az$, they are called compound quantities, whereof the simple quantities, b, d, ab, d, dy and az, are called the terms or members.

The letters by which any simple quantity is expressed may be ranged according to any order at pleasure, and yet the signification continue the same; thus ab may be wrote ba, for ab denotes the product of a by b, and ba the product of b by a; for it is well known, that when two numbers are to be multiplied together, it matters not which of them is made the multiplicand, nor which the multiplier; the product either way comes out of the same. In like manner it will appear, that abc, acb, bac, bca. cab, and cba, all express the same thing (as will be demonstrated in its proper place) but it is sometimes convenient, in long operations, to place the several letters according to the order which they stand in the alphabet.

Likewise the several members, or terms, of which any quantity is composed, may be disposed according to any order at pleasure, and yet the signification be no ways altered thereby; thus $a-2ab+5a^2b$ may be wrote $a+5a^2b\ 2ab$, or $-2ab+a+5a^2b$, &c. for all these represent the same thing; that is, the quantity which remains, when, from the sum of a and $5a^2b$, the quantity $2ab$ is deducted.

When any calculation is to be made, it is done either by Addition, Subtraction, Multiplication, or Division of quantities, which four fundamental rules I shall now proceed to explain.

ADDITION.

Addition, in Algebra, is performed by connecting the quantities by their proper signs, and joining into one sum, such as can be united; for the more ready performing of which, observe the following rules.

RULE 1. If the quantities are alike, and have all the same signs, add the coefficient of those terms together, and to their sum join the letters common to each term, prefixing the common sign.

EXAMPLES.

	E. 1.	E. 2.	E. 3.
To	$3a$	$5ab$	$18xyz$
Add	$4a$	$3ab$	$12xyz$
Sum	$7a$	$8ab$	$30xyz$

In example 1, the coefficients are 4 and 3, which, added together, make 7; to which joining a, the quantity, it is $7a$; and no sign being prefixed to either $3a$ or $4a$, the affirmative sign is understood as prefixed to both; hence $7a$ or $+7a$ is the sum required.

If

If there are two or more quantities connected by the signs + or —, and are like two or more quantities connected by the signs + or —, they are added as in the former examples, taking due care that the quantities, which compose their sum, are connected with their proper signs, according to rule 1.

E. 4.		E. 5.
To	$4a+7b$	$21ab+2cd$
Add	$6a+2b$	$3ab+3cd$
Sum	$10a+9b$	$24ab+5cd$

In example 4, there is $4a+7b$, to be added to $6a+2b$; the quantities being disposed as in the example, it follows from the former examples, that $6a$ being added to $4a$, makes $10a$, and $2b$ added to $7b$ makes $9b$; and as $7b$ and $2b$ have both the affirmative sign, to $10a$ connect $9b$ with the sign +; hence $10a+9b$ is the sum required.

E. 6. To - - - $2\sqrt{ab}+7\sqrt{bc}$
Add - - - $3\sqrt{ab}+2\sqrt{bc}$
$6\sqrt{ab}+9\sqrt{bc}$

Sum will be - - $11\sqrt{ab}+18\sqrt{bc}$

In example 6, $6\sqrt{ab}$; $3\sqrt{ab}$; $2\sqrt{ab}$, added together, make $11\sqrt{ab}$; and $9\sqrt{bc}$; $2\sqrt{bc}$; $7\sqrt{bc}$, added together, make $18\sqrt{bc}$; then connecting them with the affirmative sign +, we have $11\sqrt{ab}+18\sqrt{bc}$, the sum required.

E. 7.		E. 8.
To	$3yb+7a$	$-14y+d$
Add	$2yb+a$	$-y+2d$
Sum	$5yb+8a$	$-15y+3d$

In example 7, when you come to $+7a$ to $+a$, there being no coefficient prefixed to a, unity or 1 in such cases is always the coefficient; and by what has been already taught, $+7a$ being added to $+a$, the sum is $+8a$, as in the example.

In example 8, when $2d$ is added to d, the sum is $3d$, for the same reason.

RULE 2. When in the quantities to be added, there are like terms, whereof some are affirmative, and others negative, add together the affirmative terms (if there be more than one) and do the same by the negative ones; then take the difference of the two sums (not regarding the signs) by subtracting the coefficient of the lesser from that of the greater, and joining the letters common to each; to which difference prefix the sign of the greater.

 It

It is of no signification whether the quantity that has the greatest coefficient stands above or below.

	E. 1.	E. 2.	E. 3.
To	$6a$	$18b$	$36ad$
Add	$-3a$	-12	$-8ad$
Sum	$3a$	$6b$	$28ad$

In example 1, the coefficient 3, subtracted from 6, leaves 3, to which joining a, it is $3\,a$, and the sign of 6, the greatest coefficient, is affirmative; therefore $3\,a$, or $+\,3\,a$ is the sum required.

	E. 4.	E. 5.	E. 6.
To	$-18e$	$8ac$	$5ax$
Add	$6e$	$-14ac$	$-ax$
Sum	$-12e$	$-6ac$	$4ax$

In example 4, the coefficient 6, subtracted from 18, leaves 12, to which joining e, it is $12\,e$; but the sign of 18, the greatest coefficient, being —, prefix that sign to $12\,e$, then is $-\,12\,e$ the sum required.

E. 7.

$$\begin{array}{rrrr} 12abc & -16abd & +25acd & -72bcd \\ 16abc & +12abd & +20acd & -18bcd \\ -13abc & +26abd & -15acd & +12bcd \\ 32abc & -18abd & -10acd & +16bcd \\ \hline \text{Sum } 47abc & +4abd & +20acd & -62bcd \end{array}$$

In example 7 the coefficients 32, 16, 12, which have the affirmative sign being added together, make 60, then subtracting 13, the coefficient which hath the negative sign, leaves 47, to which joining $a\,b\,c$, it is $47\,a\,b\,c$: proceed thus through all the quantities as the rule directs, and you will find the sum to be $47\,a\,b\,c + 4\,a\,b\,d + 20\,a\,c\,d - 62\,b\,c\,d$, as in the example.

E. 8.

$$\begin{array}{l} \frac{5a}{b} - \frac{3cc}{a} + 7\sqrt{\frac{bc}{a}} - 9\sqrt{\frac{ab+cc}{a}} \\ \frac{8a}{b} + \frac{7cc}{a} - 12\sqrt{\frac{bc}{a}} + 6\sqrt{\frac{ab+cc}{a}} \\ \hline \text{Sum } \frac{13a}{b} + \frac{4cc}{a} - 5\sqrt{\frac{bc}{a}} - 3\sqrt{\frac{ab+cc}{a}} \end{array}$$

In example 8, and all others, where fractional and radical quantities are concerned, every such quantity, exclusive of its coefficient, is to be treated in all respects like a simple quantity expressed by a single letter.

RULE 3. When, in the quantities to be added, there are terms without others like to them, write them down one after the other, with the same coefficients and signs they have in the example.

The

The quantities may be ſet in any order; that is, any quantity may be ſet firſt, in the middle, or laſt, as it is not material how they are ranged, ſo as they are but connected with their proper ſigns.

	E. 1.	E. 2.
To	$3a$	$a+d$
Add	$4d$	$2x$
Sum	$3a+4d$	$a+d+2x$

E. 1. The quantities or letters being unlike, I place down $3a$ and becauſe $4d$ has the ſign $+$, therefore after the $3a$ put $+4d$, ſo is $3a+4d$ the ſum required.

E. 2. Having put down a, after that put $+d$, and after that $+2x$; ſo is $a+d+2x$ the ſum required.

	E. 3.	E 4.
To	$4a-14m$	$aa+bb$
Add	$3x+5z$	$z-7y$
Sum	$4a-14m+3x+5z$	$aa+bb+z-7y$

To	$8b+6y-a$
Add	$3e-4c+2x$
Sum	$8b+6y-a+3e-4c+2x$

Note. In rule 3, where the quantities are unlike, it is plain that ſuch quantities cannot be united into one, or otherwiſe added, than by their ſigns; thus, for example, let a be ſuppoſed to repreſent a crown, and b a ſhilling, then the ſum of a and b can be neither $2a$ nor $2b$, that is, neither two crowns nor two ſhillings, but one crown $+$ one ſhilling, or $a+b$.

Here follows a few examples, wherein all the three foregoing rules are promiſcuouſly uſed.

	E. 1.	E. 2.
To	$3a-7d+x$	$-4a+7m-21z$
Add	$2a+9d$	$11a-12m+2y$
Sum	$5a+2d+z$	$7a-5m-21x+2y$

E. 1. $2a$ added to $3a$ makes $5a$, and $-7d$ added to $9d$ makes $2d$, by rule 2; and there being no quantity like z, that muſt be placed by itſelf, by rule 3; and connecting theſe quantities with their proper ſigns, we have $5a+2d+z$, the ſum required.

$$\text{Add}\begin{cases}5\sqrt{ax}-8\sqrt{aa-xx}+12\sqrt{aa+4xx}\\ 8\sqrt{ax}+15\sqrt{aa-xx}-8\sqrt{aa+4xx}\\ 6\sqrt{aa}-7\sqrt{aa-xx}+10\sqrt{aa+4xx}\end{cases}$$

$$\text{Sum}\quad 19\sqrt{ax}\quad \div \quad +14\sqrt{aa+12xx}$$

E. 3. The coefficients 6, 8, and 5, being added together, make 19; then joining $\sqrt{ax}$, it makes $19\sqrt{ax}$; and -7 and -8 added to 15, the ſum is 0; and -8 added to $12+10$, the ſum is 14; and prefixing the

the ſign of the two greateſt coefficients, and $\sqrt{aa}$ being joined, we have $+14\sqrt{aa}$, likewiſe $4+4+4=12$, to which joining the quantity xx, and connecting the quantities with their proper ſigns, we have $19\sqrt{ax}+14\sqrt{aa}+12xx$, the ſum required.

$$\begin{array}{lll} \text{E. 4.} & \text{To} & -14m+30+\ 8a \\ & \text{Add} & -\ 8a-22+16m \\ \hline & \text{Sum} & 2m+\ 8 \end{array}$$

In example 4, $-14m$ added to $16m$, the ſum is $2m$, and $-22m$ added to 30, the ſum is 8, and $8a$ added to $-8a$, the ſum is 0; hence $2m+8$ is the ſum required.

Note. In this example, the ſame quantities are not ſet under one another, that the learner may ſee it is not material how they are placed, if the quantities are alike, for they muſt be added the ſame as if they ſtood one under the other.

That this rule may be the better underſtood by the learner, I ſhall give one more example, wherein let us ſuppoſe a to denote a pound ſterling, b a ſhilling, and c a penny, thus,

$$\begin{array}{lll} & \text{E. 5.} & \text{£.}\ \ s.\ \ d. \\ \text{To} & 7a-9b+5c = & 7-9+5 \\ \text{Add} & 3a+5b-9c = & 3+5-9 \\ \hline \text{Sum} & 10a-4b-4c = & 10-4-4 \end{array}$$

By this example may be underſtood, why addition is changed into ſubtraction, when the ſigns differ, and the ſum of the greater quantity is prefixed to the remainder: for in the ſum of 10*l.* nine ſhillings are wanting; therefore, if 5*s.* are added, the defect is leſſened, and brought to 4*s.*; and becauſe there are not 5 whole ſhillings, but 5*s.*—9*d.* to be added, the ſum 10*l.*—4*s.* exceeds the truth by 9*d.* which are therefore to be ſubtracted. Now in the upper number, to which the lower is to be added, there are 5*d.* theſe may be ſubtracted, and the other 4*d.* in the lower number, ſet down as wanting, or negative quantities; and this was the way the rule was firſt diſcovered.

SUBTRACTION.

Subtraction in algebra is performed by the following general

RULE. Change all the ſigns of thoſe quantities which are to be ſubtracted, or conceive them to be changed; then add theſe quantities to the others, according to the ſeveral rules of addition, and you will have the difference or remainder required.

$$\begin{array}{llll} & \text{E. 1.} & \text{E. 2.} & \text{E. 3.} \\ \text{From} & 6a & -8b & 12ab \\ \text{Take} & 3a & -2b & -6ab \\ \hline \text{Remains} & 3a & -6b & 18ab \end{array}$$

In example 1, there is $3a$, having the ſign $+$, to be ſubtracted, which being made, or ſuppoſed to be made $-$, then by the general rule $6a$ is to be added to $-3a$, the ſum of which is $3a$ by rule 2 of addition, and is the remainder required.

E. 4.

	E. 4.	E. 5.	E. 6.	E. 7.
From	$5\,ac$	$-\,ab$	$-7\,ad$	$5\,xy$
Take	$-\,ac$	$-\,5\,ab$	$+\,ad$	xy
Remains	$6\,ac$	$4\,ab$	$-8\,ad$	$4\,xy$
Proof	$5\,ac$	$-\,ab$	$-7\,ad$	$5\,xy$

From the four preceding examples, it may be eaſily perceived, that ſubtraction in algebra is proved as in common arithmetic, by adding the remainder to the qnantity which is ſubtracted.

	E. 8.	E. 9.
From	$-5\,zy-2\,am$	$14\,a-5\,b$
Take	$3\,zy+4\,am$	$-3\,a-5\,b$
Remains	$-\,8\,zy-6\,am$	$17\,a \quad 0$

If the quantities to be ſubtracted are unlike thoſe from which the ſubtraction is to be made, ſet down theſe with the ſame ſigns and coefficients they have in the example; after which, place the quantities to be ſubtracted with their coefficients, but change their ſigns.

	E. 10.	E. 11.
From	$3\,ac$	$5\,x^3+8\,xz$
	bd	$8\,x^2-\,x$
Remains	$3\,ac-bd$	$5\,x^3+8\,xz-8\,x^2+x$

In Example 10. having put down $3\,ac$, after which put $-\,bd$, the quantity to be ſubtracted being $+\,bd$, and $3\,ac-bd$ is the remainder required.

MULTIPLICATION.

In Multiplication there is one general rule for the ſigns, viz. when the ſigns of the factors are alike (that is, both $+$ or both $-$) the ſign of the product is $+$; but when the ſigns of the factors are unlike, the ſign of the product is $-$. This general rule will reſolve itſelf into four particular caſes, which I ſhall illuſtrate ſeparately in ſimple quantities.

Case 1. When any poſitive quantity, as $+a$, is multiplied by a poſitive quantity $+b$, the meaning is, that $+a$ is to be taken ſo many times as there are units in b, and the product is evidently b times a, or $b\,a$.

EXAMPLES,

Multiply	$+\,a$	$3\,a$	$5\,bx$	$9\,dc$
By	$+\,b$	$6\,b$	7	$8\,x$
Product	$+\,ba$	$18\,ab$	$35\,bx$	$72\,xdc$

In example 1, having joined the letters $b\,a$, and each of them having the affirmative ſign, therefore, by the rule, $b\,a$ or $+\,b\,a$ is the product required: and ſo of others.

Case 2. When $-a$ is multiplied by b, then $-a$ is to be taken as often as there are units in b, and the product muſt be b times $-a$, or $-ba$.

EXAMPLES.

EXAMPLES.

Multiply	$-a$	$-2a$	$-bx$	$-9dc$
By	b	$4b$	7	$3z$
Product	$-ba$	$-8ab$	$-7bx$	$-27dcz$

In example 1, case 2, the product of a by b is ba, and as the sign of a is $-$, and that of b is $+$, therefore to ba prefix the sign $-$, so is $-ba$ the product required.

CASE 3. As multiplication by a positive number implies a repeated addition, so multiplication by a negative implies a repeated subtraction; and therefore, when a or $+a$ is to be multiplied by $-b$, it means only, that $+a$ is to be subtracted as often as there are units in b, and therefore the product being negative, must also be $-ba$; see the following

EXAMPLES.

Multiply	$+a$	$3a$	$6bc$	$9acd$
By	$-b$	$-4b$	-8	$-4x$
Product	$-ba$	$-12ab$	$-48bc$	$36acdx$

CASE 4. When $-a$ is to be multiplied by $-b$, then $-a$ is to be subtracted as often as there are units in b; but to subtract $-a$ is equivalent to adding $+a$; therefore this case is the same in effect as case 1, and the product is evidently $+ba$, or ba.

EXAMPLES.

Multiply	$-a$	$-4a$	$-6by$	$-8xy$
By	$-b$	$-3b$	-9	$-5a$
Product	ba	$12ab$	$54by$	$40axy$

A compound quantity is multiplied by a simple one, by multiplying every term of the multiplicand by the multiplier.

EXAMPLES.

Multiply	$a+d$	$-mx-y$	$a-b+c$
By	x	$-v$	$-b$
Sum	$xa+xd$	$vmx+vy$	$-ab+bb-bc$

If there are coefficients, or numbers prefixed to the quantity, then multiply the numbers as in common arithmetic, and to their products join the products of the quantities found by the last example.

Multiply	$3a-b+2c$
By	$8b$
Product	$24ab-8bb+16bc$

Multiply	$2a^2-4ab+3ac-2bc+3b^2-2c^2$
By	$2abc$
Product	$4a^3bc-8a^2b^2c+6a^2bc^2-4ab^2c^2+6ab^3c-4abc^3$

If any algebraic quantities are to be multiplied by a pure number, this number is to be multiplied into every one of the coefficients of the

the other quantities, in all respects as before, and to each particular product set or join that quantity whose coefficient was multiplied.

EXAMPLES.

Multiply	$3a+4b$	$2x-6c$
By -	6	8
Product	$18a+24b$	$16x-48c$

Compound quantities are multiplied into one another, by multiplying every term of the multiplicand by each term of the multiplier, successively, and connecting the several products thus arising, with the signs of the multiplicand, if the multiplying term be affirmative, but with contrary signs, if negative.

EXAMPLES.

Multiply	$3a+2x$	$a^3+a^2b+ab^2+b^3$
By -	$4a+3x$	$a-b$
	$12aa+8ax$	$a^4+a^3b+a^2b^2+ab^3$
	$9ax+6xx$	$-a^3b-a^2b^2-ab^3-b^4$
Produ.	$12aa+17ax+6xx$	$a^4 \quad * \quad * \quad * -b^4$

In the above example, by striking out all the terms that destroy each other, the product becomes a^4-b^4.

Note. If the sign of any proposed term of the multiplier, in any case whatever, be affirmative, it is easy to conceive, that the required product will be greater than it would be if there was no such term, by the product of that term into the whole multiplicand; and therefore it is, that this product is to be added or wrote down with its proper signs. But if, on the contrary, the sign of the term by which you multiply be negative, then, as the required product must be less than it would be, if there were no such term, by the product of that term into the whole multiplicand, this product, it is manifest, ought to be subtracted or wrote down with contrary signs.

Hence is derived the common rule, that like signs produce $+$, and unlike signs $-$.

For, first, if the signs of both the quantities or terms to be multiplied are affirmative, it is plain, that the sign of the product must likewise be affirmative.

Secondly, if the signs of both quantities are negative, that of the product will be affirmative, because contrary to that of the multiplicand, as proved above.

Thirdly, if the sign of the multiplicand be affirmative, and that of the mulitplier negative, the sign of the product will be negative, because the same with that of the multiplicand.

Lastly, if the sign of the multiplicand be negative, and that of the multiplier affirmative, the sign of the product will be negative, because the same with that of the multiplicand.

And these are all the cases that can possibly happen with regard to the variation of signs.

Examples for the learner's exercise.

$$\begin{array}{ll}
\text{Multiply} & a^3-3a^2b+3ab^2-b^3 \\
\text{By} \quad - & a^2-2ab+b^2 \\
\hline
 & a_5-3a^4b+3a^3b^2-a^2b^3 \\
 & \quad -2a^4b+6a^3b^2-6a^2b^3+2ab^4 \\
 & \qquad\qquad +a^3b^2-3a^2b^3+3ab^4-b^5 \\
\hline
\text{Product} & a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5
\end{array}$$

$$\begin{array}{ll}
\text{Multiply} & xx+xy+yy \\
\text{By} \quad - & xx-xy+yy \\
\hline
 & x^4+x^3y+x^2y^2 \\
 & \quad -x^3y-x^2y^2-xy^3 \\
 & \qquad\qquad x^2y^2+xy^3+y^4 \\
\hline
\text{Product} & x^4 \quad * \quad +x^2y^2 \quad * \quad +y^4
\end{array}$$

DIVISION.

In division of algebraic quantities, the rule for the signs is the same as in multiplication, viz. if the signs of the divisor and dividend are alike, the sign of the quotient must be $+$; but if they are unlike, the sign of the quotient must be $-$.

This is a general rule for all operations in division, which are only the reverse of multiplication, and therefore will be easy to understand, when illustrated by examples.

EXAMPLES.

$$\begin{array}{lll}
\text{Divide} & acd & -mad \\
\text{By} \quad - & ac & -md \\
\hline
\text{Quotient} & d & a
\end{array}$$

In the first example, because ac is in the dividend and divisor, reject it, and put down d for the quotient.

The truth of these operations in division may be proved like those in arithmetic; for the quotient and divisor being multiplied, the product will be the dividend, if the work is true; thus, in the second example, by multiplying a the quotient into $-md$ the divisor, the product is mda, or adm, or mad, to which must be prefixed the sign $-$, because the signs of md and a are unlike; hence the product with its sign is $-mad$, the given dividend.

E. 3. $-a)+ab(-b$ E. 4. $2ab)2ab(1$

E. 5. $2a)6ab(3b$
$6ab$
$\overline{\quad * \quad}$

E. 6. $a)aa+ab(a+b$
aa
$\overline{\quad +ab \quad}$
$+ab$
$\overline{\quad * \quad}$

E. 7

E. 7. $4a-b)4aa-7ab-2bb(a-2b$

$$4aa+ab$$

$$8ab-2bb$$

$$8ab-2bb$$

$$* \quad *$$

The truth of these examples are proved as in common arithmetic.

E. 8. $a+x)a^3+5a^2x+5ax^2+x^3(a^2+4ax+x^2$

$$a^3+a^2x$$

$$4a^2x+5ax^2$$

$$4a^2x+4ax^2$$

$$ax^2+x^3$$

$$ax^2+x^3$$

$$* \quad *$$

To work the above example, say, how often is a contained in a^3; the answer is a^2, which I write down in the quotient, and multiply the whole divisor $a+x$ thereby, and there arises a^3+a^2x; which, subtracted from the two first terms of the dividend, leaves $4a^2x$; to this remainder I bring down $+5ax^2$, the next term of the dividend, and then seek again how many times a is contained in $4a^2x$; the answer is $4ax$, which I also put down in the quotient, and by it multiply the whole divisor, and there arises $4a^2x+4ax^2$, which subtracted from $4a^2x+5ax^2$, leaves ax^2 to which I bring down x^3, the last term of the dividend, and seek how many times a is contained in ax^2, which I find to be x^2, which I also put in the quotient, and by it multiply the whole divisor, and then, having subtracted the product from ax^2+x^3, I find there is nothing remains.

E. 9.

$$x+3ax^2-x^3$$

$$a^2-2ax+x^2)a^5-5a^4x+10a^3x^2-10a^2x^3+5ax^4-x^5(a^3-3a^2$$

$$a^5-2a^4x+a^3x^2$$

$$-3a^4x+9a^3x^2-10a^2x^3$$

$$-3a^4x+6a^3x^2-3a^2x^3$$

$$3a^3x^2-7a^2x^3+5ax^4$$

$$3a^3x^2-6a^2x^3+3ax^4$$

$$-a^2x^3+2ax^4-x^5$$

$$-a^2x^3+2ax^4-x^5$$

$$* \quad * \quad *$$

E. 10.

$3a-6)6a^4-96(2a^3+4a^2+8a+16$, &c. *ad infinitum*, by adding (the power of a.

$$\begin{array}{l}
6a^4-12a^3 \\
\hline
12a^3-96 \\
12a^3-24a^2 \\
\hline
24a^2-96 \\
24a^2-48a \\
\hline
48a-96 \\
48a-96 \\
\hline
\end{array}$$

E. 11. *

$4x-5a)48x^3-76ax_2-64a^2x+105a^3(12x^2-4ax-21a^2$

$$\begin{array}{l}
48x^3-60ax^2 \\
\hline
16ax^2-64a^2x \\
16ax^2+20a^2x \\
\hline
-84a^2x+105a^3 \\
-84a^2x+105a^3 \\
\hline
\end{array}$$

*

It may be proper to obſerve to the learner, that the work will not always terminate without a remainder, in which caſe this method is of little uſe. In all ſuch caſes it will be moſt proper to expreſs the quotient in the manner of a fraction, by writing the diviſor under the dividend, with a line between them, as in the following examples.

	E. 1.	E. 2.	E. 3.
Divide	$8my$	$3a+4b$	$14db-5xz$
By .	z	$6m$	$6y$
Quotient	$\frac{8my}{z}$	$\frac{3a+4b}{6m}$	$\frac{14db-5xz}{6y}$

INVOLUTION.

Involution is the raiſing of powers from any propoſed root, and is therefore performed by multiplication; for the given quantity being multiplied by itſelf, will be the ſquare of that quantity, and that product being multiplied by the given quantity, will be the cube of that quantity; and ſo on as in common arithmetic.

EXAMPLES.

Required to find the cube of a $2a$

$$\begin{array}{lcc}
\text{Required to find the cube of} & a & 2a \\
 & a & 2a \\
\hline
\text{the uare of } a = & aa & 4aa \\
 & a & 2a \\
\hline
\text{the cube of } a = & aaa & 8aaa
\end{array}$$

In

In like manner, any other single quantity may be raised to any required power.

If there are two or more quantities connected by the signs + or —, to be raised to any given power, it is still performed by common multiplication.

Two quantities, when connected by the sign +, are commonly called a binomial; and by the sign —, a residual. Let it be required to raise the binomial, $a+b$ to the fourth power.

$$\begin{array}{l}
a+b \\
a+b \\
\hline
aa+ab \\
\quad\; ab+bb \\
\hline
aa+2ab+bb, \text{ the square, or 2d power.} \\
a+b \\
\hline
a^3+2a^2b+ab^2 \\
\qquad a^2b+2ab^2+b^3 \\
\hline
a^3+3a^2b+3ab^2+b^3, \text{ the cube, or 3d power.} \\
a+b \\
\hline
a^4+3a^3b+3a^2b^2+ab^3 \\
\qquad a^3b+3a^2b^2+3ab^3+b^4 \\
\hline
a^4+4a^3b+6a^2b^2+4ab^3+b^4, \text{ the 4th power.}
\end{array}$$

Let it be required to involve, or raise the residual $a-b$ to the fourth power.

$$\begin{array}{l}
a-b \\
a-b \\
\hline
aa-ab \\
\quad -ab+b^2 \\
\hline
a^2-2ab+b^2, \text{ the 2d power.} \\
a-b \\
\hline
a^3-2a^2b+ab^2 \\
\quad -a^2b+2ab^2-b^3 \\
\hline
a^3-3a^2b+3ab^2-b^3, \text{ the 3d power.} \\
a-b \\
\hline
a^4-3a^3b+3a^2b^2-ab^3 \\
\quad -a^3b+3a^2b^2-3ab^3+b^4 \\
\hline
a^4-4a^3b+6a^2b^2-4ab^3+b^4, \text{ the 4th power.}
\end{array}$$

The

The products may be found by making two geometrical progressions, the one beginning at the desired power of the first part of the root, and ending at an unit; and the other beginning at an unit, and ending at the power of the other part of the root: to find the 6th power of $a+b$, write the powers.

$$\text{Thus} \begin{cases} a^6 a^5 a^4 a^3 a^2 a 1 \\ 1 b b^2 b^3 b^4 b^5 b^6 \end{cases}$$

Then $a^6+a^5b+a^4b^2+a^3b^3+a^2b^4+ab^5+b^6$, will be the terms in the 6th power of $a+b$, by multiplying the powers above by those below; and to find their unciæ or coefficients, that of the first term is always an unit, and that of the second is the exponent of the first; and of the third is the exponent of a in the second term, multiplied by the affixed unciæ, and divided by $2=15$; and of the third is the exponent of a in the third term, multiplied by the prefixed unciæ 15, and divided by 3, and so of the fourth, &c.

EXAMPLE.

Let it be required to complete all the terms of the aforesaid several powers viz. $a^6+a^5b+a^4b^2+a^3b^3+a^2b^4+ab^5+b^6$.

1. The index of a^6, the first term, will be the unciæ of the second term. Thus a^6+6a^5b.

2. Then half the second term's index into its unciæ, viz. $\frac{6\times5}{2}=15$, will be the third term's unciæ.

Thus $a^6+6a^5b+15a^4b^2$ will be the three first terms.

Third, again $\frac{15\times4}{3}=20$, the unciæ of the fourth term.

Then it will be $a^6+6a^5b+15a^4b^2+20a^3b^3$.

Fourth, and $\frac{20\times3}{4}=15$, the unciæ of the fifth term.

Then $a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4$, &c. until all the terms are compleated with their respective unciæ's, and then they will stand thus:

$$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6.$$

If the work of the preceding example be well understood, it will be found very easy to raise any power from a binomial or residual root, to what height you please, without the trouble of a continued involution, or without the help of a table of powers, as proposed by several authors.

EXAMPLE.

Let it be required to raise $a+b$ to its cube or third power, both in numbers and species, where $a=10$, and $b=2$?

$a+b$

$$
\begin{array}{llll}
a+b & = & 10+2 & \\
a+b & = & 10+2 & \\
\hline
a^2+ab & = & 100+20 & 12 \\
\quad ab+b^2 & = & \quad 20+4 & 12 \\
\hline
a^2+2ab+b^2 & = & 100+40+4=144=\text{Square.} & \\
a+b & = & 10+2 & \\
\hline
a^3+2a^2b+ab^2 & = & 1000+400+40 & 144 \\
\quad a^2b+2ab^2+b^3 & = & \quad 200+80+8 & 12 \\
\hline
a\ +3a^2b+3ab^2+b^3 & = & 1000+600+120+8=1728=\text{Cube.} &
\end{array}
$$

EVOLUTION.

Evolution is the extraction of roots, and therefore opposite to involution, or raising of powers, and is performed by converse operations, viz. by the division of the indices.

To extract the root of any simple quantity, consider how many times the letter is repeated, or how high the power of it is; and if the given power have no numbers prefixed to it, and its index can be divided by the index of the root required, the quotient will be the index of the root sought.

Thus the square root of a^6, by dividing the exponent by 2, is found to be a^3, and the cube root of a^6 is a^2; also the biquadratic root of $\overline{a+y}|^8$, will be $\overline{a+y}|^2$; likewise the cube root of $\overline{xx+yy}|^{\frac{1}{2}}$ will be $\overline{xx+yy}|^{\frac{2}{3}}$.

If the given powers have coefficients, then you must extract their respective roots, as in vulgar arithmetic.

Thus the square root of $81a^4$, is $9a^2$; and the square root of $1296a^8b^8$, is $36a^4b^4$; and so of others.

If the quantity given be a fraction, its root will be extracted, by extracting the root of each particular factor.

Thus the square root of $\dfrac{a^2b^2}{c^2}$ will be $\dfrac{ab}{c}$ and that of $\dfrac{81\times aa\times\overline{aa+xx}|^4}{16\times\overline{a-x}|^2}$

will be $\dfrac{9a\times\overline{aa+xx}|}{4\times\overline{a-x}}$; and so of others.

If we cannot extract the square root of both the numerator and denominator, the given quantity is a surd, and must have the sign of the root required prefixed to it.

Thus, suppose it was required to extract the square root of $xx+2xn-nn$; the answer will be $\sqrt{xx+2xn-nn}$, the square root required.

In

In the above example there are three quantities, and two different letters, x and n; two of theſe three quantities, viz. xx and nn, are pure powers of x and n; but both theſe powers have not the ſign $+$, for it is $-nn$, therefore, I conclude, that the given quantity is a ſurd quantity, whoſe ſquare root cannot be extracted any otherwiſe than by prefixing the ſign $\surd$ to it as above, which expreſſes the ſquare root.

Evolution of compound quantities is performed by the following

RULE. Place the ſeveral terms, whereof the given quantity is compoſed, in order, according to the dimenſions of ſome letter therein, as ſhall be judged moſt proper; then let the root of the firſt term be found, and placed in the quotient, which term being ſubtracted, let the firſt term of the remainder be brought down, and divided by twice the firſt term of the quotient, or by three times its ſquare, or four times its cube, &c. according as the root to be extracted is a ſquare, cubic, or biquadratic one, &c. and let the quantity thence ariſing be alſo wrote down in the quotient, and the whole raiſed to the ſecond, third, or fourth power, &c. according to the aforeſaid caſes, reſpectively, and ſubtracted from the given quantity, and if any thing remains, let the operation be repeated, by always dividing the firſt term of the remainder by the ſame diviſor, found as above.

EXAMPLES.

It is required to extract the ſquare root of $x^2+2xy+y^2$?

Thus, $x^2+2xy+y^2(x+y$, the root required.

$2x)\ 2xy$

$x^2+2xy+y^2$, ſecond power of $x+y$.

*

The ſquare root of compound quantities may be extracted according to the common method of extracting the ſquare root in numbers.—— See the laſt example.

Thus, $x^2+2xy+y^2(x+y$, root as before

x^2

$2x+y)\quad 2xy+y^2$

$2xy+y^2$

*

It is required to extract the ſquare root of $a^4-2a^3x+3a^2x^2-2ax^3+x^4$?

$a^4-2a^3x+3a^2x^2-2ax^3+x^4\ (a^2-ax+x^2$, root.

$2a^2)-2a^3x$.

$a^4-2a^3x+a^2x^2$, ſecond power of a^2-ax.

$2a^2)\ 2a^2x^2$, firſt term of the remainder.

$a^4-2a^3x+3a^2x^2-2ax^3+x^4$, ſquare of a^2-ax+x^2.

* * *

Of

Or thus, as in common arithmetic.

$$\begin{array}{ll}
 & a^4-2a^3x+3a^2x^2-2ax^3+x^4(a^2-ax+x^2, \text{ root as before.} \\
 & a^4 \\
\hline
2a^2-ax) & -2a^3x+3a^2x^2 \\
 & -2a^3x+a^2x^2 \\
\hline
2a^2-2ax+x^2) & 2a^2x^2+2ax^3+x^4 \\
 & 2a^2x^2+2ax^3+x^4 \\
\hline
\end{array}$$

It is required to extract the square root of $a^4+4a^3x+6a^2x^2+4ax^3+x4$?

$$\begin{array}{l}
a^4+4a^3x+6a^2x^2+4ax^3+x^4(a^2+2ax+x^2, \text{ root.} \\
2a^2)\ 4a^3x \\
a^4+4a^3x+4a^2x^2 \\
\hline
\quad 2a^2)\ \ 2a^2x^2 \\
a^4+4a^3x+6a^2x^2+4ax^3+x^4 \\
\hline
\end{array}$$

Or thus, as in common arithmetic.

$$\begin{array}{ll}
 & a^4+4a^3x+6a^2x^2+4ax^3+x^4(a^2+2ax+x^2, \text{ root as before.} \\
 & a^4 \\
\hline
2a^2+2ax) & 4a^3x+6a^2x^2 \\
 & 4a^3x+4a^2x^2 \\
\hline
2a^2+4ax+x^2) & 2a^2x^2+4ax^3+x^4 \\
 & 2a^2x^2+4ax^3+x^4 \\
\hline
\end{array}$$

It is required to extract the cube root of $a^3-6a^2x+12ax^2-8x^3$?

$$\begin{array}{ll}
 & a^3-6a^2x+12ax^2-8x^3\ (a-2x, \text{ root.} \\
3a^2\ ) & -6a^2x \\
 & a^3-6a^2x+12ax^2-8x^3 \\
\hline
\end{array}$$

To prove the above example.

$$\begin{array}{l}
a-2x, \text{ supposed root.} \\
a-2x \\
\hline
a^2-2ax \\
\quad -2ax+4x^2 \\
\hline
a^2-4ax+4x^2, \text{ square, or second power.} \\
a\ -2x \\
\hline
a^3-4a^2x+4ax^2 \\
\quad -2a^2x+8ax^2-8x^3 \\
\hline
\end{array}$$

Proof $a^3-6a^2x+12ax^2-8x^3$, Product.

Same as the given quantity; which proves the cube root to be as above.

Required to extract the biquadratic root of

$$16x^4-96x^3y+216x^2y^2-216xy^3+81y^4(2x-3y.$$
$$32x^3)-96x^3y$$
$$16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$$

And in the same manner the root may be determined in any other case, where it is possible to be done; bnt after all, if there is a remainder, the root is to be expressed in the manner of a surd.

FRACTIONS.

The reduction of fractional quantities is of use in changing an expression to the most simple form it is capable of.

To change different fractions into one denomination, retaining the same value.

RULE. Multiply all the denominators into each other for a new denominator, and each numerator into all the denominators except its own, for a new numerator.

EXAMPLES.

Reduc $\frac{a}{b}\ \frac{b}{c}$ and $\frac{c}{d}$ to a common denominator.

$$\text{Thus}\left\{\begin{array}{l} a\times c\times d=acd \\ b\times b\times d=bbd \\ c\times b\times c=bcc \end{array}\right\}\text{Numerators.}$$

And $b\times c\times d=bcd$ common denominator. Therefore $\frac{a}{b}\ \frac{b}{c}\ \frac{c}{d}$ becomes $\frac{acd}{bcd}\ \frac{bbd}{bcd}\ \frac{bcc}{bcd}$ the fractions required; and so of others.

When the denominators have a common divisor, instead of multiplying the terms of each fraction by the denominator of the other, you only multiply by that part which arises by dividing by the common divisor.

E. 2. Reduce $\frac{b^2}{ad}$ and $\frac{ab}{cd}$ to a common denominator.

$$\begin{array}{ll} \text{Thus} & bb\times c=bbc \\ \text{And} & ab\times a=aab \end{array}\Big\}\text{Numerators.}$$

Also $a\times c\times d=acd$ common denominator.

So $\frac{b^2}{ad}\ \frac{ab}{cd}$ become $\frac{bbc}{acd}\ \frac{aab}{acd}$ the fractions required.

To reduce fractional quantities into their lowest terms.

RULE. Divide both the numerator and denominator by the greatest common divisor.

Thus $\frac{ab}{bc}$, by dividing by b, is $\frac{a}{c}$, and $\frac{2abc}{abb}$, by dividing by ab, is $\frac{2c}{b}$; also, $a+\frac{bdc}{bc}=a+d$.

In

In ſuch ſingle fractions as theſe, the common diviſors (if there be any) are eaſily diſcovered, but the compound diviſors, whereby a fraction can be reduced to lower terms, are not ſo eaſily diſcovered, for which reaſon I have laid down the following

RULE. Divide the numerator by the denominator until nothing remains, when that can be done; or find their common meaſure by dividing the denominator by the numerator, and the numerator by the remainder, and ſo on, as in vulgar fractions.

EXAMPLES.

Reduce $\frac{a^2c-a^2d}{cd-d^2}$ to its loweſt terms.

Thus $cd-d^2)a^2c-a^2d(\frac{aa}{d}$ the fraction required.
a^2c-a^2d

Let it be required to reduce $\frac{a^3-ab^2}{a^2+2ab+b^2}$ to its loweſt terms,

$a^2+2ab+b^2)a^3-0 \quad -ab^2(a$
$a^3+2a^2b+ab^2$

Remainder $-2a^2b-2ab^2)a^2+2ab+b^2(-\frac{1}{2b}-\frac{1}{2a}$
a^2+ab

$ab+b^2$
$ab+b^2$

Hence it appears, that $-2a^2b-2ab^2$ is the common meaſure, by which $a-2ab^2$ being divided,

viz. $-2a^2b-2ab^2)a^3-0-ab^2(-\frac{a}{2b}+\frac{1}{2}$
a^3+a^2b

$-a^2b-ab$
$-a^2b-ab^2$

Then $-\frac{a}{2b}+\frac{1}{2}=-\frac{a+b}{2b}$, the new numerator, and $-\frac{1}{2b}-\frac{1}{2a}=\frac{-a-b}{2ba}$, the new denominator.

Let both be multiplied with $2ba$, and we ſhall have $-a^2+ab$ numerator.

$-a-b$ denominator. Or change the ſigns of all the quantities, it will be $\frac{a^2-ab}{2ba}$ the new fraction required, that is, $\frac{a^2-ab}{a+b}=\frac{a^3-ab}{a^2+2ab+b^2}$.

Besides these, there are other sorts of reductions, which some authors have treated of under the head of fractions; but as they are of little use in the solution of problems, I shall pass them by, and proceed to addition.

ADDITION AND SUBTRACTION.

RULE. Add or subtract their numerators as occasion requires, and to their sum or difference subscribe the common denominator, as in vulgar fractions.

EXAMPLES IN ADDITION.

Add $\frac{a}{b}$ to $\frac{c}{d}$, first reduce them to a common denominator, and they they will be $\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$, the sum required.

Add $\frac{a}{b}+\frac{c}{d}+\frac{d}{e}$ into one sum. First, these reduced to a common denominator will be $\frac{ade}{bde}+\frac{bce}{bde}+\frac{ddb}{bde}=\frac{ade+bce+ddb}{bde}$ the sum required.

EXAMPLES IN SUBTRACTION.

From $\frac{bb+aa}{c}$ take $\frac{bb}{c}$. Thus, $\frac{bb+aa}{c}-\frac{bb}{c}=\frac{aa}{c}$, the difference.

From $\frac{a}{2}$ take $\frac{a}{3}$. Thus reduced, $\frac{3a}{6}$ and $\frac{2a}{6}$, then $\frac{3a}{6}-\frac{2a}{6}=\frac{a}{6}$, the difference required.

From $\frac{b^2+a^2}{c}$ take $\frac{bb}{c}$. Thus, $\frac{b^2+a^2}{c}-\frac{b^2}{c}=\frac{a^2}{c}$, the difference.

From $\frac{2b}{d+a}$ take $\frac{a+b-d}{d+a}$. Thus, $\frac{2b}{d+a}-\frac{a+b-d}{d+a}=\frac{a-b+d}{d+a}$

MULTIPLICATION.

First, prepare the quantities as directed in vulgar fractions, then multiply the numerators together for a new numerator; and the denominators together, for a new denominator.

EXAMPLE

EXAMPLES.

Multiply $\frac{a}{b}$ by $\frac{c}{d}$. Thus $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, the product.

And $\frac{2ab}{3c} \times \frac{5ad}{3f} = \frac{10a^2bd}{9cf}$, the product.

Likewise $\frac{2ac+b-25c}{c} \times \frac{3b+4c}{1} =$

$\left.\frac{6bac+3bb-75bc+8acc+4bc-100cc}{c}\right\}$ the product.

DIVISION.

One fraction is divided by another, by multiplying the denominator of the divisor into the numerator of the dividend, for a new numerator; and the numerator of the divisor into the denominator of the dividend, for a new denominator.——EXAMPLES;—

Divide $\frac{a}{b}$ by $\frac{c}{d}$. Thus $\frac{a}{b} \div$ by $\frac{c}{d} = \frac{ad}{bc}$, the quotient:

And $\frac{5ax}{3c} \div \frac{6bc}{7d} = \frac{35adx}{18bcc}$, the quotient.

Also $\frac{6a^2b}{5x} \div \frac{5ab^2}{3x} = \frac{18a^2bx}{25ab^2x}$, the quotient.

In cases like the last, where the two numerators, or the denominators, have factors common to both, the conclusion will become more neat by first casting off such common factors.

Thus, casting away ab out of the two numerators, and x out of both denominators, we have $\frac{6a}{5}$ to be divided by $\frac{5b}{3}$ whereof the quotient is $\frac{18a}{25b}$ and so of others.

To divide any quantity by a compound divisor, let the division be made the common way continually, till it appear the terms in the quotient will come out infinitely, remembering the rules of changing the signs when multiplying, subtracting, or dividing.

Let it be required to divide b by $a+c$?

$$a+c)\,b\,\left(\frac{b}{a} - \frac{bc}{aa} + \frac{bc^2}{a^3} - \frac{bc^3}{a^4}\right., \text{ \&c. infinitely.}$$

$$\begin{array}{l} b + \frac{bc}{a} \\ \hline -\frac{bc}{a} \\ \hline -\frac{bc}{a} - \frac{bc^2}{a^2}, \text{ \&c. infinitely.} \end{array}$$

In this example, b divided by a, the quotient is $\frac{b}{a}$, the product of $\frac{b}{a}$ into $a+c$ is $\frac{ab}{a}+\frac{bc}{a}=b+\frac{bc}{a}$, which being taken from the dividend b, leaves $-\frac{bc}{a}$; again, if $-\frac{bc}{a}$ be divided by a, the quotient will be $-\frac{bc}{aa}$, then the product of $a+c$ into $-\frac{bc}{aa}$ is $-\frac{abc}{aa}-\frac{bcc}{aa}$ or $-\frac{bc}{a}-\frac{bc^2}{a^2}$. Thus it appears how the division is to be continued.

SURDS.

Surds are ſuch numbers as cannot be exactly expreſſed in figures, and as they ariſe in the ſolution of algebraic queſtions, I ſhall explain to the young algebraiſt ſo much of them only as is neceſſary to the preſent deſign.

ADDITION OF SURD QUANTITIES.

Case 1. When the quantities under the radical ſigns are alike, add the rational quantities, or thoſe which are without the radical ſigns together, by the rules of addition, and to this join the ſurd quantities, and this will be the ſum required.

If there be no rational quantities without the radical ſign, then unity, or 1, is always ſuppoſed to be the rational quantity.

EXAMPLES.

To	$\sqrt{ab}$	$2\sqrt{xy}$	$5y\sqrt{dm+z}$
Add	$\sqrt{ab}$	$\sqrt{xy}$	$y\sqrt{dm+z}$
Sum	$2\sqrt{ab}$	$3\sqrt{xy}$	$6y\sqrt{dm+z}$

In example 1, there being no rational quantities, therefore unity or 1 is the rational quantity to each. Now 1 added to 1 makes 2, to which joining the ſurd $\sqrt{ab}$, we have $2\sqrt{ab}$, the ſum required.

To	$-15m\sqrt{da-zy}$	$16ab\sqrt{14+x}$
Add	$7m\sqrt{da-zy}$	$-12ab\sqrt{14+x}$
Sum	$-8m\sqrt{da-zy}$	$4ab\sqrt{14+x}$

Case. 2.

CASE 2. When the letters under the radical ſigns are different, place them down one after the other, with the ſame ſigns they have in the queſtion.

EXAMPLES.

$$\begin{array}{ll} \text{To} & \sqrt{a} \\ \text{Add} & \sqrt{b} \\ \hline \text{Sum} & \sqrt{a}+\sqrt{b} \end{array} \qquad \begin{array}{l} a\sqrt{bx+y} \\ a\sqrt{z} \\ \hline a \;\; \overline{bx+y} : +a\sqrt{z} \end{array}$$

SUBTRACTION.

CASE 1. When the letters under the radical ſigns are alike, ſubtract the rational quantities from the rational quantities, and to the difference join the ſurd quantities, which will be the remainder required.

EXAMPLES.

$$\begin{array}{ll} \text{From} - & 5\sqrt{ad} \\ \text{Take} - & 3\sqrt{ad} \\ \hline \text{Remains} & 2\sqrt{ad} \end{array} \qquad \begin{array}{l} 6n\sqrt{nz} \\ 3n\sqrt{nz} \\ \hline 3n\sqrt{nz} \end{array}$$

$$\begin{array}{ll} \text{From} - & 12y\sqrt{d-n} \\ \text{Subtract} & -3y\sqrt{d-n} \\ \hline \text{Remains} & 15y\sqrt{d-n} \end{array}$$

The truth of theſe operations are proved as in ſubtraction of common numbers.

CASE 2. When the letters under the radical ſigns are different, ſet them down one after the other, but care muſt be taken to change the ſigns of thoſe quantities that are to be ſubtracted.

EXAMPLES.

$$\begin{array}{ll} \text{From} & 2a\sqrt{en} \\ \text{Subtract} & y\sqrt{rv} \\ \hline \text{Remains} & 2a\sqrt{en}-y\sqrt{rv} \end{array} \qquad \begin{array}{l} 5y\sqrt{a} \\ -3\sqrt{b} \\ \hline 5y\sqrt{a}+3\sqrt{b} \end{array}$$

Theſe operations are proved in the ſame manner as in the laſt caſe, by adding the remainder to the quantity that was ſubtracted.

MULTIPLICATION.

Case 1. When there are no rational quantities joined to the surd, multiply the surd quantities together, and to their product prefix the radical signs.

EXAMPLES.

Multiply	$\sqrt{a}$	$\sqrt{xy}$	$\sqrt{a+b}$	$\sqrt{ax-ay}$
By -	$\sqrt{b}$	$\sqrt{d}$	$\sqrt{x}$	$\sqrt{n}$
Product	$\sqrt{ab}$	$\sqrt{xyd}$	$\sqrt{ax+xb}$	$\sqrt{axn-ayn}$

Case. 2. If rational quantities be joined to the surds, then multiply the rational into the rational, and the surd into the surd, and join the products together.

EXAMPLES.

Multiply	$a\sqrt{x}$	$8a\sqrt{3x}$	$n\sqrt{a+y}$	$\sqrt{n+b}$
By -	$b\sqrt{y}$	$3b\sqrt{2y}$	$a\sqrt{b}$	$a\sqrt{z}$
Product	$ab\sqrt{xy}$	$24ab\sqrt{6xy}$	$na\sqrt{ab+by}$	$a\sqrt{zn+zb}$

DIVISION.

Case 1. When there are no rational quantities joined with the surd quantities, reject all those quantities in the dividend and divisor that are alike, and set down the remainder, to which prefix the radical signs, and this will be the quotient sought.

EXAMPLES.

Divide	$\sqrt{abx}$	$\sqrt{bxd}$	$\sqrt{ypa}$
By -	$\sqrt{x}$	$\sqrt{bd}$	$\sqrt{yp}$
Quotient	$\sqrt{ab}$	$\sqrt{x}$	$\sqrt{a}$

In example 1, because x is in both dividend and divisor, reject it, and put down ab with the sign $\sqrt{}$ before it, and $\sqrt{ab}$ is the quotient required; and so of others.

Divide	$\sqrt{bn+ba}$	$\sqrt{mz-mp}$
By -	$\sqrt{b}$	$\sqrt{m}$
Quotient	$\sqrt{n+a}$	$\sqrt{z-p}$

Case 2. When there are rational quantities joined with the surds, divide the rational quantities by the rational quantities, and to their quotient join the quotient of the surds found by the last case.

EXAMPLES.

Divide	$ay\sqrt{mn}$	$ba\sqrt{ayn}$	$bx\sqrt{anp}$
By -	$a\sqrt{m}$	$a\sqrt{ay}$	$x\sqrt{an}$
Quotient	$y\sqrt{n}$	$b\sqrt{n}$	$b\sqrt{p}$

The truth of these operations are proved by multiplying the quotient by the divisor, for if that produces the dividend, the work is true, otherwise it is erroneous.

EQUATIONS.

EQUATIONS.

An equation is, when two equal quantities, differently expressed, are compared together by means of the sign $=$ placed between them.

Thus $6-2=4$ is an equation expressing the equality of the quantities $6-2$ and 4: and $x=a+b$ is an equation, shewing that the quantity represented by x is equal to the sum of the two quantities represented by a and b.

Equations are the means whereby we come at such conclusions as answer the conditions of a problem, wherein, from the quantities given, the unknown ones are determined, and this is called the reduction of equations.

REDUCTION OF SINGLE EQUATIONS.

Single equations are such as contain only one unknown quantity, which must be so ordered by addition, subtraction, multiplication, division, &c. of equal quantities, that a just equality between the two parts thereof may be still preserved, and that there may result, at last, an equation wherein the unknown quantity stands alone on one side, and all the known ones on the other; the best manner of doing which will be obtained by the following rules.

RULE 1. Any term of an equation may be transposed to the contrary side, if its sign be changed.

EXAMPLE 1. Thus, $x+8=18$, then will $x=18-8=10$.

In this equation $x+8=18$, which by transposition becomes $x=18-8=10$, by only subtracting the number 8 from both sides.

E. 2. If $x+19=107$, what is the value of x?

By transposition the above equation is changed into this $x=107-19$, therefore $107-19=88$ the value of x.

E. 3. Given $x-107=19$, required the value of x?

By transposition of -107 to the other side of the equation, and changing the sign, the equation stands thus, $x=19+107$, therefore $19+107=126$, the value of x.

E. 4. If $20-3x-8=60-7x$, what is the value of x?

By transposing $7x$, we shall have $-3x+7x=60-20+8$,

or $4x=48$, therefore $x=\frac{48}{4}=12$ the value of x.

For $\overline{20-12}\times3-8=\overline{60-12}\times7=-24$ proof.

RULE 2. If there is any quantity by which all the terms of the equation are multiplied, let them all be divided by that quantity; but if all of them be divided by any quantity, let the common divisor be cast way.

E. 1. Suppose $ax=ab$, then by the rule $x=b$; also $10x=60$, reduced $x=6$; and by the latter part of the rule $\frac{x}{a}=\frac{b}{a}$ is reduced to $x=b$.

 E. 2.

E. 2. Required the value of x, which $36-\frac{4x}{9}=8$.

By multiplying both sides by 9, we have $324-4x=72$, therefore $4x=324-72=252$, consequently $x=\frac{252}{4}=63$.

E. 3. If $6x^2-20x=16x+2x^2$, what is the value of x?

By dividing by $2x$ we have $3x-10=8+x$, and by transposition $3x-x=8+10$, that is $2x=18$, therefore $x=\frac{18}{2}=9$ answer.

RULE 3. If there are reduceable fractions, let the whole equation be multiplied by the product of all their denominators, or, which is the same, let the numerator of every term in the equation be multiplied by all the denominators, except its own, supposing such terms (if any there be) that stand without a denominator, to have an unit subscribed.

E. 1. If $z+\frac{z}{2}+\frac{z}{4}=10$, what is z equal to?

By multiplying the equation by 8, the product of the two denominators 2 and 4, we have $8z+4z+2z=80$, or $14z=80$; therefore $z=80\div 14=5,728$.

E. 2. Let $z\div 5+z\div 3=z-7$, required the value of z?
This reduced will become $3z+5z\div 15=8z\div 15=-7$, consequently $8z=15z-105$, whence $7z=105$, therefore $z=105\div 7=15$ the answer.

RULE 4. If in your equation there is an irreduceable surd, wherein the unknown quantity enters, let all the other terms be transposed to the contrary side (by rule 1) and then, if both sides are involved to the power denominated by the surd, an equation will arise free from radical quantities, unless there happens to be more surds than one, in which case the operation is to be repeated.

Thus, $\sqrt{x}+6=10$, by transposition becomes $\sqrt{x}=10-6=4$, which, by squaring both sides, gives $x=16$.

So likewise $\sqrt{aa+xx}-c=x$ becomes $\sqrt{aa+xx}=c+x$, which, squared, gives $aa+xx=cc+2cx+xx$, or $aa-c=2cx$, per rule 1.

E. 1. If $\sqrt{\frac{5x}{3}+12}=17$, what is x?

By transposition becomes $\sqrt{\frac{5x}{3}}=17-12=5$, and $\sqrt{5x}=5\times 3=15$; then by involving 15 to the power denominated by the surd, we have $5x=225$, therefore $x=\frac{225}{5}=45$.

E. 2. What

E. 2. What is the value of x when $\sqrt{12+x}=2+\sqrt{x}$?

By squaring both sides, we have $12+x=4+4\sqrt{x}+x$, and by transposition $4\sqrt{x}=8$, consequently $x=4$.

E. 3. Suppose $\sqrt{4x+16}=12$, *Query* the value of x;

By squaring both sides we have $4x+16=144$, and by transposition $4x=144-16=128$, therefore $x=\dfrac{128}{4}=32$.

E. 4. If $\sqrt{x}+6=10$, what is the value of x?

By transposition becomes $\sqrt{x}=10-6=4$, and by squaring both sides we have $x=16$.

E. 5. Suppose $\sqrt{ax+b^2}-c=d$, what is the value of x?

Then $\sqrt{ax+b^2}=d+c$, and by squaring we have $ax+b^2=d^2+2dc+c^2$, whence $x=\dfrac{d^2+2dc+c^2-b^2}{a}$.

RULE 5. Having, by the preceding rules, cleared your equations of fractional and radical quantities, and so ordered it by transposition, that all the terms wherein the known quautities are found may stand on the same side thereof, let the whole be divided by the coefficients, or the sum of the coefficients of the highest power of the said unknown quantity,

E. 1. If $12y=48$, what is y equal to?

by dividing the whole by the coefficient of y, we have $y=48\div12=4$.

E. 2. Suppose $5y=79$, what is y equal to?

To disengage y, you must take away the 5, and place it under the 79, thus $y=\frac{79}{5}=15,8$.

E. 3. What is the value of y, when $5y-16=3y+12$?

By transposition becomes $5y-3y=12+16$, whence $y=\frac{28}{2}=14$.

To reduce two or more equations to a single one.

RULE. Let the given quantities or equations be multiplied or divided by such numbers or quantities, whether known or unknown, that the term which involves the highest power of the unknown quantity to be exterminated, may be the same in each equation, and then, by adding or subtracting the equations, as occasion requires, that term shall vanish, and a new equation emerge, wherein the number of dimensions (if not the number of unknown quantities) will be diminished.

E. 1. Given $\left\{\begin{array}{l}5x+8y=106\\4x-5y=5\end{array}\right\}$ *Query* the value of x & y?

Here, by multiplying the first equation by 4, and the second by 5, in order that the coefficients of x may be the same in both, we have,

$$20x+32y=424$$
$$20x-25y=25$$

 By

By ſubtracting the latter from the former, we have $57y=399$: hence $y=\frac{399}{57}=7$. And ſo by the firſt equation $x=\frac{5\times 8}{4}=\frac{40}{4}=10$.

E. 2. Given $\left\{\begin{array}{l}5x-3y=90\\2x+5y=160\end{array}\right\}$ *Query* the value of x and y?

Here, by multiplying the firſt equation by 2, and the ſecoud by 5, in order that the coefficients of x may be the ſame in both, there ariſes

$$10x-6y=180$$
$$10x+25y=800$$

By ſubtracting the former from the latter, we have $31y=620$; hence $y=\frac{620}{31}=20$. And ſo by the firſt equation $x=\frac{90+3y}{5}=\frac{90+60}{5}=30$.

E. 3. Given $\left\{\begin{array}{l}\frac{x}{2}+\frac{y}{3}=16\\\frac{x}{5}-\frac{y}{9}=2\end{array}\right\}$ *Query* x and y.

Here the equations cleared of fractions will be

$$3x+2y=96$$
$$9x-5y=90$$

Now, if from the triple of the former, the latter be ſubtracted, we have $6y+5y=288-90$, that is, $11y=198$; hence $y=\frac{198}{11}=18$; and $x=\frac{96-2y}{3}=\frac{96-36}{3}=20$.

E. 4. Let $\left\{\begin{array}{l}x+y=13\\x+z=14\\y+z=15\end{array}\right\}$ *Query* x, y, and z.

Here, by ſubtracting the firſt equation from the ſecond, in order to exterminate x, we have $z-y=1$, to which the third equation being added, y will likewiſe be exterminated, there coming out $2z=16$, or $z=8$; whence $y=z-1=7$, and $x=13-y=6$.

E. 5. Given $\left\{\begin{array}{l}x+100=y+z\\y+100=2x+2z\\z+100=3x+3y\end{array}\right\}$ *Query* x, y, and z.

To the double of the firſt, let the ſecond equation be added, ſo ſhall the x's, on the contrary ſides, deſtroy each other, and we ſhall have $300+y=2y+4z$, or $y+4z=300$.

Moreover, to the triple of the firſt let the third equation be added, whence will be had $z+400=6y+3z$, or $2z+6y=400$.

Now, if from the double of this laſt equation (viz. $4z+12y=800$) the former (viz. $y+4z=300$) be ſubtracted, there will come out $11y=500$;

500; therefore $y=\frac{500}{11}=45\frac{5}{11}$, and $z=\frac{300}{4}=75-\frac{y}{4}=75-11\frac{4}{11}=63\frac{7}{11}$, and $x=y+z-100=109\frac{1}{11}-100=9\frac{1}{11}$.

Questions producing Simple Equations.

When a question is proposed to be solved algebraically, its true design and signification ought to be perfectly understood, so that it may be abstracted from all ambiguous and unnecessary phrases, and the conditions thereof exhibited in the clearest light possible, this being done, and the several quantities therein concerned being denoted by proper signs, let the true sense and meaning of the question be translated from the English into algebra, and the conditions thus expressed in algebraic terms will, if it be properly limited, give as many equations as is necessary to its solution; but these things will be best understood by examples.

Question 1. What two numbers are those, whose difference is 14, and sum 48;

Let - - - - - $v=$ the lesser number.
Then the greater number will be $v+14$
Which by addition gives - $2v+14=48$ ℘ question.
Therefore by transposition - $2v=48-14=34$
And - - - - - $v=\frac{34}{2}=17$ the lesser number.
Consequently - - - $17+14=31$ the greater number.
For - - - - - $31-17=14$
And - - - - - $31+17=48$ proof.

Quest. 2. Four men, A, B, C, and, D built a ship, which cost 5214*l.* whereof B paid twice as much as A, C paid as much as A and B, and D paid as much as C and B; what did each pay?

Suppose A paid v pounds
Then B paid $2v$
C - - - $3v$
And D - $5v$

Whence the whole sum paid is - $11v=5214l.$ by question.
Therefore - - - - - $v=\frac{5214}{11}=474l.$ A's share
Consequently - - - - - - $2v=948l.$ B's share
$3v=1422l.$ C's share
And - - - - - - - $5v=2370l.$ D's share
Proof £. 5214

Quest. 3. A borrowed of B as much money as A had, and spent 6*d.* to treat him; after which, meeting with C, A borrowed of him twice as much money as he had left, and treated him with 12*d.*; lastly, A borrowed of D three times as much money as he had left, and spent on him 18*d.* after which he had 30*d.* left; what had he at first?

Suppose he had x pence at first,
Then he borrowed x pence of B,
And after spending 6*d.* had $2x-6$ left,

Then

Then he borrowed - $4x - 12$ of C,
And after ſpending 12*d.* had $6x - 30$ left;
Then he borrowed - $18x - 90$ of D,
And after ſpending 18*d.* had $24x - 138$ left:
But - $24x - 138 = 30$ by the queſtion.
Therefore, - $24x = 168$

And - - $\frac{168}{24} = 7d.$ the anſwer.

Queſt. 4. A charitable lady, relieving four poor perſons, gave among them 6*s.* 8*d.* to the ſecond ſhe gave twice, to the third thrice, and to the fourth four times as much as to the firſt; what did ſhe give to each?

Suppoſe ſhe gave - x pence to the firſt,
Then ſhe gave - $2x$ pence to the ſecond
———————— $3x$ ——— to the third,
And ———————— $4x$ ——— to the fourth;
And ſhe gave in all $10x (= 6s.\ 8d.) = 80$, by the queſtion.
Therefore $x = \frac{80}{10} = 8d.$ the ſum the firſt had.
For $8d. + 16d. + 24d. + 32d. = 80d.$ the proof.

Queſt. 5. In a lump of mixed metal, weighing 29℔. there was 2℔. of ſilver more than of gold, 4℔. of copper more than of ſilver, and 3℔. of braſs more than of copper; how many pounds were there of each?

Suppoſe there were - x ℔ of gold,
Then there were - - $x + 2$ of ſilver,
And - - - - - - $x + 6$ of copper,
Alſo - - - - - - $x + 9$ of braſs:
But their ſum is - $4x + 17 = 29$ ℘ queſtion.
Therefore by ſubtraction, $4x = 29 - 17 = 12$;
Conſequently $x = \frac{12}{4} = 3$℔. of gold, the anſwer.
For $3 + 5 + 9 + 12 = 29$℔, Proof.

Queſt. 6. Being to buy a ſuit of clothes for each of my ſix children, I propoſe to lay out four times as much on the eldeſt as I do on the youngeſt, and to beſtow twelve ſhillings a ſuit leſs on each than on the next elder; what will each ſuit coſt?

Suppoſe the youngeſt's ſuit coſt x ſhillings,
Then the ſecond's will coſt - $x + 12$,
——— the third's - - - $x + 24$,
——— the fourth's - - - $x + 36$,
——— the fifth's - - - $x + 48$,
And the eldeſt's - - - $x + 60$,
But - - - - - - - - - $x + 60 = x4$ ℘ queſtion.
Therefore - - - - - - $3x = 60$,
Conſequently $x = \frac{60}{3} = 20s.$ what the youngeſt's ſuit coſt.
For $20 + 12 + 12 + 12 + 12 + 12 = 80$, and $\frac{80}{4} = 20$, Proof.

Queſt. 7.

Queſt. 7. The paving of a ſquare, at 2*s.* a yard, coſt as much as the incloſing it at 5*s.* a yard; the ſide of that ſquare is required?

Let - x=ſide of the ſquare,
Then - - $4x$=yards of incloſure,
And - - xx=yards of pavement,
Whence - $20x=(4x\times5=)$price of incloſing,
And - - $2xx=(xx\times2=)$price of paving:
But - $2xx=20x$ by the queſtion.
∴ - - $xx=10x$ } by diviſion, the Anſwer.
And - - $x=10$ }
For $10\times4\times5=100\times2=200$, Proof.

Queſt. 8. A general diſpoſing his army into a ſquare battle, finds he has 284 men more than a perfect ſquare, but increaſing the ſide by one man, he will want 25; how many had he?

Let - - - - - - - x=ſide of the firſt ſquare,
Then - - - $xx+284$=army,
And - $\overline{x+1}\times\overline{x+1}-25$=army,
Hence - - $xx+2x-24=xx+284$;
Then - - - - $2x = 308$;
∴ - - - - - - - $x = 154$;

Conſequently he had $(154\times154+284=)24000$ men, the anſwer.

Queſt. 9. A perſon being aſked how old he was, anſwered, that the product of $\frac{1}{20}$ of the years he had lived, being multiplied by $\frac{5}{8}$ of the ſame, would be his age; what was it?

Suppoſe his age was x years,

Then - - - $\frac{x}{20}\times\frac{5x}{8}=x$ per queſtion.

That is $\left(\frac{5xx}{20\times8}=\right)\frac{xx}{4\times8}=x$,

But by multiplication $xx=32x$,
∴ by diviſion - $x=32$, his age.

Queſt. 10. A man dying, left his wife in a pregnant ſtate ordering by will, that if the child proved a daughter, then his wife ſhould have $\frac{2}{3}$ and the child $\frac{1}{3}$ of his eſtate, but if it was a ſon, then he ſhould have $\frac{2}{3}$ and the mother $\frac{1}{3}$ thereof; now it happened that the mother was delivered of a ſon and daughter; how muſt the eſtate (which was 6300*l.*) be divided among them?

Suppoſe the daughter's ſhare was x£.
Then the mother's would be $2x$,
And the ſon's - - - - $4x$,

For then { the ſon's ſhare is to the mother's / the mother's to the daughter's } as $\frac{2}{3}$ to $\frac{1}{3}$:

But the whole eſtate $6300=7x$ per queſtion.

∴ $\left(\frac{6300}{7}=\right)900=x$, the daughter's ſhare.

QUADRATIC.

QUADRATIC EQUATIONS.

A simple quadratic equation is that which involves the square of the unknown quantity only.

An adfected quadratic equation is that which involves the square of the unknown quantity, together with the product that arises by multiplying it by some known quantity.

Of these equations there are three forms, viz.

$$1.\quad x^2+ax=b$$
$$2.\quad x^2-ax=b$$
$$3.\quad x^2-ax=-b$$

To find the value of x in each of these equations, observe the following rules:

1. Transpose all the terms that involve the unknown quantity to one side of the equation, and the known terms to the other side, and let them be ranged according to their dimensions.

2. When the square of the unknown quantity has any coefficient prefixed to it, let all the rest of the terms be divided by that coefficient.

3. Add the square of half the coefficient of the second term to both sides of the equation, and that side which involves the unknown quantity will then be a complete square.

4. Extract from both sides of the equation, and the value of the unknown quantity will be determined.

EXAMPLE 1. Suppose $x^2+4x=140$, what is the value of x? First, $x^2+4x+4=140+4=144$, by completing the square, Then $\sqrt{x^2+4x+4}=\sqrt{144}$ by evolution,
$\therefore x=12-2=10$.

E. 2. Suppose $x^2-6x+8=80$, what is the value of x?
First, $x^2-6x=80-8=72$ by transposition,
Then $x^2-6x+9=72+9=8$; by completing the square,
And $x-3=\sqrt{81}=9$ by extracting the root,
$\therefore x=9+3=12$.

E. 3. Suppose $2x^2+8x-20=70$, what is x equal to?
First, $2x^2+8x=70+20=90$ by transposition
Then $x^2+4x=45$ by dividing by 2,
And $x^2+4x+4=49$ by completing the square,
Whence $x+2=\sqrt{49}=7$ by extracting the root,
Consequently $x=7-2=5$.

E. 4. Suppose $3x^2-3x+6=5\frac{1}{3}$, *Query* x?
Here $x^2-x+2=1\frac{7}{9}$ by dividing by 3,
And $x^2-x=1\frac{7}{9}-2$ by transposition.
Also $x^2-x+\frac{1}{4}=1\frac{7}{9}-2+\frac{1}{4}=\frac{1}{36}$ by completing the square,
And $x-\frac{1}{2}=\sqrt{\frac{1}{36}}=\frac{1}{6}$ by extracting the root,
$\therefore x=\frac{1}{6}+\frac{1}{2}=\frac{2}{3}$.

Questions

Questions for exercise,

Quest. 1 What two numbers are those, whose difference is 8, product 240?

Let v = the lesser number,
Then will $v+8$=the greater,
And $v^2+8v=240$ by the question,
Whence $v^2+8v+16=240+16=256$ by completing the square,
Also $v+4=\sqrt{256}=16$ by extracting the root,
$\therefore v=16-4=12$ the lesser number
And $v+8=20$ the greater.

Quest. 2. It is required to divide 100 into two such parts, that if they be multiplied together, the product shall be 2100.

Let the greater part above $50=v$.
Then will $50+v$=the greatest part,
And $50-v$=the lesser.
$\therefore \overline{50+v}\times\overline{50-v}$, or $2500-v^2=2100$ by the question,
Whence $v^2=400$, and $v=\sqrt{400}=20$ by evolution,
$\therefore 50+v=70$ the greater part,
And $50-v=30$ the lesser part,
For $70\times30=2100$ proof,

Quest. 3. What number is that, whose third part added to its fourth part will be 21?

Let v=the given number,
Then $\frac{v}{3}+\frac{v}{4}=21$ by the question,
$\therefore 7v=252$, consequently $v=36$ the number required,
For $\frac{36}{3}+\frac{36}{4}=12+9=21$ proof.

Quest. 4. Sold a piece of cloth for 24*l.* and gained as much *per cent.* as the cloth cost me; what was the price of the cloth?

Let v = pounds the cloth cost,
Then $24-v$ = the whole gain,
But $100 : v :: v : \overline{24-v}$ per question,
Or $v^2=100\times\overline{24-v}=2400-100v$
That is, $v^2+100v=2400$,
Whence $v^2+100v+2500=2400+2500=4900$ by completing the square.
And $v+50=\sqrt{4900}=70$ by evolution,
Consequently $v=70-50=20$ the price of the cloth.

Quest. 5.

A Landed Man two Daughters had, they both were very fair,
He gave to each a piece of land, one round, the other square;
At twenty shillings an acre just, each piece its value had,
The shillings that encompass'd it, for it exactly paid;
If 'cross a shilling be an inch as it is very near,
Which had the greater fortune she, who had the round or square?
The learned Youth who this explains,
Shall have the wealthiest for his pains.

Solution for the square piece.

Put x=the side of the square in inches,
Then $4x$=the peremiter. and x^2=the area,

Then ℘ quest. $\frac{x^2\times20}{6272640}=4x$ (6272640=the square inches in an acre.

Hence $x=\frac{6272640}{5}=1254528$ inches,,

$\overline{1254528}|^2=\frac{1573840502784}{6272640}=250905,6$ acres in the square piece.

Which at 20 shillings an acre amounts to 250905*l.* 12*s.* 0*d.* the portion of she who had the square.

Solution for the round piece.

Let x=the diameter, then $3,1416x$=the peremiter, and $,7854x^2$ the area

Hence ℘ quest. $\frac{,7854x^2\times20}{6272640}=3,1416x$

Therefore $x=\frac{19706125,8240}{15,708}=1254528$ inches

And $\overline{1254528}|^2=1573840502784\times,7854=1236094330886,5536$ inches, $\frac{1236094330886,5536}{6272640}=197061,258$ acres in the round piece, which at 20 shillings, ℘ acre=197061*l.* 5*s.* 2*d.* the portion of she who had the round.

		£.	s.	d.
The value of the square piece is,	-	250905	12	0
The value of the round piece is,	-	197061	5	2
In favour of she who had the square,	-	53844	6	10

Quest. 6.

Quest. 6. *Success to the persons, who think and take pains,*
More for good of mankind, than lucre or gain;
If th' fill not their purses, yet honor they'll get,
Men in ages to come will remain in their debt.
Then why should the vulgar sound learning despise,
By learning we're taught to be happy and wise;
Had J....son of Lichfield, ne'er rambl'd in thought;
A God made of matter, he ne'er had found out.
And how such a God, could more matter create,
To more than myself, may appear intricate;
In deep obstruse learning, so far he has gone,
He has almost found out th' philosopher's stone.
How this may go down, with the bishop of Cloyne,
A great virtuoso; I cannot divine,
That matter or body, did ever exist,
He flatly denies, and believes it a jest,
And boldly maintains, it's no other esse,
Then what he is pleas'd, for to call it percipe.
Some people may think, such a tale appears odd,
But if it be true, where is Will J....son's God.
Th' learn'd bishop Burnet, condemns his creator,
And says, he's n't wisely dispos'd of his matter;
And thinks that if he had the architect been,
A world more commodious, we soon should have seen.*
A learned divine, called Tristram Shandy,
Has written good books, for t' read on a Sunday;
By these with th' assistance of Priestley and Hume,
A short way to Heaven, is found I persume.
Kiel says, that the earth on its axis turns round,
And W—hurst, clock-maker, has been under ground,
To see if the wheels it moves by, could be found.
He there had discover'd, how islands were made,
And stratas of different matter are laid;
How mountains were rais'd by the heat of a fire,
And had it been hotter, they'd risen much higher.
He grants, by projection, earth moves round the sun,
But says not, why round on its axis does run,
I humbly beg leave, I a thing may propose,
Which he, or some other, I hope will disclose.
Let the earth be as round as a globe, and suppose,
Its diameter in miles as the margin here shows;* *8050
What time must it take, just once round to revolve,
On its axis ye skilful be pleas'd t' resolve,
Its force centrifugal, at th' equinox line,
To attraction as one t' five score mayn't decline;
Also, in the latitude†, here set below,
The proportion they bear to each other pray show.
And in the same latitude, please to disclose,
What five score pounds weight, by such whirling would lose?

* See Derham's Physico-Theology, page 47. † Latitude 53 degrees, 20 minutes.

Let $d = 42504000$ feet the supposed diameter of the earth, and $s = 16\frac{1}{12}$: then by *Art.* 1189 of *Martin's Institutions* we have $\sqrt{\frac{dxs}{100}} = 2614,58$ feet the velocity ℔ second, when the centrifugal force is $= \frac{1}{100}$ part of gravity; and by uniform motion $2614,58 : 1'' :: 133530566,4$ feet the earth's circumference : 51071 seconds $= 14h.\ 11'\ 11''$ the time of one revolution: now as the centrifugal force is to gravity as 1 to 100, and the same force being every where as the distance from the earth's axis; it follows that 99 pounds under the equinoctial will be 99,4028 in latitude 53° 20′, and under the pole 100 pounds.

Quest. 7. *From the Welch Cross in Birmingham, two men they set out,*
Resolving to travel the whole world about;
The one, he went easterly steering his way,
The other went north, as some people do say;
The first travell'd 7 $\frac{455}{21535}$ miles every day,
The other 11 $\frac{143}{3307}$, but now, I pray,
How many times round the world must they go,
And how many miles will each travel also,*
And how many days must they be to obtain,
To meet at the Cross aforesaid again?

Put $7\frac{455}{21535} = a$, $11\frac{143}{4307} = b$, r for the circumference of the earth $= 21600$ miles, x and y (which must be whole numbers) for the respective number of times that each man must travel round the earth before they meet at the same point from which they set out; then rx = the miles travelled by the first man, ry = those travelled by the second man. Hence $\frac{rx}{a}$ or $\frac{ry}{b}$ = the common time of both their travelling $\therefore \frac{rx}{a} = \frac{ry}{b}$, or $\frac{x}{a} = \frac{y}{b}$, that is, $bx = ay$, in numbers $6512184y = 10233432x$, consequently $y : x :: 10233432 : 6512184 = \frac{6512184}{10233453} = \frac{7}{11}$ in its lowest terms, whose numerator shews the number of times travelled round the earth by the first man, and the denominator those of the second man.

Quest. 8. A ball descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time; required the tower's height, and the whole time of descent?

Let t = the whole time of descent, so will $t - 1$ = the time of descent through the first half of the tower's height, and therefore (the spaces described being always as the squares of the times) we have $tt : \overline{t-1}|^2 :: 2 : 1$, whence $tt - 2 + 1 = \frac{1}{2}tt$, from which $t = 2 + \sqrt{2} = 3,414$, and the tower height $= 187,48$ feet.

* Reckoning 360 degrees, each 60 English miles, according to vulgar computation.

BOOK-KEEPING.

BOOK-KEEPING.

PART IX.

SECTION LXXXII.

BOOK-KEEPING, BY SINGLE ENTRY.

IN book-keeping by ſingle entry, two books are indiſpenſibly neceſſary, viz. Day Book and Ledger; the forms of which may be ſufficiently known by inſpection.

In the Day Book every perſon is written down Debtor to the things he receive, from you on truſt, and Creditor by thoſe which you receive from him.—In the margin of the Day Book are written the pages where the accounts ſtand in the Ledger. Inſtead of theſe marginal figures, ſome make only a daſh with the pen, to ſhew that the account has been poſted, that is, entered in the ledger; but it is better to uſe the figures, for they ſhew not only that the account has been poſted, but likewiſe where to find it in the Ledger, without looking into the alphabet. I have entered in the Day Book what is received, as well as what is delivered, which is very neceſſary in teaching; for the learner ought to make out his Ledger from his Day Book.

There are ſeveral other books kept by moſt merchants, as the Caſh Book; the Book of Houſe Expences, the Invoice Book, the Bill Book, &c. &c.

Directions for the Reader.

Your books being ruled in the proper form, copy into your Day Book one month's accounts; then calculate them upon your ſlate, to find if they be rightly caſt up. Next, rule your ſlate in the form of the Ledger, and upon it poſt the accounts that were copied in the Day Book, with their dates prefixed, obſerving to put on the Debtor ſide of each perſon's account, thoſe accounts to which he is Debtor in the Day Book, and on the Creditor ſide, thoſe by which he is Creditor, and if any accounts conſiſt but of one article, you are to expreſs it particularly with its money in the columns; but if of ſeveral, write *to* or *by* ſundries, placing the ſum of the amounts of all the articles in the columns. After the accounts are properly placed, tranſcribe them into your Ledger, leaving a proper ſpace under each perſon's name, to receive more accounts.

Then under the proper letters in the alphabet, enter thoſe names with the pages where they ſtand in the Ledger; and laſtly, write the Ledger pages to the ſeveral accounts in the Day Book,

Do the ſame with the next month's accounts, and ſo on till the whole be finiſhed: You muſt not enter any perſon's name down again, which has been entered before, till the ſpace firſt aſſigned to it ſhall be filled with articles, and then the account muſt be transferred to a new place; and at the end of the old Ledger, draw out a balance account; placing your Debts on one ſide and your Credits on the other.

THE

THE DAY BOOK.

		£.	s.	d.
	January 1, 1800.			
1	*Mr. John Holland, of York,* Dr.			
	To 7 yards of fine broad cloth, at 18*s*. 6*d*. - -	6	9	6
	— 20 ——— superfine ditto, - 19*s*. 8*d*. - -	19	13	4
	——— 1. ———	26	2	10
1	*George Birch, Esq. of Bath,* Dr.			
	To 6 gallons palm sack, at 8*s*. 6*d*. per gal. - -	2	11	0
	— 9 ——— port, red, - 5 8 - - - -	2	11	0
	— 9 ——— claret, - 8 9 - - - -	3	81	9
	——— 4. ———	9	0	9
1	*Mrs. Sarah Moore,* Dr.			
	To 2lb. green tea, at 18*s*. 0*d*. - - - -	1	16	0
	— 2½ congou, - 9 6 - - - -	1	3	9
	— ½ stone of sugar, 5 0 per stone - - -	0	2	6
	— a lump of sugar, weight 20lb. at 10*d*. per pound	0	16	8
	——— 9. ———	3	18	11
2	*Sir Joseph Johnson,* Dr.			
	To a silver punch bowl, weight 23*oz*. at 5*s*. 10*d*. per *oz*.	6	14	0
	— a tankard, weight 10*oz*. 10*dwts*. - 6 0 -	3	3	0
	——— 20. ———	9	17	0
1	*Sir John Mosley,* Dr.			
	To a ream of fine post paper - - - -	1	5	0
	——— 27. ———			
2	*Mr. John Summers, schoolmaster,* Dr.			
	To 6 cyphering books, at 1*s*. 2*d*. each - -	0	7	0
	— 3 dozen of copy books, 2 4 per dozen - -	0	7	0
	— 4 quires of fools cap, 0 9 per quire - -	0	3	0
	— 1 quire of thin post - - - -	0	0	8
	——— February 5. ———	0	17	8
1	*Mr. Anthony Archer,* Dr.			
	To a ledger, ruled - - - - - - -	1	0	0
	— 5 hundred quils, at 2*s*. per hundred - -	0	10	0
	— 4 reams of thick post, at 1*l*. 2*s*. 4*d*. per ream -	4	9	4
	— 8 reams of fool's cap - 1 1 0 - - - -	8	8	0
	——— 12. ———	14	7	4
1	*Mr. William Grove,* Dr.			
	To 4 gallons of rum, at 12*s*. 0*d*. per gallon -	2	8	0
	— 2 gallons of brandy, 8 0 - - - - -	0	16	0
	— 3 gallons English gin, 4 6 - - - - -	0	13	6
		3	17	6

		£.	s	d.
	February 20.			
2	*William Warner, Esq.* Dr.			
	To 10 *oz.* of nutmegs, at 1*s.* 2*d.* per *oz.* - -	0	11	8
	— 4 pounds coffee, - 4 0 per *lb.* - -	0	16	0
	— 5 —— almonds, - 1 2 —— - - -	0	5	10
	— 8 —— raisins, - 0 8 —— - - -	0	5	4
	27.	1	18	2
1	*Sir John Mosley,* Cr.			
	By cash received of him in full - - - -	1	5	0
	March 22.			
1	*George Birch, Esq. of Bath,* Dr.			
	To 8 gallons sherry, at 6*s.* 4*d.* per gallon - -	2	10	8
	— 12 —— rhenish, - 6 6 —— - -	3	18	0
	— 8 —— Lisbon, - 4 2 —— - -	1	17	6
	April 24.	8	6	2
1	*Mrs. Sarah Moore,* Cr.			
	By cash received of her in full, - - - -	3	18	11
	May 3.			
1	*Mr. John Holland, of York,* Dr.			
	To 25 yards of yard-wide cloth, at 5*s.* 2*d.* per yard	6	9	2
	— 8 ditto drugget, —— - 5 8 ——	2	5	4
	— 9 —— serge, —— - 2 6 ——	7	9	6
	— 36 —— shalloon, —— - 1 8 ——	3	0	0
	14.	12	17	0
2	*Mr. John Flint, of Nottingham,* Dr.			
	To 12 pair worstead stockings, at 4*s.* 2*d.* per pair -	2	10	0
	— 5 — silk ditto -16 4 —— -	4	1	8
	— 16 — thread ditto - 5 0 —— -	4	0	0
	June 3.	10	11	8
3	*Mr. James Davies,* Dr.			
	To 8 quarters of wheat, at 2*l.* 8*s.* 0*d.* per quarter -	19	4	0
	— 4 —— rye, - 1 8 2 —— -	5	12	8
	— 20 —— oats, - 0 10 9 —— -	10	10	0
	12.	35	6	8
2	*Sir Joseph Johnson,* Cr.			
	By a bank note, received of his servant, - - - -	5	0	0
	17.			
1	*Mrs. Sarah Moore,* Dr.			
	To 6 pounds of hard soap, at 6½*d.* per *lb.* - -	0	3	3
	— 4 —— soft ditto, - 6 - - - - -	0	2	0
	— 5 —— starch, - 5 - - - - -	0	2	1
	— 4 dozen of candles, - 6 - - - - -	0	4	0
		0	11	4

		£.	s.	d.
	June 21.			
1	*Mrs. Sarah Moore, Cr.*			
	By cash received in full —— ——	1	11	4
	July 7.			
2	*Mr. John Summers, Schoolmaster, Cr.*			
	By cash received in full —— ——	0	17	8
	28.			
3	*Mr. James Davies, Dr.*			
	To 12 bushels of peas, at 2*s.* 6*d.* per bushel - -	1	10	0
	— 8 ——— beans, 2 2 —— - -	0	17	4
	— 10 ——— malt, 5 0 —— - -	2	10	0
	August 1.	4	17	4
2	*William Warner, Esq. Dr.*			
	To 9 gross of bottles, at 1*l.* 13*s.* 0*d.* per gross -	14	17	0
	— 2 ——— small ditto, 0 12 0 —— -	1	4	0
	— 4 decanters, - - 0 1 4 each - -	0	5	4
	7.	16	6	4
1	*Mr. Anthony Archer, Cr.*			
	By a note upon Mr. John Steventon, for - -	10	0	0
	— cash in full - - - - - -	4	7	4
	21.	14	7	4
1	*Mr. Charles Jones, of Shrewsbury, Dr.*			
	To 24lb. of cochineal, at 1*l.* 2*s.* 6*d.* per lb. -	27	0	0
	— 3 opium 0 8 0 ——	1	4	0
	— 12 rose pink 0 0 10 ——	0	10	0
	September 4.	28	14	0
2	*Mr. John Summers, schoolmaster. Dr.*			
	To 12 schoolmaster's guides, at 2*s.* 2*d.* each -	1	6	0
	— 3 dozen copy books, 2 6 per dozen	0	7	6
	— ream fool's cap, —— ——	1	0	0
	9.	2	13	6
2	*Mr. John Flint, Cr.*			
	By a bank note for —— ——	5	0	0
2	*Mr. John Summers, Dr.*			
	To 6 dozen of Dyche's spelling book's at 10*s.* per dozen	3	0	0
	October 2.			
3	*Mr. Samuel Taylor, Dr.*			
	To 20*lb.* of flax, at 1*s.* per *lb.* ——	1	0	0
	15.			
2	*Mr. John Johnson, of Great Haywood, Dr.*			
	To 4½ *cwt.* Iron, at 18*s.* per *cwt.* ——	4	1	0
	21.			
3	*Mrs. Phebe Young, Cr.*			
	By 60 yards of Irish cloth, at 2*s.* 6*d.* per yard	7	10	0

		£.	s.	d.
	October 27.			
1	*George Birch, Esq.* Cr.			
	By cash in full	17	6	11
	30.			
3	*Mr. Samuel Taylor,* Dr.			
	To 12*lb.* of flax, at 1*s.* 0*d.* per *lb.*	0	12	0
	— 10 ——— 0 6	0	5	0
	November 13.	0	17	0
1	*Mr. John Holland,* Cr.			
	By a bill for	20	0	0
	15.			
3	*Mr. James Davies,* Cr.			
	By cash in full	40	4	0
	22.			
3	*Mr. Thomas Green.* Dr.			
	To 2 dozen knives and forks, at 12*s.* per dozen	1	4	0
	— a set of china	3	10	6
	— a mahogany tea board	0	12	0
	26.	5	6	6
3	*Mr. Thomas Green,* Cr.			
	By 30 ells of holland, at 5*s.* 2*d.* per ell	7	15	0
	28.			
2	*Sir Joseph Johnson,* Cr.			
	By cash in full	4	17	2
	December 1.			
2	*Mr. John Summers, schoolmaster,* Cr.			
	By cash in full	5	13	6
	3.			
1	*Mr. Anthony Archer,* Dr.			
	To a cask of rum	10	0	0
	6.			
2	*William Warner, Esq.* Cr.			
	By cash in full	18	4	6
	8.			
3	*Mr. John Hunter, of Friesely,* Dr.			
	To 4 tons of coals, at 7*s.* 6*d.* per ton	1	10	0
	10.			
1	*Mr. William Grove,* Cr.			
	By cash in full	3	17	6
	12.			
3	*Mr. Carless,* Dr.			
	To a tun of oil, containing 236 gallons, at 2*s.* 6*d.* per gallon	29	10	0
	13.			
2	*Mr. John Johnson,* Cr.			
	By cash in full	4	1	0

		£.	s.	d.
	December 18.			
2	*Mrs. Hill, Dr.*			
	To a lump of sugar, wt. 26*lb.* at 12*d.* per *lb.* - -	1	6	0
	23.			
3	*Mr. John Young, Dr.*			
	To 4 *cwt.* 2 *qrs.* cheese, at 32*s.* per *cwt.* - -	7	4	0
2	*Mrs. Hill, Cr.*			
	By cash in full - - - - - - - -	1	6	0

LEDGER.

The Alphabet.

A		**B**		**C**	
Mr. Anth. Archer	1	George Birch, Esq.	1	Mr. Carless - -	3
D		**E**		**F**	
Mr. James Davies	3			Mr. John Flint -	3
G		**H**		**I**	
Mr. William Grove	2	Mr. John Holland	1	Sir Joseph Johnson	2
Mr. John Grove	1	Mrs. Hill - -	2	Mr. Charles Jones	1
Mr. Thomas Green	3			Mr. John Johnson	2
K		**L**		**M**	
				Mrs. Sarah Moore	1
				Sir John Mosley -	1
N		**O**		**P**	
Q		**R**		**S**	
				Mr. John Summers	2
T		**V**		**W**	
Mr. Samuel Taylor	3			Wm. Warner, Esq.	2
X		**Y**		**Z**	
		Mrs. Phebe Young	3		
		Mr. John Young	3		

		£.	s.	d.			£.	s.	d.
1) 1800	*Dr.* Mr. *John*				1800	*Holland, of York,* *Cr.*			
January 1	To sundries - - - - - -	26	2	10	Nov. 13	By a bill for - - - - -	20	0	0
May 3	To sundries - - - - - -	12	17	0		By cash remains to balance - -	18	19	10
		38	19	10			38	19	10
1800	*Dr.* *George*				1800	*Birch, Esq. of Bath,* *Cr.*			
July 1	To sundries - - - - - -	9	0	9	Oct. 27	By cash in full - - - - -	17	6	11
March 22	To sundries - - - - - -	8	6	2					
		17	6	11					
1800	*Dr.* *Mrs. Sarah*				1800	*Moore,* *Cr.*			
January 4	To sundries - - - - - -	3	18	11	April 24	By cash in full - - - - -	3	18	11
June 17	To sundries - - - - - -	1	11	4	June 21	By cash in full - - - - -	1	11	4
		5	10	3			5	10	3
1800	*Dr.* *Sir John*				1800	*Mosley,* *Cr.*			
Jan. 20	To a ream of paper - - -	1	5	0	Feb. 27	By cash in full - - - - -	1	5	0
1800	*Dr.* *Mr. Anthony*				1800	*Archer,* *Cr.*			
February 5	To sundries - - - - - -	14	7	4	August 7	By sundries - - - - - -	14	7	4
Dec. 3	To a cask of rum - - - - -	10	0	0		By cash remains to balance - -	10	0	0
		24	7	4			24	7	4
1800	*Dr.* *Mr. William*				1800	*Grove,* *Cr.*			
Feb. 12.	To sundries - - - - - -	3	17	6	Dec. 10	By cash in full - - - - -	3	17	6
1800	*Dr.* *Mr. Charles*					*Jones, of Shrewsbury,* *Cr.*			
August 21	To sundries - - - - - -	28	14	0		By cash remains to balance - -	28	14	0

Date	Dr.	£	s.	d.	Date	Cr.	£	s.	d.
2) 1800	*Dr.* *Mr. John*	£.	s.	d.	1800	*Summers, Schoolmaster,* *Cr.*	£.	s.	d.
Jan. 27	To sundries - - - -	0	17	8	July 7	By cash in full - - -	0	17	8
Sept. 4	To sundries - - - -	2	13	6	Dec. 1	By cash in full - - -	5	13	6
— 9	To 6 dozen of spelling books, at 10*s*	3	0	0					
							6	10	2
		6	10	2					
1800	*Dr.* *William*				1800	*Warner, Esq.* *Cr.*			
Feb. 10	To sundries - - - -	1	18	2	Dec. 6	By cash in full - - -	18	4	6
August 1	To sundries - - - -	16	6	4					
		18	4	6					
1800	*Dr.* *Sir Joseph*				1800	*Johnson,* *Cr.*			
January 9	To sundries - - - -	9	17	0	June 12	By a bank note - - -	5	0	0
					Nov. 28	By cash in full - - -	4	17	0
							9	17	0
1800	*Dr.* *Mr. John*				1800	*Flint, of Nottingham,* *Cr.*			
May 14	To sundries - — —	10	11	8	Sept. 9	By a bank note - - -	5	0	0
						Cash remains to balance -	5	11	8
							10	11	8
1800	*Dr.* *Mrs.*				1800	*Hill,* *Cr.*			
Dec. 18	To a lump of sugar, wt. 26*lb* at 12*d*	1	6	0	Dec. 23	By cash in full - - -	1	6	0
1800	*Dr.* *Mr. John*				1800	*Johnson,* *Cr.*			
Oct. 15.	To 4½ *cwt* of iron, at 18*s* —	4	1	0	Dec. 13	By cash in full - - -	[illegible]	1	0

		£.	s.	d.			£.	s.	d.
3) 1800	*Dr.* *Mr. James*					*Davies,* *Cr.*			
June 3	To ſundries - - -	35	6	8	1800				
July 28	To ſundries - - -	4	17	4	Nov. 15	By caſh in full - - - -	40	4	0
		40	4	0					
1800	*Dr.* *Mr. Samuel*					*Taylor,* *Cr.*			
October 2	To 20 pounds of flax, at 1*s* -	1	0	0					
30	To ſundries - - -	0	17	0		Caſh remains to balance - - -	1	17	0
		1	17	0					
1800	*Dr.* *Mrs. Phebe*				1800	*Young,* *Cr.*			
	To balance - - -	7	10	0	Oct. 21	By 60 yards of Iriſh cloth, at 2*s.* 6*d.*	7	10	0
1800	*Dr.* *Mr. Thomas*				1800	*Green,* *Cr.*			
Nov. 22	To ſundries - - -	5	6	6	Nov. 26	By 30 ells of Holland, at 5*s.* 2*d.* -	7	15	0
	Caſh remains to balance - -	2	8	6					
		7	15	0					
1800	*Dr.* *Mr. John*					*Hunter,* *Cr.*			
Dec. 8	To 4 tun of coals, at 7*s* 6*d* -	1	10	0		Caſh remains to balance - - -	1	10	0
1800	*Dr.* *Mr.*					*Carleſs,* *Cr.*			
Dec. 12	To a tun of oil - - -	29	10	0		Caſh remains to balance - - -	29	10	0
1800	*Dr.* *Mr. John*					*Young,* *Cr.*			
Dec. 23	To 4 hundred 2 quarters cheeſe, at 32*s*	7	4	0		Caſh remains to balance - - -	7	4	0

BALANCE.

1800	Dr. Balance.	£.	s.	d.	Cr.	£.	s.	d.
	To Mr. John Holland, due to me -	18	19	10	By Mrs. Phebe Young - - -	7	10	0
	To Mr. Anthony Archer - -	10	0	0	By Mr. Thomas Green - - -	2	8	6
	To Mr. John Flint - - -	5	11	8		9	18	6
	To Mr. Samuel Taylor - - -	1	17	0				
	To Mr. John Hunter - - -	1	10	0				
	To Mr. Carlefs - - - -	29	10	0				
	To Mr. John Young - - -	7	4	0				
		75	2	6				

BOOK-KEEPING BY DOUBLE ENTRY,

ACCORDING TO

THE ITALIAN METHOD.

THIS method was firſt invented in Italy, for which reaſon it is called the Italian Method; and it is ſaid to be by Double Entry, becauſe every article is twice entered in the Ledger.

The Books generally uſed in this way of keeping accounts, are three, viz. the Waſte Book, the Journal, and Ledger, of all which I ſhall give a ſhort account.

1. OF THE WASTE BOOK.

The Waſte Book contains a complete memorial of every tranſaction in buſineſs, recorded promiſcuouſly as they happen with repect to time.

This book opens with an inventory of the perſon's money, effects, and debts, which at his firſt ſetting out in trade, are to be gathered from the particulars that make up his real eſtate,

2. OF THE JOURNAL.

The Journal agrees with the Waſte Book in the form or manner of ruling, dating, and order or ſucceſſion of accounts, according to their dates; and differs from it by having the Debtors and Creditors of all accounts ſpecified.

On the right hand margins of each folio, or page, of the Journal and Waſte Book, are ruled three columns for pounds, ſhillings, and pence; and on the left hand margins, a column to receive the figures expreſſing the folios, or pages, where the ſame accounts are entered in the ſucceeding book; viz. in the Waſte Book margin are ſet the correſponding Journal pages, and in the Journal margin, the Ledger pages.

3. OF THE LEDGER.

The Ledger is a large volume, containing all the tranſactions of a man's affairs, in ſuch order, that thoſe belonging to every different ſubject, lie together in one place, making ſo many diſtinct accounts.—In this book, all the accounts diſperſed in the Journal, are drawn out and tituled at the top Debtor and Creditor.

To form each accouut, two pages are required, oppoſite to each other; that on the left hand ſerving for Debtor, the other for Creditor; by which means, at any time, the merchant may be ſatisfied how any particular account ſtands: And for the more readily finding any particular account, the Ledger has always an alphabet prefixed to it: The right

right hand margin of each page is ruled into three columns for money, and one for the figures expreffing the folio's where the fame articles ftand, on the other folio of fome other accounts; and on the left hand margin is formed a column, for the dates of the articles.

In any entry, to know what to make Debtor, and what Creditor, obferve the following RULES.

1. What money, goods, and wares you have in poffeffion, or are owing to you, make each particular account Debtor to Stock, and Stock Creditor by each account.

2. What you owe to any perfon, make Stock Debtor for fo much to the perfon, and the fame perfon Creditor by Stock.

3. What money is owing to you, make the perfon owing Debtor, and Stock Creditor.

In buying and felling GOODS.

1. To enter goods bought in, for ready money, make the goods bought Dr. and cafh Cr.

2. To enter goods bought upon truft, make the goods Dr. and the feller Cr.

3. To enter goods bought for part ready money and part truft, make the goods Dr. and the feller Cr.

4. To enter goods fold for ready money, make the cafh Debtor, and the goods Creditor.

5. To enter goods fold upon truft, make the buyer Debtor, and the goods Creditor, the fame when feveral payments are to be made to you, mentioning in the journal the feveral times of payment.

6. To enter goods fold for part ready money and part truft, make the buyer Debtor for the whole, and the goods Creditor; then make the cafh received Debtor, and the buyer Debtor, for what remains unpaid.

BARTERING.

1. When you give one fort of goods for another fort of equal value, make the goods received Debtor, and thofe you part with Creditor.

2. When you give one fort of goods for feveral other forts of equal value, make each particular fort of goods received Debtor for its refpective value, and the goods delivered by fundry accounts Creditor for the whole.

BORROWING AND LENDING.

1. Make cafh Creditor for what you lend, and the perfon that borrows Debtor.

2. What money you borrow, make cafh Debtor, and the perfon lending Creditor.

BILLS.

1. When you draw bills of exchange upon your factor, and receive the contents, Debtor cafh, and Creditor the factor's account current.

2. When

2. When your factor draws bills of exchange upon you for goods bought by him abroad, and you pay the contents, make the drawer Debtor and cash Creditor.

3. When bills of exchange are drawn by one of my factors on another, and I receive the contents at usance, I Debtor the accepting man, and Creditor the factor drawing.

PROFIT AND LOSS.

1. What money you gain, win, or receive gratis, make cash Debtor to profit and loss, and profit and loss Creditor by its value.

2. What money or goods you give away, lose, or is spoiled, &c. is Creditor, and profit and loss is debtor for its value.

FOREIGN TRADE.

1. When goods are sent to your factor abroad, make the voyage Debtor and the goods Creditor.

2. When you have advice that your factor has received the goods, then he becomes Debtor to the voyage, and the voyage Creditor. If he gains by selling the goods, he becomes Debtor to profit and loss, on account of the gain.

3. If he returns the goods he is Creditor, and the goods are Debtor.

HOUSE EXPENCES.

When you pay servants wages, house expences, &c. make profit and loss Debtor, and cash Creditor for its value.

ERRORS.

If you have entered any thing in your ledger under a wrong title, or false, you need not blot it out, but make this mark (X) in the margin against it, and write on the contrary side *error per contra*, with the sum against it, and the same mark in the margin; and the account will be right.

Directions for the learner.

Having ruled your books according to the forms of the following specimens, copy into your waste book the first month's transactions as they stand in the following waste books, omitting the left hand marginal figures, which are to be inserted according to the following directions.

Enter these articles one by one in the journal, according to the journal form, and when any article is entered in the journal, turn to the same article in the waste book, and directly against it, in the margin, write the number of the folio where it stands in the journal.

THE WASTE BOOK.

			£.	s.	d.
	Birmingham, January 6, 1800.				
	An inventory of the money, goods, and debts due to or by me, A.... B....				
1	I have in ready money - - -	100*l* 0*s* 0*d*			
	— 10 bags of hops, each 1*cwt* at 3*l* per bag	30 0 0			
	— 4 pipes of wine, at 20*l* 5*s* per pipe -	81 0 0			
	— 6 pieces of broad cloth, at 25*l* 10*s* per piece	153 0 0			
	— Thomas Rigby owes me on demand	10 10 0			
			374	10	0
	I owe as follows,				
1	To John Fletcher on bond - -	16*l* 0*s* 0*d*			
	To Samuel Turner, on account - -	2 0 0			
			18	0	0
	January 1, 1800.				
1	Bought of John Jackson half a ton of cheese, at 20*l* 5*s* per ton, for which I paid ready money - -		10	2	6
	3.				
1	Bought of Richard Perks 4 hogsheads of cyder, at 2*l* 5*s* per hogshead				
	paid in ready money - -	5*l* 0*s* 0*d*			
	remains due - -	4 0 0			
			9	0	0
	5.				
1	Bought of Samuel Tonks 2 hogsheads of tobacco, at 10*l* 6*s* per hogshead, to pay in three months -		20	12	0
	7.				
1	Sold to John wheeler 4 bags of hops, at 4*l* 10*s* per bag, for which I received ready money -		18	0	0
	10.				
1	Sold to Samuel Tonkes 2 pipes of wine, at 25*l* per pipe				
	received in part -	30*l* 0*s* 0*d*			
	remains due on demand -	20 0 0			
			50	0	0
	16.				
1	Sold John Jackson a piece of broad cloth, at 28*l* 10*s* to be be paid an one month - - - -		28	10	0
	19.				
1	Bartered 4 hogsheads of cyder, at 3*l* per hogshead, for half a tun of cheese at the same value -		12	0	0
	22.				
1	Lent Abraham Taylor the sum of 5*l* to be paid on demand		5	0	0
	26.				
1	Drawn a bill on Thomas Rigby, to be paid at sight		5	0	0
	28.				
	Thomas Rigby paid the bill when I drew on him		5	0	0

		£	s.	d.
	January 30.			
2	John Fletcher has drawn a bill on me, payable at sight	0	0	0
	February 5.			
2	Paid the bill to John Fletcher - - - -	0	0	0
	7.			
2	Borrowed of James Steventon the sum of ten pounds	10	0	0
	9.			
2	Paid John Fletcher a quarter's interest, due at Christmas last	6	4	0
	10.			
2	This day dined with the Honorable B. C. Esq. and gave his servants - - -	0	5	0
	11.			
2	Won to-day at Quadrille - -	2	2	0
	13.			
2	Houshold expences last month - -	4	15	0
	14.			
2	Paid my book-keeper a quarter's salary, and other expences	5	0	0
	16.			
2	Shipped 2 *cwt* of Gloucester cheese, in the Diligence, John Lowe, master, consigned to William Cartwright at the Hague - - - -			
	the cheese valued at - 2*l* 10*s* 0*d*			
	paid freight and custom - 0 12 0			
		3	2	0
	20.			
2	Received advice that my factor has received the cheese safe at the Hague, - - -			
	sold for - - 6*l* 10*s* 0*d*			
	charges - - 0 18 0			
	being deducted makes -	5	11	6
	23.			
2	Received from my factor, William Cartwright, at the Hague, a chest of sugar, weight net 3 *cwt* 2 *qrs* valued at 4*l* 8*s* 0*d*			
	paid freight and custom here - 1 3 6			
		5	11	6
	25.			
2	Received a legacy left me by my uncle	5	0	0
	27.			
2	Paid church and poor - -	0	2	6

THE JOURNAL.

Birmingham, January 1, 1800.

	£	s.	d.	£	s.	d.
Sundries Dr. to Stock - -	£.374	10	0			
Cash in ready money - - - -	100	0	0			
Hops, for 10 bags, at 3*l.* per bag - -	30	0	0			
Wine, for 4 pipes, at 20*l.* 5*s.* per pipe -	81	0	0			
Broad cloth, for 6 pieces, at 15*l.* 10*s.* per piece	153	0	0			
Thomas Rigby, per note on demand -	10	10	0			
				374	10	0
Stock Dr. to sundries - - -	18	0	0			
To John Fletcher, on bond - - - -	16	0	0			
To Samuel Turner, on account - -	2	0	0			
				18	0	0
January 1, 1800.						
Cheshire cheese Dr. to Cash - -	£.10	2	6			
For half a tun - - - - - - - -				10	2	6
3.						
Cyder Dr. to sundry accounts - -	9	0	0			
To cash paid - - - - - - -	5	0	0			
To Richard Perks, remains due at 1 month	4	0	0	9	0	0
5.						
Tobacco Dr. to Samuel Tonks - -	20	12	0			
For two hogsheads, at 10*l.* 6*s.* per *hhd.* to pay in 3 months				20	12	0
7.						
Cash Dr. to Hops - - - - - -	18	0	0			
Received for 4 bags, at 4*l.* 10*s.* per bag - -				18	0	0
10.						
Sundry accounts Dr. to Wine - -	50	0	0			
Cash received in part for 2 pipes - -	30	0	0			
Samuel Tonks, for the rest on demand -	20	0	0			
				50	0	0
16.						
J. Jackson Dr. to broad cloth, for 1 *piece value*	28	10	0			
To be paid in one month - - - - - -				28	10	0
19.						
Cheese Dr. to Cyder - - - -	12	0	0			
For 4 hogsheads; received in barter half a tun of Gloucester cheese, of the same value - - - - -				12	0	0
22.						
Abraham Taylor Dr. to Cash - -	5	0	0			
To pay on demand - - - - - - -				5	0	0
26.						
Bills received Dr. to Thomas Rigby						
For one drawn on him, to be paid at sight - -				5	0	0

		£.	s.	d.
January 28.				
Cash Dr. to bills receivable				
Received the bill of Thomas Rigby - -		5	0	0
30.				
John Fletcher, Dr. to bills payable				
To one to be paid at sight - - -		10	0	0
February 5.				
Bills payable Dr. to cash -	£.10 0 0			
Paid John Fletcher the bill - - -		10	0	0
7.				
Cash Dr. to James Steventon for -	10 0 0			
Borrowed of him - - - -		10	0	0
9.				
Profit and loss Dr. to cash -	4 0 0			
Paid John Fletcher a quarter's interest - -		0	4	0
10.				
Profit and loss Dr. to cash -	0 5 0			
For 5*s*. given to B. C. Esq. servants - -		0	5	0
11.				
Cash Dr. to profit and loss -	2 2 0			
Won at quadrille - - - -		2	2	0
13.				
Profit and loss Dr. to Cash -	4 15 0			
For one month's house expences - -		4	15	0
14.				
Profit and loss Dr. to cash	15 0 0			
For a quarter's salary and other expences -		15	0	0
16.				
Voyage to the Hague Dr. to sundry accounts	3 2 0			
To cheese, 2*cwt* shipped on board, valued at	2 10 0			
Paid freight and custom - -	0 12 0			
		3	2	0
20.				
William Cartwright, Dr. to sundry accounts	5 11 6			
To a voyage to the Hague -	3 2 0			
To profit and loss gained by selling the goods	2 9 6			
		5	11	6
23.				
Sugar Dr. to sundry accounts -	5 11 6			
To William Cartwright, for one chest received weight 3*c* 2*qrs* valued at -	4 8 0			
To cash paid freight and custom -	1 3 6			
		5	11	6
25.				
Cash Dr. to profit and loss -	5 0 0			
For a legacy left me by my uncle - -		5	0	0
27.				
Profit and loss Dr. to cash -	0 2 6			
Paid to church and poor - - -		2	0	6

THE LEDGER, 1800.

Index or Alphabet.

A		**B**		**C**	
		Broad cloth folio	1	Cash - - - -	1
		Bills receivable -	2	Cheese - - - -	2
		Bills payable - -	2	Cyder - - - - -	2
		Balance - - -	3	Cartwright, William	2
D		**E**		**F**	
				Fletcher, John -	1
G		**H**		**I**	
		Hops - - - -	1	Jackson, John -	2
K		**L**		**M**	
N		**O**		**P**	
				Perks, Richard -	2
				Profit and Loss -	3
Q		**R**		**S**	
		Rigby, Thomas -	1	Stock - - - -	1
				Steventon, James -	3
				Sugar - - - -	3
T		**V**		**W**	
Tonks, Samuel -	2	Voyage to the Hague	3	Wine - - - -	1
Tobacco - - -	2				
Taylor, Abraham -	2				
X		**Y**		**Z**	

		Fol.	£.	s.	d.
1) 1800	*Stock* Dr.				
Jan. 1	To sundry accounts as per journal -	1	18	0	0
	Cash Dr.				
Jan. 1	To stock - - -	1	100	0	0
7	To hops - - -	1	18	0	0
10	To wine - - -	1	30	0	0
28	To bills receivable - -	2	5	0	0
Feb. 7	To James Steventon - -	3	10	0	0
11	To profit and loss - -	3	2	2	0
25	To profit and loss - -	3	5	0	0
			170	2	0
1800	*Hops* Dr.				
Jan. 1	To stock 10 bags, at 3*l* per bag -	1	30	0	0
	To profit and loss gained -	3	6	0	0
			36	0	0
	Wine Dr.				
Jan. 1	To stock 4 pipes, at 20*l* 5*s* per pipe	1	81	0	0
	To profit and loss gained -	3	9	10	0
			90	10	0
	Broad cloth Dr.				
Jan. 1	To stock 6 pieces, at 25*l* 10*s* per piece	1	153	0	0
	To profit and loss gained -	3	3	0	0
			156	0	0
	Thomas Rigby Dr.				
Jan. 1	To stock per note on demand -	1	10	10	0
	John Fletcher Dr.				
Jan. 30	To bills payable for one drawn on me payable at sight - - -	2	10	0	0
	To balance remains due to me -	3	6	0	0
			16	0	0

1)1800	*Per contra* Cr.	Fol.	£.	s.	d.
Jan. 1.	By sundry accounts as per journal -	1	374	10	0
	Per contra Cr.				
Jan. 1	By cheese paid for one ton — —	2	20	10	0
3	By cyder — — —	2	5	0	0
22	By Abraham Taylor — —	2	5	0	0
Feb. 5	By bills payable — — —	2	10	0	0
9	By profit and loss — —	3	0	4	0
10	By profit and loss — —	3	0	5	0
13	By ditto ditto — —	3	4	15	0
14	By ditto ditto — —	3	15	0	0
16	By voyage to the Hague paid freight and custom — — —	3	0	12	0
23	By freight and custom — —	3	1	3	6
27	By profit and loss — —	3	0	2	6
	By balance remaining in hand —	3			
1800	*Per contra* Cr.				
Jan. 7.	By cash for 10 bags, at 4*l.* per bag —	1	18	0	0
	By balance remaining in hand, 6 bags at 3*l.* per bag — — — —	3	18	0	0
			36	0	0
	Per contra Cr.				
Jan. 10.	By sundry accounts as per journal —				
	By cash received in part for 2 pipes £.30 0 0	1			
	By Samuel Tonks, remains due on demand —— 20 0 0				
			50	0	0
	By balance remains 2 pipes — —	3	40	10	0
			90	10	0
	Per contra Cr.				
Jan. 16.	By John Jackson one piece, to be paid in one month — — —	1	28	10	0
	By balance remains 5 pieces —	3	127	10	0
			156	0	0
	Per contra Cr.				
Jan. 26.	By bill receivable for one drawn on him, to be paid at sight — — —	2	5	0	0
	By balance remains due on demand —	3	5	10	0
			10	10	0
	Per contra Cr.				
Jan. 1.	Due on bond — — —	1	0	16	0

2) 1800			Fol.	£.	s.	d.
		Samuel Tonks, Dr.				
Jan.	10	To wine on demand — —	2	20	0	0
		To balance due to him — —	3	2	12	0
				22	12	0
		Cheese Dr.				
Jan.	1	To cash paid for half a ton of Gloucester	1	10	2	6
	19	To cyder bartered 4 hogsheads, at 3*l.* per hhd. for half a ton of Cheshire cheese, at the same value — — —	2	12	0	0
		To profit and loss gained — —	3	0	2	0
				22	4	6
		Cyder Dr.				
		To sundry accounts as per journal —				
Jan.	3.	To cash paid in part for 4 hogsheads £. 5 0 0	1			
		To Richard Perks, remains due 4 0 0	2			
				9	0	0
		To profit and loss gained — —	3	3	0	0
				12	0	0
		Richard Perks, Dr.				
Jan.	3.	To balance remains due to him —	3	4	0	0
		Tobacco Dr.				
Jan.	5.	To Samuel Tonks 2 hogsheads, to be paid in 3 months, at 10*l.* 6*s.* per hhd —	2	20	12	0
		John Jackson Dr.				
Jan.	16.	To broad cloth for one piece, to be paid in one month — — —	1	28	10	0
		Abraham Taylor Dr.				
Jan.	22.	To cash 5*l.* lent him to be paid on demand	1	5	0	0
		Bills receivable Dr.				
Jan.	26.	To Thomas Rigby, for one drawn on him, to be paid at sight — —	1	5	0	0
		Bills payable Dr.				
Feb.	5	To cash paid John Fletcher, his bill drawn on me payable at sight — —	1	10	0	0
		William Cartwright, Dr.				
Feb.	20.	To sundry accounts as per journal —				
		To a voyage to the Hague — £. 3 2 0	3			
		To profit and loss gained by selling the goods — — — 2 9 6	3	5	11	6

3 U

			Fol.	£.	s.	d.
2) 1800		*Per contra* Cr.				
Jan.	1	By stock due on account -	1	2	0	0
	5	By tobacco 2 hogsheads, at 10*l* 6*s* per hogshead	2	20	12	0
				22	12	0
		Per contra Cr.				
Jan.	19	By a voyage to the Hague, shipped on board 2 *cwt* of Cheshire cheese -	3	2	10	0
		By balance remains 2 *cwt* of Gloucester	3	10	2	6
		By balance remains 8 *cwt* of Cheshire	3	9	12	0
				22	4	6
		Per contra Cr.				
Feb.	16	By Cheshire cheese half a ton received in barter for 4 hhds. at 3*l* per hhd. -	2	12	0	0
		Per contra Cr.				
Jan.	3	By cyder remains due at 3 months, in part for 4 hogsheads - -	2	4	0	0
		Per contra Cr.				
Jan.	5	By balance remains 2 hhds. at 10*l* 6*s* per hhd.	3	20	12	0
		Per contra Cr.				
		By balance remains due to me -	3	28	10	0
		Per contra Cr.				
		By balance remains due to me -	3	5	0	0
		Per contra Cr.				
Jan.	28	By cash received the bill -	1	5	0	0
		Per contra Cr.				
Jan.	30	By John Fletcher, for one bill drawn on me, to be paid at sight - - -	1	10	0	0
		Per contra Cr.				
Feb.	23	By sugar received 1 chest, net weight 3 *cwt* 2 *qrs* valued at - - £4 8 0	3			
		By balance remains due to me - 1 3 6	3			
				5	11	6

			Fol.	£.	s.	d.
3) 1800	*Profit and loss*	*Dr.*				
Feb. 9	To cash paid John Fletcher	—	1	0	4	0
10	To cash given to B. C. Esq. servants	—	1	0	5	0
13	To cash for one month's houshold expences		1	4	15	0
14	To cash for a quarter's salary to my book-keeper and other expences —		1	15	0	0
27	To cash paid to church and poor	—	1	0	2	6
	To stock gained by trade —	—	1	10	17	0
				31	3	6
	James Steventon	*Dr.*				
	To balance remains due on demand	—	3	10	0	0
	Voyage to the Hague	*Dr.*				
Feb. 16	To sundry accounts as per journal	—	3	3	2	0
	Sugar	*Dr.*				
Feb. 23.	To sundry accounts as per journal	—				
	To William Cartwright for one chest, received net weight 3*c.* 2*qrs.* valued at	—	3	4	8	0
	To cash paid freight and custom	—	1	1	2	6
				5	11	6

BALANCE.

			Fol.	£.	s.	d.
1800	*Balance*	*Dr.*				
	To cash remaining in my hands	—	1	117	17	6
	To hops, 6 bags remains, at 3*l.* per bag		1	18	0	0
	To wine, remains 2 pipes, at 20*l.* 5*s.* per pipe		1	40	10	0
	To broad cloth, remains 5 pieces, at 25*l.* 10*s.* per piece — —	—	1	127	10	0
	To Thomas Rigby, remains due on demand		1	5	10	0
	To Gloucester cheese, remains 10 cwt. at 20*l* 5*s.* per ton — —	—	2	10	2	6
	To Cheshire cheese, remains 8 cwt. at 24*l.* per ton — —	—	2	9	12	0
	To tobacco, remains 2 hhds. at 10*l.* 6*s.* per hhd.			20	12	0
	To John Jackson, remains due to me	—	2	28	10	0
	To Abraham Taylor, remains due to me		2	5	0	0
	To William Cartwright, remains due to me		2	1	3	6
	To sugar, remains in my hands	—	3	5	11	6
				389	19	0

Date	Per contra Cr.	Fol.	£.	s.	d.
3) 1800	*Per contra* Cr.	Fol.	£.	s.	d.
Feb. 11.	By cash won at quadrille — —	1	2	2	0
20	By William Cartwright — —	3	2	9	6
25	By profit and loss for a legacy left by my uncle	1	5	0	0
	By hops gained — — —	1	6	0	0
	By wine gained — — —	1	9	10	0
	By broad cloth — — —	1	3	0	0
	By cheese — — —	2	0	2	0
	By cyder — — —	2	3	0	0
			31	3	6

Date	*Per contra* Cr.	Fol.	£.	s.	d.
Feb. 7.	By cash borrowed — — —	1	10	0	0

Date	*Per contra* Cr.	Fol.	£.	s.	d.
Feb. 20.	By William Cartwright, who has received the goods — — —	2	2	2	0

Date	*Per contra* Cr.	Fol.	£.	s.	d.
23	By balance remains one chest, net weight 3cwt. 2qrs. valued with charges —	3	5	11	6

Date	*Per contra* Cr.	Fol.	£.	s.	d.
1800.	By John Fletcher, remains due to him —	1	6	0	0
	By Samuel Tonks, remains due to him —	2	2	12	0
	By Richard Perks, remains due to him —	2	4	0	0
	By James Steventon, remains due to him —	3	10	0	0
	By Stock, for my neat estate — —	1	367	7	0
			389	19	0

Forms of Acquittances, Notes, Bills of Exchange, &c.

ACQUITTANCES UPON RECEIPT OF MONEY.

A general Receipt.

RECEIVED March 12, 1800, of Mr. John Smith, the sum of thirty pounds, in full of all demands.

£.30 0 0 By me, W. T.

An Acquittance, proper to be given by a servant, when he receives money for the use of his master,

RECEIVED March 21, 1800, of Mr. Thomas Brown, nine pounds, six shillings, for the use of my master, Daniel Young, on account.

£.9 6 0 By me, A. B.

A Receipt or Acquittance for Rent paid.

RECEIVED Nov. 16, 1800, of Mr. John Simpson, twenty pounds, for a quarter's rent, due at Michaelmas last.

£.20 0 0 By me, B. C.

An Acquittance for Debt, received of a third hand.

RECEIVED the 1st day of March, 1800, of Mr. A. C. by the hands of Mr. G. D. the sum of eight pounds, ten shillings, in full, for certain goods, &c. bought by the said A. C. of me, in full of all demands.

£.8 10 0 By me, W. P.

A Receipt for Interest due on Bond.

RECEIVED this, &c. of Mr. A. B. the sum of ten pounds. in full for one year's interest of 200*l*. due to me at Christmas last, on bond, from the said A. B.

£.10 0 0 By me, C. D.

An Acquittance for a Legacy.

RECEIVED this, &c. of A. B. executor of the last will and testament of C. D. late of ——, &c. deceased, the sum of fifty pounds, in full of a legacy bequeathed to me, in and by the last will and testament of the said C. D. in full of all demands

£.50 0 0 By me, Y. R.

A Receipt

A Receipt for Writings entrusted in a Person's Hands.

RECEIVED this, &c. of A. B. of ——, &c. two several deeds or conveyances, one thereof being a lease, and the other a release, made between ——, &c. for which several deeds or writings, I hereby promise to be accountable, and to re-deliver the same to the said A. B. on demand.

Witness, *Hugh Whatley.*

PROMISSORY NOTES.

The form of one payable on Demand.

I promise to pay to A. B. or order, the sum of thirty pounds, on demand, for value received. Witness my hand this first day of March, 1800.

£30 0 0 B. C.

Note. All promissory notes, bills of exchange, or drafts, being negotiable or transferable, for the payment of twenty shillings, or any sum of money above that sum and less than five pounds, must specify the names and places of abode of the persons respectively, to whom, or to whose order, the same shall be made payable, and shall bear date before or at the time of drawing or issuing thereof, and shall be made payable within the space of twenty-one days next after the day of the date thereof, and shall not be transferable or negociable after the time thereby limited for the payment thereof.

The form of one payable at a certain time.

Birmingham, 1st of March, 1800.

TWENTY-ONE days after date, I promise to pay to A. B. or his order, the sum of four pounds, for value received.

Witness, J. K. J. T.

The Indorsements,

12th of March, 1800.

Pay the contents to E. F. of Birmingham, or his order.

Witness, T. B. A. B.

Note. Promissory notes and book debts, if not legally demanded in the space of six years, cannot be recovered by law.

INLAND BILLS OF EXCHANGE.

Form of one payable at sight.

£.100 0 0 *Birmingham, Jan.* 1, 1800.

AT sight pay Mr. R. B. or order, the sum of one hundred pounds, the value received of B. C. and place it to account, as per advice from

To Mr. A. B. merchant,
Bath-street, Bristol. E. D.

Form of one payable after sight.

£.96 12 6 *Birmingham, June* 1, 1800.

AT ten days sight pay Mr. R. S. or order, the sum of ninety-six pounds, twelve shillings and six-pence, for value received of T. L. and place it to account, as per advice from

To Mr. James Cox, jeweller, W. T.
High-street, Liverpool.

Form

Form of one payable after date.

£.300 0 0 — *Birmingham, June 3, 1800.*

TWO months after date pay Mr. A. Y. or order, three hundred pounds, value received of G. B. Esq. and place it to account, as per advice from

To Mr. O. M. at the Angel, High Green, Wolverhampton. — F. T.

Another at Sight.

£.2 18 6 — *Birmingham, April 8, 1800.*

At sight hereof pay Thomas Hurd, or order, two pounds eighteen shillings and six-pence, for value received, as advised.

To Mr. William Shepard, Leeds. — *Per John Gray.*

FOREIGN BILLS OF EXCHANGE.

For crowns 600, at usance. — *London, March 10, 1800.*

London on Paris. First bill. } AT usance pay this first bill of exchange to C. D. or order, six hundred crowns, for the value here received of Sir A. B. and place it to account, as per advice from

To Mr. X. Y. merchant, at Paris. — C. F.

For crowns 600, at usance. — *London, Feb. 4, 1800.*

Second bill. } AT usance pay this my second of exchange (my first not paid) to C. D. or order, six hundred crowns, for value received of Sir A. B. and place it to account, as per advice

To Mr. X. Y. merchant, at Paris. — C. F.

For crowns 400, at 34*d*. per crown. — *Paris, June 4, 1800.*

Paris on London. The bill. } AT double usance pay this my only bill of exchange to myself, the sum of four hundred crowns, exchange at thirty-four pence sterling per crown, the value received of Monf. E. C. and place it, as per advice, to the accompt of

To Mr. P. L. merchant, in London. — T. T.

FINIS.

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1800.

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